HW 1

10/10.

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worked w/ Ariel

We know

(*) $$\frac{d^2\vec{r}}{dt^2} = \frac{-\gamma}{r^{\alpha+1}}\hat{e}_r$$

describes the motion of a mass about the origin in the plane

Thus, $\vec{r}(t) \neq 0$ as our object is going around but is never at our origin

We postulate the circular ansatz $\vec{r}(t) = Ae^{i\omega t}$ for a solution

$\Rightarrow$

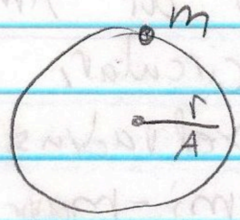
$$\dot{r} = Ai\omega e^{i\omega t}$$

$$\ddot{r} = Ai^2\omega^2 e^{i\omega t} = -A\omega^2 e^{i\omega t}$$

$$\vec{e}_r = \frac{\vec{r}}{r} = \frac{Ae^{i\omega t}}{r}$$

But since we've postulated our solution as a circle,

$A = r$



and thus

$$\vec{e}_r = \frac{Ae^{i\omega t}}{A} = e^{i\omega t}$$

So (*) $\Rightarrow$ $$-A\omega^2 e^{i\omega t} = \frac{-\gamma}{A^{\alpha+1}} e^{i\omega t}$$

$$\Rightarrow -\omega^2 = \frac{-\gamma}{A^{\alpha+2}} \Rightarrow \omega^2 = \gamma A^{-(\alpha+2)}$$

(B) Let $\alpha = 1$

Then

$$\omega^2 = \gamma A^{-(1+2)} = \gamma A^{-3}$$ ✓

We know

$$\vec{r}(t) = Ae^{i\omega t}$$

describes our position, so we have a period

$$P = 2\pi/\omega$$

Thus $P^2 = (2\pi/\omega)^2 = 4\pi^2/\omega^2 = 4\pi^2/\gamma A^3$

$$\Rightarrow P^2 = \frac{4\pi^2 A^3}{\gamma}$$ ✓ yes; good!

Since γ is a constant, we have

$$P^2 = KA^3 \quad \text{where } K = 4\pi^2/\gamma \text{ is a positive constant}$$

Thus our Period squared is directly proportional to our Amplitude cubed, and since our motion is circular, our Amplitude is the same as our fixed radius, which is the same as our semi-major axis of orbit, which is the special case of Keplers 3rd Law.

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Math 130

1c We prove the contrapositive

Assume $\exists$ a rest point for the CFP'
So, $\exists t \in \mathbb{R}$ s.t. $\dot{r}(t) = 0$
Thus, $\int \dot{r}(t)\,dt = \int 0\,dt = K = r(t)$

We consider

$$\left.\begin{aligned} \dot{r} &= v \\ \dot{v} &= -f(r)\frac{\vec{r}}{r_3} \end{aligned}\right\} \quad (\text{CFP}')$$

where $f(r) : \mathbb{R}^3/0 \to \mathbb{R}$

a) our objects goes around, never through the origin

Thus $\vec{r} \neq 0 \Rightarrow \vec{r}(t)$ is a nonzero constant.

But,

$$\dot{r}(t) = 0 = v$$

$$\Rightarrow \ddot{r}(t) = 0 = \dot{v}$$

$$\Rightarrow \dot{v} = -f(r)\frac{\vec{r}}{r} = 0$$

$$\frac{\vec{r}}{r} \neq 0 \Rightarrow -f(r) = 0$$

So $\exists$ a rest point for CFP'
$\Rightarrow f(\vec{r})$ is zero

Excellent

Thus also
$f(\vec{r})$ never zero $\Rightarrow \exists$ no rest points