

HW 1.

Kirrilov. Exercises 2.2-2.5.

HW 2. Here you will prove that $SO(4)$ is essentially $SU(2) \times SU(2)$.

Identify \mathbb{R}^4 with the quaternions \mathbb{H} in the usual way, and $SU(2)$ with the group of unit quaternions as per lecture or Hsiang. Define a representation ρ of $SU(2) \times SU(2)$ on $\mathbb{R}^4 = \mathbb{H}$ by declaring that $\rho(g_1, g_2)(q) = g_1 q \bar{g}_2$ for $q \in \mathbb{H}$.

A) Show that $\|\rho(g_1, g_2)(q)\| = \|q\|$ where $\|q\|$ is the positive square root of $q\bar{q}$.

B) From (A) conclude that ρ takes values in $O(4) \subset GL(4, \mathbb{R})$.

C) Use connectedness to show that the image of ρ is all of $SO(4)$.

D) Compute the kernel of ρ .

E) Conclude that $SO(4) = SU(2) \times SU(2) / \pm(1, 1)$