Reading and Lecture schedule. Lie Groups. W 2017. UCSC. Montgomery

week 1. *Reading. Kirrilov, ch. 2. HSIANG, lecture 2, p. 20. Le Donne, 1st few pages; J. Lawson.*

Lecture 1. Basic definitions and examples. The Lie algebra of a Lie group. HW assigned.

Lecture 2. $SO(3)$ and the dynamics of a rigid body. *Reading: Landau-Lifshitz, vol 1, ch VI. Arnol'd. ch. 6, sec. 28. Montgomery: By how much does a rigid body rotate.*

week 2. *Reading. Kirrilov, ch. 2. HSIANG, lecture 2.*

Lecture 3. $SU(2)$: the group of unit quaternions. As a sphere. As the universal cover of $SO(3)$. As the total space of the Hopf fibration. *Reading: HSIANG, lecture 2, p. 21-26 ;*

Lecture 4. The exponential map, ct'd. *Reading: HSIANG, ch 2; J. Lawson; Stillwell , 'Naive Lie Theory'*

week 3. *Reading: HSIANG, ch 3*

Maximal torus. Haar measure. Bi-invariant metrics, ...

Lecture 5. Statement and examples of the maximal torus theorem. Tools for proof: Haar measure. Bi-invariant metrics. Equality of the group and the Riemannian exponential for bi-invariant metrics.

Lecture 6. Transformation-theoretic proof of the Maximal torus theorem. Appearance of Weyl group. Go back and develop representation theory as needed.

week 4. Representation theory of S^1 and of $SU(2)$.

lecture 7. S^1 and its representations. $SU(2)$'s representations. The list, via homogeneous polynomials in two complex variables. *Reading: Hsiang , ch. 1*

lecture 8. $SU(2)$; ct'd angular momentum and spin in quantum mechanics. *Reading: Dirac, Principles of Quantum Mechanics; Folland "Quantum Field Theory, a Tourist's Guide*

week 5. $SU(3)$ and its reps.

Lecture 9. Representations of a torus. Roots and weights: generally. For $SU(3)$. The basic hexagon. *Reading. Fulton and Harris: Representation theory. Cahn: Semi-simple Lie algebras and their representation theory. Hsiang.*

Lecture 10. Representation theory of $SU(3)$. Weyl formula for dimensions of irreps. Background: positive roots. Weyl chamber. Half the sum of the positive roots. ... *Reading. Fulton and Harris: Representation theory. Cahn: Semi-simple Lie algebras and their representation theory. Duistermaat-Kolk: Lie groups*

! the rest of this outline is more tentative! we may, for example, take another week on $SU(3)$

week 6. Heisenberg group.

Lecture 11. Its appearance and definition. Various realizations. SubRiemannian structure. *Reading: Ch 1 of my book.*

Lecture 12. Nilpotent groups. Carnot groups, generally.

Reading: My book.

week 7. The unitary reps of the Heisenberg group.

Lecture 13. Weyl representation, Stone-Von Neumann theorem.

Reading: Folland. Mackey.

Lecture 14. $SL(2, \mathbb{R})$ arising as linear symmetries of commutation relations. Metaplectic representation.

week 8. Symplecticity and Poisson structures.

Symplectic group. Symplectic manifolds. Co-adjoint orbits. Symplectic G- manifolds.

week 9 Isotropy representation. G-spaces. Equivariant tubular nbhd theorem. Perhaps Gils YM example(s).

week 10 Classiification theory for compact simple Lie groups = classification theory for complex simple Lie algebras. The list and Dynkin diagrams.

Further topics as time permits. The space forms and how their isometry groups are diffeo to their frame bundles. Various special isomorphisms: $Sl(2, \mathbb{C})$ with the Lorentz group. More Homogeneous spaces. Symmetric spaces. Examples. The case of CP^2. Its isotropy rep. The various kinds of 2-planes. Diffeomorphism groups and other "infinite pseudogroups" . Momentum maps. Clifford algebras and the construction of $Spin(n)$. G_2.

Additional Papers ; some for presentations... . Arnol'd on Fluids.

Me: By How Much Does a Rigid Body Rotate.

Milnor: On left invariant metrics on Lie Groups.
Arnol'd, Mechanics: appendix on systems with symmetry.
My paper on singular extremals on LIe groups...