HOMEWORKS. , CELESTIAL MECHANICS, W 2017, UCSC.
DUE 1ST FRIDAY OF CLASS.

Homework Assignment 1. Due 1st Friday of class, so Jan 13.

Problem 1. [Cf Geiges HW 1.7] A mass m moves about the origin in the plane with respect to a power law central force law

$$\frac{d^2\vec{r}}{dt^2} = -\frac{\gamma}{r^{\alpha+1}}\vec{e}_r$$

where $\gamma > 0$ is a positive constant, $\alpha > 0$ is a positive constant exponent, and where $\vec{e}_r = \vec{r}/r$ is the unit radial vector field. Postulate the circular ansätz [1]

$$\vec{r}(t) = Aexp(i\omega t)$$

for a solution.

A) Derive an algebraic relation between the amplitude A and frequency ω which guarantee that the ansätz satisfies the differential equation.

B) From your algebraic relation, derive the spacial case of Kepler's 3rd law for the Newtonian force law which is valid for circular motion.

Problem 2. Verify that if $f(\vec{r})$ is never zero, then there are no rest points for the central force problem (CFP') of p. 2. [2]

Homework Assignment 2. Due 2nd Wed of class. So, the 18th of Jan.

Problem 3. Assuming that $\vec{r}$ satisfies (CFP) of p 2, G, with $f(\vec{r}) = f(r)$, compute the second derivative of $\langle \vec{r}, \vec{r} \rangle = r^2$ with respect to time. Use this to prove that indeed $\ddot{r} = -\frac{d}{dr}V_{eff}(r)$ where c^2 is the squared length of the angular momentum vector and where $V_{eff}(r) = \frac{1}{2}\frac{c^2}{r^2} + V(r)$ with $dV/dr = -f(r)$ of (CFP).

Problem 4. G: 3.2

Homework Assignment 3. Due 3rd Fri of class, so Jan 27.

Problem 3. POLLARD. EXER 2.1.

Problem 4. [Cf G. HW 2.3 and G. 3.4 b, eq (3.11)] Prove that a linear equation in x, y and $r = \sqrt{x^2 + y^2}$, such as $Ar + Bx + Cy = D$ defines a conic in the plane.

Problem 5. Geiges HW 2.1.

Homework assignment 4. Due , probably , 1st Fri of Feb.

Problem 6. In class we derived the radial motion equations $r(t)$ for a particle moving in a central force with potential $V(r)$. We did this by rewriting the energy as $E = \frac{1}{2}(\dot{r}^2 + \frac{c^2}{r^2}) + V(r)$ where c^2 is the length of the angular momentum vector.

A) Solve this equation for $\dot{r}$ to get a first order differential equation for r depending parametrically on E and c^2.

[1] Recall $exp(i\theta) = \cos(\theta) + i\sin(\theta)$ and that $\mathbb{C}$ and $\mathbb{R}^2$ are identified by identifying $x + iy$ with (x, y), so that when viewed as a vector on the plane $\mathbb{R}^2$ we have that $exp(i\theta) = (\cos(\theta), \sin(\theta))$

[2] so no solutions $\vec{r}(t) = const.$

B) Take as initial conditions a value $r = r_0$ for which $V(r_0) = E$ and show your differential equations has two different solutions, one being $r(t) = r_0$.

C) Look up the fundamental existence and uniqueness theorem of differential equations and explain why B does not contradict the uniqueness part of this theorem. (You can find this theorem stated in the preface of the book as the 'Picard-Lindelöf existence and uniqueness theorem).