Uncertainty, search, and information in fisheries

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We consider the problem of modelling uncertainty in the location of schools of fish and the effect of search by fishing vessels in reducing the uncertainty. Our methods involve a preliminary period of searching and fishing, followed by a Bayesian update of information and a reallocation of vessels. The first search problem that we study is how to determine the optimal allocation of search effort over several historical fishing grounds. in which the current abundance of fish has a known prior probability distribution. In our second application, we consider the case of a single fishing ground and determine the optimal allocation of search effort over time. We assume that fishermen are profit maximizers (although this can be relaxed) and compare the value of competitive fishing strategies with cooperative ones in which search effort is optimized.

Introduction

Until recently, most bioeconomic fishery models presented in the literature were of a deterministic nature (e.g. Clark, 1976). Considerations of "uncertainty" have been limited (with some notable exceptions) to the introduction of random "noise", or fluctuations, either in the biological system (Reed, 1979; Beddington and May, 1977; Ludwig and Varah, 1979), or in the economic system (Andersen, 1982; Wilson, 1980).

Most actual fisheries, however, exhibit severe forms of uncertainty in the deeper meaning of the word, implying "ignorance" – at least partially. Estimates of stock abundance, for example, are usually highly uncertain in this sense, as are estimates of the parameters of biological processes such as recruitment, growth, and natural mortality rates. The latter type of uncertainty has been discussed in the setting of "adaptive" management policies by Walters and Ludwig (1981), and Ludwig and Walters (1982), using data from the Pacific salmon fisheries.

From the point of view of the individual fisherman, one of the most important sources of uncertainty concerns the location of fish. It is this question, which has received very little study, that we take up in the present

paper. (See Bockstael and Opaluch (1981) for an empirical study of fishermen's response to uncertainty regarding relative abundance of alternative stocks.)

Specifically, we consider here the problem of modelling uncertainty regarding the location of fish concentrations, and the effect of search by fishing vessels in reducing such uncertainty. We show how to determine the optimal allocation of search effort between several "historical fishing grounds". The current abundance of fish on each ground is assumed unknown, but the probability distribution for abundance (the so-called prior probability distribution) for each ground is known from the historical record of catch and effort. After a preliminary period of searching (and fishing), the abundance estimates are updated by a Bayesian formula, and vessels may then be reallocated according to the results.

A second searching problem that we analyse is one in which the fishing ground is far from the home port of the fleet, so that a trip to the ground is costly. For this case, we show how to find an optimal allocation of search effort over time.

These problems, which are computationally difficult (mainly because of the "curse of dimensionality" in stochastic dynamic programming, see Bertsekas. 1976) are solved here only under a number of quite restrictive

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Table 1. Characteristics of search in four selected fisheries.

Fishery	Size of cells km ²	No. of cells	No. of vessels	Quality of order information	Approximate fishing time per trip in hours
Eastern tropical Pacific purse seine British Columbia salmon purse seine Gulf of Carpentaria prawn trawl California shrimp trawl	111×111	~300	~300	Poor	1 000
	18× 18	30	~ 50¹	Good	20
	111×111	13	~200	Poor	1 000
	37× 1·8	30	< 50	Poor	10-100

¹Coordinated group.

assumptions. The possibility of relaxing these assumptions is considered in a concluding section of the paper.

A question of interest for fishery management is whether fishermen, acting competitively, will be motivated to allocate searching effort in an approximately optimal manner, or whether a cooperative (or regulated) solution would be more productive. The competitive case will be referred to as the EMFH ("every man for himself") strategy, whereas the cooperative case will be called AFOAOFA ("all for one and one for all"). In general EMFH will be a suboptimal strategy, owing to the information externality (see Peterson (1975), who argues that the use of EMFH strategies results in severe underexploration for oil deposits).

Some examples of exploration and search in fisheries

In this section, we give four examples of fisheries in which significant effort is devoted to search. Unlike other natural resources where search is important (such as oil exploration), the fisherman's problem is complicated by the movement of fish. Thus, even if the fish are known to be present, they still must be found.

As examples, the fisheries of interest here are: i), the eastern tropical Pacific (ETP) tuna-purse-seine fishery; ii), the British Columbia salmon-purse-seine fishery; iii); the Australian Gulf of Carpentaria prawn fishery; and iv), the California-Oregon-Washington shrimp fishery. These fisheries are all linked by the problem of locating fish, but it will be seen that the length, time, and number scales of the variables characterizing the problem are very different. Some of the parameters of the fisheries are summarized in Table 1.

Eastern tropical Pacific tuna-purse-seine fishery

The ETP fishery operates off the coast of Mexico, Central, and South America, taking yellowfin tuna (*Thunnus albacares*) and skipjack (*Katsuwonus pelamis*), primarily using purse-seine vessels. In a large region near the coast, fishing was, until 1978, regulated by the

Inter-American Tropical Tuna Commission (IATTC). Outside the regulated area, fishing was not controlled. For purposes of data analysis and presentation, the commission considers effort and catch data in 1° cells; at the latitudes of interest this cell is about 60 nautical miles (111.2 km) square. More than 350 vessels have operated in the regulated region at a time (Calkins, 1975). Tuna vessels generally remain at sea for long periods, moving from cell to cell. Tuna are migratory species, and there is usually considerable uncertainty regarding the location of the fish. The apparent year-toyear fluctuations in population size also appear to be large. Search for tuna has become a very complex process, sometimes involving helicopters and other forms of airborne sensing (Woods, Stevenson, and Gandy, 1975). The location of tuna schools is related to the position of thermoclines and other hydrographic factors. The distribution of information over a season has been recognized as important (Delpierre and Thibaudeau, 1975). The explicit inclusion of search as part of the fishing process has also been recognized (Pella and Psaropulos, 1975). Since the cells are large, the reallocation of boats is a time-consuming, and therefore costly process.

A number of distinct searching modes are used in this fishery. For example, tuna are often found associated with logs or other debris, and with dolphins. When seeking associated tuna, fishermen tend to search at the horizon, but when looking for unassociated tuna the search is concentrated near the vessel (Greenblatt, 1976). The search can also be described as active or passive. For example, a vessel steaming from home to a ground where fish are known to be present may passively search on the way out by looking for signs indicating the presence of tuna. If signs are found, then an active search for tuna may begin.

The IATTC has begun a study of search effort and catch rates for this fishery (R. Allen, personal communication). One discovery arising from statistical analysis concerns the tail of the distribution of times to detection. Since it provides a way of introducing some of our ideas, we will briefly digress to discuss it here. One might assume that the times between detection of schools are exponentially distributed, i.e. that Pr (detection given t hours of search) = $1 - \exp(-\lambda t)$ and then

try to estimate the parameter λ . When this is done, it is found on comparison with data that the theoretical form $1 - \exp(-\lambda t)$ falls off much more rapidly than the data for the large t. One possible explanation of this phenomenon is that λ itself has a distribution associated with it; e.g., suppose that

$$\Pr\left[\lambda \in (x, x+dx)\right] = dx \times \alpha^{\nu} e^{-\alpha x} x^{\nu-1} / \Gamma(\nu),$$

i.e. a gamma density. The expression $1 - \exp(-\lambda t)$ is viewed as a conditional probability of success and the unconditional one is found by averaging over the gamma density. If this is done, we find that Pr (detection given t hours of search) = $1 - \alpha^{\nu}/(\alpha + t)^{\nu}$, which has a tail that decays more slowly than $\exp(-\lambda t)$. Thus, such an integration may yield a result that fits the data better. One interpretation of such an integration is that the data from the fishery involve mixing a large number of expressions of the form $1 - \exp(-\lambda t)$ with frequency of occurrence

$$a^{\nu}e^{-a\lambda}\lambda^{\nu-1}/\Gamma(\nu)$$
.

British Columbia salmon purse seiners

The salmon fishing areas on the coast of British Columbia have been divided into 30 regions and eight aggregated regions (Hilborn and Ledbetter, 1979). The cells used in this fishery are relatively small, about 10 nautical miles (18 km) square. Five species of Pacific salmon are taken: sockeye (Oncorhynchus nerka), chinook (O. tshawytscha), coho (O. kisutch), pink (O. gorbuscha), and chum (O. keta). We shall describe the activities of purse seiners, although other vessel types are also used in the salmon fishery.

Although thousands of vessels may participate, one expects no more than a maximum of 50 vessels to participate in information sharing: the larger fishing companies may own about 50 vessels. In general, the fishing week consists of an opening of two days, and closure of the fishery for the remaining five days. Unlike the tuna, salmon runs are fairly predictable, hence the prior information on the location of fish is better. In general, the following characteristics apply. On any opening day, there may be about three cells in which salmon runs are likely, based on historical data. In addition, aerial surveys during the closed days may provide up-to-date information. Although salmon may be present in all three cells, the main problem is to pick the cell in which the run is highest. Hence, before the fishery opens, attempts are made to allocate the vessels so as to maximize the expected harvest and to insure that a big run is not missed. After the first day of the opening, the vessels may be reallocated across the cells, based on the information gained during the first day.

The actual mode of fishing in the different cells may differ. In some cases, the seiners move around looking

for fish. In others, they wait in queues at the mouth of the bay or strait. Each seiner gets to lay one set and then returns to the end of the line. In such cases the length of the queue may be important in allocating vessels to cells.

Gulf of Carpentaria prawn fishery

This fishery, which occurs in the tropical waters of northern Australia, was recently studied by Clark and Kirkwood (1979), who developed a bioeconomic model for the fishery, but did not include search explicitly. Two types of vessels (brine and freezer demersal trawlers) and at least three kinds of prawns (banana, Penaeus merguiensis; tiger, P. esculentus; and endeavour, Metapenaeus endeavouri) are involved in the fishery. We shall, however, concentrate here on a different aspect of the fishery. For the brine vessels, a large cost is associated with travelling from home port to the fishing grounds, a distance of over 1800 km. For example, for brine trawlers Clark and Kirkwood (1979) assumed a cost of steaming to and from the fishing ground at about A \$ 2000, so that at A \$ 1/kg at least 2000 kg must be caught to make the trip profitable. (Other considerations, including lost income opportunities, increase this figure. Clark and Kirkwood (1979) suggest that brine trawlers require at least A \$ 8000 expected net revenue per trip to induce owners to travel to the Gulf.)

The banana prawns, in particular, form dense schools (called boils), but the stock fluctuates considerably and unpredictably from year to year: the current year's abundance only becomes clear as the season develops and catches are recorded.

If information sharing does not occur, then the only way that the individual fishermen can obtain information about catch rates is by steaming out to the grounds themselves and spending a certain amount of time searching for prawns. If information sharing occurs, then only k of N vessels might go out for some initial time, with costs, gains, and information shared. After preliminary explorations, more vessels could go out to sea or, in bad years, some of the k vessels could return.

We note that the fishing ground is large, e.g. it consists of about 13 cells of the order of $1^{\circ} \times 1^{\circ}$ (60 nautical miles or 111 · 2 km square). Reallocation of vessels between cells in various years is a possibility, but it is also costly.

California ocean-shrimp fishery

The fishery for ocean shrimp (Pandalus jordani) can be viewed as operating in three large, distinct areas along the coast of California (Eales and Wilen, 1982) associated with the ports of Crescent City, Eureka, and Fort Bragg, California. Each area is approximately 20

nm (about 37 km) long and 1 nm (about 1·8 km) wide. The narrowness of the fishing areas appears to be the result of the concentration of shrimps along a certain depth contour (Eales and Wilen, 1982). Each of the large aggregate areas contains a number of smaller fishing regions.

Otter trawls, which became legal in 1963, are now used exclusively in the shrimp fishery (Abramson and Tomlinson, 1972). In general, between 17 and 50 vessels participate in the fishery in a given year. The vessels remain at sea for only a few days before returning to port. Catch rates of about 310 kg/h (standard deviation about 75 kg/h) were experienced between 1958 and 1969 (Abramson and Tomlinson, 1972).

The sharing of information between fishermen is presently being investigated. It is thought that too many fishermen in a given area "spoil" the ground, inasmuch as shrimp concentrations are scattered by fishing, and catch rates are thereby reduced with each drop of the net. Hence there is an incentive to limit the number of fishermen sharing information on the location of shrimp concentrations.

A model of two independently fluctuating fish stocks

The models of search that we consider in this paper are stylized simplifications of the search process in a real fishery. The underlying problem is, how can fishermen make rational decisions on where to search for fish? Also, what advantages, if any, are there to cooperative searching?

The fisherman must make his decisions on the basis of the information available to him at any given time. On the other hand, fishing itself (including searching) provides additional information regarding the local abundance of fish. In our models we shall adopt the standard Bayesian approach to the question of continual "updating" of past information as new information becomes available.

Let λ_i be a parameter proportional to the average density of detectable fish "clumps" (schools or other aggregations) in a given fishing area A_i , during a given fishing season. We shall assume, in our first model, that λ_i remains constant throughout a given fishing season (this assumption is relaxed later), but that λ_i varies in a random manner from one season to the next. Thus λ_i is a random variable; we denote its probability distribution by $f(\lambda_i)$.

Searching for fish in area A_i is itself a random process, which we model by means of the Poisson process with parameter λ_i (see Koopman, 1980; Shotton, 1973; Swierzbinski, 1981). In area A_i we have (for small dt)

Pr(one encounter, by one vessel,

in time interval
$$t, t+dt$$
) = $\lambda_i dt$ (1)

where Pr(...) represents the probability of the event (...). By an "encounter" we simply mean that the vessel locates one "clump" of fish. Thus the *expected* rate of encounter in area A_i , per vessel, equals λ_i "clumps" per unit time. To keep the calculations reasonably simple, we shall assume that all "clumps" are the same size.

Suppose now that k_i vessels search independently in area A_i , for a unit time period $\Delta = 1$. If n_i denotes the number of clumps encountered during the period, we have

$$\Pr(n_i) = \frac{(k_i \lambda_i)^{n_i}}{n_i!} e^{-k_i \lambda_i}.$$
 (2)

We now look at the "inverse" question: given that n_i encounters with fish clumps have occurred on A_i in the given time period, what additional information does this provide about the current season's value of λ_i ? Let $f_i(\lambda_i|n_i)$ denote the probability distribution for λ_i , given that n_i clumps have been encountered. We then have, by Bayes's theorem

$$f_i(\lambda_i|n_i) = \frac{\Pr(n_i|\lambda_i)f_i(\lambda_i)}{\int_0^\infty \Pr(n_i|\lambda)f_i(\lambda) d\lambda}$$
(3)

where $Pr(n_i|\lambda_i)$ is given by Equation (2). In the language of statistical decision theory, the original probability distribution $f_i(\lambda_i)$ is referred to as the "prior" distribution, and $f_i(\lambda_i|n_i)$ is the "updated" or "posterior" distribution. We shall henceforth adopt this terminology.

A particularly simple expression occurs if we adopt the gamma distribution as our prior probability distribution:

$$\gamma(\lambda;\alpha,\nu) = \frac{\alpha^{\nu}}{\Gamma(\nu)} \lambda^{\nu-1} e^{-\alpha\lambda} (\lambda \ge 0). \tag{4}$$

The gamma distribution involves two positive parameters α , ν ; the mean and standard deviation are given by

mean
$$\bar{\lambda} = \nu/\alpha$$
 standard deviation $\sigma = \sqrt{\nu/\alpha}$. (5)

A straightforward calculation establishes that if the prior distribution $f_i(\lambda)$ is given by the gamma distribution $\gamma(\gamma;\alpha_i,\nu_i)$, then the posterior distribution following n_i Poisson encounters by k_i vessels during time $\Delta = 1$ unit, as given by Equation (3), becomes

$$\Pr[\lambda \in (\lambda^0, \lambda^0 + d\lambda^0) | n_i \text{ encounters}]$$

$$= \gamma(\lambda^0; n_i + \nu_i, \alpha_i + k_i) d\lambda_0. \tag{6}$$

In other words, the posterior distribution is once again a gamma distribution, with updated parameters

$$v_i' = v_i + n_i$$
 and $\alpha_i' = \alpha_i + k_i$.

We can now address the problem of optimal allocation of search effort. Imagine that the N vessels of a certain fishing fleet exploit stocks of fish on two fishing areas A_1 , A_2 . The abundance parameters λ_1 , λ_2 have prior distributions as described above; these distributions are assumed to be independent (but this assumption could be relaxed – see the concluding section).

For the first period k_i vessels search in area A_i , where $k_1 + k_2 = N$, resulting in updated distributions $f_i(\lambda_i|n_i)$ which depend on the number of encounters n_i . At the end of this first period, vessels may be reallocated among the two fishing grounds, depending on the updated probabilities. The process is repeated during period 2, and a further reallocation undertaken if indicated. Suppose there are M such periods constituting the total annual fishing season.

We shall assume that the objective to be maximized (the so-called objective function) is simply the expected total net seasonal revenue, equal to the expected total value of the catch, minus the cost incurred in moving vessels from one ground to the other. Our optimization problem is to determine the allocation of vessels $k_{1}^{(i)}$, $k_{2}^{(i)}$ for i = 1, 2, ..., M, so as to maximize this expected value. This can be formulated as a problem in stochastic dynamic programming.

Consider the situation where the updated parameters v_i , α_i have just been obtained, and the reallocation decision is about to be made. Let there be n periods of fishing remaining in the current season, and Q vessels currently allocated to ground A_1 .

To start with, take n = 1. Let $J_1(Q, \nu_1, \alpha_1, \nu_2, \alpha_2)$ denote the maximum expected net return with one period remaining:

$$J_{1}(Q, \nu_{1}, \alpha_{1}, \nu_{2}, \alpha_{2}) = \max_{k_{1}+k_{2}=N} E[p(k_{1}n_{1} + k_{2}n_{2}) - c|Q - k_{1}|]$$
 (7)

where p is the value of one clump of fish (all clumps have the same value!) and c is the cost of switching one vesses from one ground to the other.

Since $E(n_i) = \bar{\lambda}_i = \nu_i/\alpha_i$, Equation (7) reduces to

$$J_{1}(Q, v_{1}, \alpha_{1}, v_{2}, \alpha_{2})$$

$$= \max_{0 \leq k_{1} \leq N} \{ p[k_{1}\bar{\lambda}_{1} + (N-k_{1})\bar{\lambda}_{2}] - c|Q-k_{1}| \}$$

$$= p[k_{1}^{*}\bar{\lambda}_{1} + (N-k_{1}^{*})\bar{\lambda}_{2}] - c|Q-k_{1}^{*}|$$
(8)

where k_1^* , the optimal number of vessels on A_1 , is given by

$$k_{1}^{*} = \begin{cases} N & \text{if } \bar{\lambda}_{2} < \bar{\lambda}_{1} - c/p \\ Q & \text{if } \bar{\lambda}_{1} - c/p < \bar{\lambda}_{2} < \bar{\lambda}_{1} + c/p \\ 0 & \text{if } \bar{\lambda}_{1} + c/p < \bar{\lambda}_{2}. \end{cases}$$
(9)

This simply says: if there are no switching costs and only one fishing period remaining, all vessels should be allocated to the more likely ground: otherwise vessels are reallocated if the expected increase in catch value exceeds the switching cost.

When more than one period remains (n > 1), the above rule may not be valid, because searching yields improved information about the location of fish. In general let $J_n(Q, \nu_1, \alpha_1, \nu_2, \alpha_2)$ denote the maximum expected value of net returns when n fishing periods remain, and where ν_1 , α_1 represent the most recent updates. We then have the easily derived Bellman equation:

$$J_{n+1}(Q,\nu,\alpha) = \max_{\substack{k_1+k_2=N\\}} \{p_1(k_1\bar{\lambda}_1+k_2\bar{\lambda}_2) - c|Q-k_1| + E[J_n(k_1,\nu_1+n_1,\alpha_1+k_1,\nu_2+n_2,\alpha_2+k_2)]\}$$
(10)

the expected value being taken with respect to n_1 and n_2 . (If gamma priors are not assumed, the notation must be changed to represent the updated distributions.)

The expected-value expression in Equation (10) can be written as:

$$E[\dots] = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} J_n(k_1, \nu_1 + n_1, \alpha_1 + k_1, \nu_2 + n_2, \alpha_2 + k_2) \Pr(n_1) \Pr(n_2)$$
(11)

where

$$Pr(n_i) = \int_0^\infty Pr(n_i|\lambda)\gamma(\lambda;\nu_i, \alpha_i) d\lambda$$

$$= \frac{k_i^{n_i}}{n_i!} \frac{\alpha_i^{\nu_i}}{(\alpha_i + k_i)^{(n_i^{-1} + \nu^{-1})}} \frac{\Gamma(n_i + \nu_i)}{\Gamma(\nu_i)} (k_i \neq 0).$$
(12)

$$\Pr(n_i) = \begin{cases} 1 & n_i = 0 \\ 0 & n_i > 0 \end{cases} \quad (k_i = 0). \tag{13}$$

In the decision-theory literature Equation (11) would be called the "pre-posterior" expectation.

The computational complexity of our allocation problem is now apparent. First, the double series Equation (11) generally contains a large number of significant terms (i.e. converges slowly), because $Pr(n_i)$ is significant for many values of n_i , and also because the factors $k_i/(\alpha_i + k_i)$ are usually close to 1 (generally α_i is of the order of 1). But more important, at each iteration the previous value function J_n must be evaluated for every pair (n_1, n_2) used in the (truncated) summation. These values must be obtained by interpolation from the (large) array of values of J_n stored at the *n*th stage. We thus encounter the usual "curse of dimensionality" of dynamic programming compounded by the need at every step to sum a slowly converging double series. It is to be doubted whether this approach can be developed into a practical fishing management tool!

Let us, therefore, adopt a further simplification. Assume that updating of probabilities occurs only after the *first* fishing period; vessels are then reallocated to their

final destinations for the remainder of the season. (This would introduce the problem of optimizing the length of the initial search period, but we will not study this problem here.) Unless the cost of switching vessels is low, this assumption does not seem to be unreasonable. We call this strategy "myopic Bayes", in analogy with the work of Walters and Ludwig (1981).

Under this new assumption, the expression J_1 for the second-period return becomes

$$\tilde{J}_{1}(Q,\nu,\alpha) = \max_{0 \le k_{1} \le N} \{(M-1) \times p[k_{1}\bar{\lambda}_{1} + (N-k_{1})\bar{\lambda}_{2}] - c|Q-k_{1}|\}$$
(14)

(for which the optimal k_1 is easily determined, as in Equation (9)). Here M denotes the total number of periods in the season. The total expected value is therefore

$$\tilde{J}(\nu,\alpha) = \max_{0 \le k_1 \le N} \left\{ p \left[k_1 \tilde{\lambda}_1 + (N - k_1) \tilde{\lambda}_2 \right] + E \left[\tilde{J}_1(k_1, \nu_1 + n_1, \alpha_1 + k_1, \nu_2 + n_2, \alpha_2 + N - k_1) \right] \right\}.$$
(15)

Here we assume that the cost of initial allocation of the vessels is independent of their destination, and can be ignored. This assumption could easily be altered if one ground is more remote than the other.

In order to give an indication of the significance of searching and updating of information, we performed some sample calculations using artificial data. The results are illustrated in Figures 1, 2, and 3.

In Figure 1 the two grounds are taken to be equivalent, with $\nu_1 = \nu_2 = 0 \cdot 1$ and $\alpha_1 = \alpha_2 = 1 \cdot 0$. Thus $\bar{\lambda}_1 = \bar{\lambda}_2 = 10$. There are N = 10 vessels; switching costs were ignored: c = 0. The expected catch in period 1 is 100 clumps of fish, independent of vessel allocation. The figure shows the expected catch in period 2 following appropriate reallocation of vessels, as a function of the number k_1 of vessels allocated to ground A_1 in period 1.

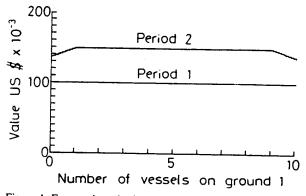


Figure 1. Expected catch plotted against number of vessels on ground 1 for parameter values: $\alpha_1 = 0 \cdot 1$; $\alpha_2 = 0 \cdot 1$; $\nu_1 = 1 \cdot 0$; $\nu_2 = 1 \cdot 0$; c = 0; and p = \$1000.

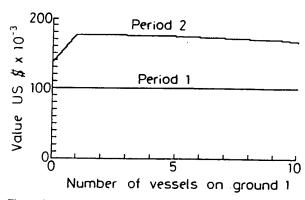


Figure 2. Expected catch plotted against number of vessels on ground 1 for parameter values: $\alpha_1 = 0.01$; $\alpha_2 = 0.1$; $\nu_1 = 0.1$; $\nu_2 = 1.0$; $\alpha_3 = 0.0$; and $\alpha_4 = 0.0$

As expected, the maximum is achieved for $k_1 = N/2 = 5$, and this results in a 50 % increase in expected catch. But almost the same degree of improvement (49 % or better) arises for $1 \le k_1 \le 9$. Only if no searching takes place on one of the grounds ($k_1 = 0$ or 10) does performance in period 2 fall off, and even then there is a 37 % improvement due to updating.

The difference between the period 2 and period 1 values can be interpreted as the value of information obtained from the first-period observations. In Figure 1, where the coefficient of variation σ_i/λ_i is 1·0 on each ground, the value of information amounts to 50 % of the expected catch for each subsequent period. (A non-myopic strategy would employ the additional information obtained during periods 2.3, ...)

In Figure 2, the parameters are altered so that, while $\bar{\lambda}_1 = \bar{\lambda}_2$, the stock abundance on A_1 fluctuates more than that on A_2 . (The values are $\alpha_1 = 0.01$ and $\nu_1 = 0.1$, $\alpha_2 = 0.1$ and $\nu_2 = 1.0$. Thus $\bar{\lambda}_1 = \bar{\lambda}_2 = 10$ as before, but the coefficient of variation on A_1 is now 3.16.) In this case the optimal allocation k_1^* is one vessel on the "uncertain ground" A_1 and nine on A_2 ; this leads to a 76% increase in expected catch in later periods. Again, however, the degree of improvement is not very sensitive to k_1 , provided some searching occurs on the uncertain ground.

Finally, in Figure 3 we supposed that expected catches on A_1 are twice those on A_2 . The optimum allocation in period 1 is k_1^* = nine vessels on A_1 , but the actual choice of k_1 is not very critical.

Three conclusions seem to follow from these sample calculations. First, information obtained from "sampling" of fish stocks can lead to significant increases in expected seasonal catches, particularly where stock fluctuations are large. Second, the amount of information obtained depends on the initial allocation of vessels. But third, the exact allocation may not be critical, provided that *some* searching takes place on each ground.

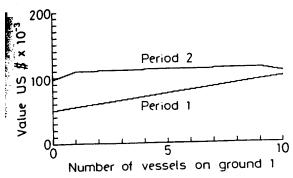


Figure 3. Expected catch plotted against number of vessels on ground 1 for parameter values: $\alpha_1 = 0 \cdot 1$; $\alpha_2 = 0 \cdot 2$; $\nu_1 = 1 \cdot 0$; $\nu_2 = 1 \cdot 0$; c = 0; and p = \$1000.

One distant fishing ground

We now consider a second search problem in which N vessels of a fishery operate on one historical ground that is far from port. We assume that all vessels start at the port, that the cost of travel to the ground is high, and that the myopic Bayes strategy is used.

For this problem to be meaningful, depletion must be included in the model. Hence, we now replace Equation (1) by

Pr (one encounter, by one vessel, in time t, t+dt, given that n schools were already encountered)

$$= (\lambda - n\delta) dt. \tag{18}$$

In Equation (18), we interpret δ as the amount that the encounter rate λ is reduced by one set; thus λ/δ represents the total number of schools initially present.

Instead of Equation (2), we find that (Feller, 1968) Pr [k vessels encounter n schools in (0,t)]

$$= P_n(k,t|\lambda) = \begin{cases} \left[\frac{\lambda/\delta}{n}\right] (1 - e^{-k\delta t})^n e^{-kt(\lambda - n\delta)} & 0 \le n \le \lambda/\delta \\ 0 & n > \lambda/\delta. \end{cases}$$
(19)

For a given λ , the expected harvest H_c is simply

$$H_c = \frac{\lambda}{\delta} (1 - e^{-k\delta t}). \tag{20}$$

As $\delta \to 0$, the results in Equations (18) and (20) approach the limiting form of a Poisson process. If the prior density on λ is a gamma density as before, then we have that the unconditional harvest, H is simply

$$H = \frac{\nu}{a\dot{\delta}} (1 - e^{-k\delta t}). \tag{21}$$

In order to do our pre-posterior analysis, we rewrite Equation (19) as

$$P_n(k,t|\lambda) = \begin{cases} \frac{1}{n!} \left(\frac{\lambda}{\delta}\right)_n (1 - e^{-k\delta t})^n e^{-kt(\lambda - n\delta)} & 0 \le n \le \lambda/\delta \\ 0 & n > \lambda/\delta \end{cases}$$
(22)

where the notation $(\lambda/\delta)_n$ is due to Feller (1968):

$$\left(\frac{\lambda}{\delta}\right)_n = \left(\frac{\lambda}{\delta} - n + 1\right)\left(\frac{\lambda}{\delta} - n + 2\right)\dots\left(\frac{\lambda}{\delta}\right) \quad \lambda/\delta \ge n. \tag{23}$$

We note the recurrence

$$(\lambda/\delta)_{n+1} = (\frac{\lambda}{\delta} - n)(\lambda/\delta)_n \tag{24}$$

and se

$$(\lambda/\delta)_n = \sum_{j=1}^n A(j,n)(\lambda/\delta)^j.$$
 (25)

For the coefficients A(j,n), the following properties are easily derived:

$$A(n,n) = 1$$

$$A(k,n+1) = A(k-1,n) - nA(k,n)$$

$$k = 2,3, ..., n-1,n$$

$$A(1,n+1) = -nA(1,n).$$
(26)

Using Equation (25) in Equation (22) gives us

$$P_{n}(k,t|\lambda) = \begin{cases} \frac{1}{n!} \left(1 - e^{-k\delta t}\right)^{n} e^{-kt(\lambda - n\delta)} \times \\ \times \sum_{j=1}^{n} A(j,n) \left(\frac{\lambda}{\delta}\right)^{j} & 0 \le n \le \lambda/\delta \\ 0 & n > \lambda/\delta. \end{cases}$$
(27)

If the prior density on λ is a gamma density with parameters ν and α , then the unconditional probability of finding n schools is

$$P_{n}(k,t) = \frac{1}{n!} (1 - e^{-k\delta t})^{n} e^{ktn\delta} \times$$

$$\times \int_{n\delta}^{\infty} \frac{e^{-k\lambda t} e^{-\alpha \lambda} \lambda^{\nu-1} \alpha^{\nu}}{\Gamma(\nu)} \sum_{j=1}^{n} A(j,n) (\frac{\lambda}{\delta})^{j} \quad n \ge 1$$
 (28)

$$P_0(k,t) = \alpha^{\nu}/(\alpha+kt)^{\nu}.$$

Hence we find that

$$P_{n}(k,t) = \frac{1}{n!} \left(1 - e^{-k\delta t}\right)^{n} e^{kin\delta} \frac{\alpha^{\nu}}{\Gamma(\nu)} \times \sum_{j=1}^{n} \frac{A(j,n)}{\delta^{j}} \frac{\Gamma[j + \nu, (\alpha + kt)n\delta]}{(\alpha + kt)^{j+\nu}} \qquad n \ge 1$$
 (29)

$$P_0(k,t) = \alpha^{\nu}/(\alpha+kt)^{\nu}.$$

In what follows, we will only write the more complicated expression in Equation (29), understanding what it means for n = 0. In Equation (29) $\Gamma(\mu, x)$ is the incomplete gamma function

$$\Gamma(\mu, x) = \int_{x}^{\infty} e^{-s} s^{\mu - 1} ds.$$
 (30)

Many of its properties are described in Abramowitz and Stegun (1965) or Gradshteyn and Rhyzik (1980). For our purposes, the most important properties are these:

$$\Gamma(\mu,x) = \Gamma(\mu) - x^{\mu} \Gamma(\mu) e^{-x} \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(\mu+n+1)}$$

$$\Gamma(\mu+1,x) = \mu \Gamma(\mu,x) + x^{\mu} e^{-x}.$$
(31)

The posterior density is obtained by Bayes's formula. At the end of the interval (0,t), we compute

Pr [λ was between $(\lambda_0, \lambda_0 + d\lambda_0)|k$ vessels found n schools]

$$= \begin{cases} 0 & \lambda < n\delta \\ e^{-(\alpha+kt)\lambda_0} \lambda_0^{\nu-1} \sum_{j=1}^n A(j,n) \left(\frac{\lambda_0}{\delta}\right)^j d\lambda_0 \\ \frac{\sum_{j=1}^n \frac{A(j,n)}{\delta^j} \frac{\Gamma[j+\nu,(\alpha+kt)n\delta]}{(\alpha+kt)^{j+\nu}} & \text{if } \lambda_0 \ge n\delta \end{cases}$$

and we note that

Pr [current value of λ is between $(\lambda_0, \lambda_0 + d\lambda_0)|k$ vessels found n schools]

= Pr [prior value of λ was between $(\lambda_0 + n\delta$. $\lambda_0 + n\delta + d\lambda_0)|k$ vessels found n schools]

$$= \frac{e^{-(\alpha+kt)(\lambda_0+n\delta)}}{\sum_{j=1}^n \frac{A(j,n)}{\delta^j} \frac{\Gamma[j+\nu,(\alpha+kt)n\delta]}{(\alpha+kt)^{j+\nu}}} \times \left(\lambda_0+n\delta\right)^{\nu-1} \sum_{j=1}^n A(j,n) \left(\frac{\lambda_0+n\delta}{\delta}\right)^j d\lambda_0.$$
(33)

The updating formula now follows from this:

$$\bar{\lambda}' = E(\lambda'|k, n)$$

=
$$\int_0^\infty \lambda_0 \Pr[\lambda \text{ between } \lambda_0 \text{ and } \lambda_0 + d\lambda_0 | k, n] d\lambda_0$$

where the probability expression is given by Equation (32).

We now consider the myopic Bayes strategy and assume that at the end of the interval (0,t), we decide on the optimal number of vessels for the rest of the season. of maximum length T-t; here T is the total length of the fishing season. If l vessels are sent out (with l < 0 implying that vessels are called back to port), the conditional harvest for the remainder of the season is

$$H_c = \frac{\bar{\lambda}'}{\delta} \left[1 - e^{-(k+l)\delta(T-t)} \right]. \tag{34}$$

We find that:

expected harvest [given that k vessels found n schools in (0,t) and k+l vessels out in (t,T)] $\equiv H(n,k,l,T-t;\nu,\alpha)$

$$= \frac{\frac{1}{\delta} [1 - e^{-(k+l)\delta(T-t)}]}{\sum_{j=1}^{n} \frac{A(j,n)}{\delta^{j}} \frac{\Gamma[j+\nu,(\alpha+kt)n\delta]}{(\alpha+kt)^{j+\nu}}} \times \sum_{j=1}^{n} \frac{A(j,n)}{\delta^{j}} \frac{\Gamma[j+\nu+1,(\alpha+kt)n\delta]}{(\alpha+kt)^{j+\nu+1}}.$$
 (35)

For simplicity we write this as

$$H(n,k,l,T-t;\nu,\alpha) = \frac{1}{\delta} \left[1 - e^{-(k+l)\delta(T-t)}\right]R$$
(36)

where R is the quotient of the two series.

We can now formulate the economic optimization problem. Assume that p is the price per school, c_1 is the fixed cost of sending a vessel to the ground, and c_2 is the cost per unit time of operating a vessel on the ground. Suppose that k vessels found n schools in the first period and that l vessels are sent out in the second period. Fishing in the second period will stop when the marginal rate of return is zero. This occurs at the time t_7 such that

$$\frac{d}{dt_{s}} \left[pH(n, k, l, t_{s}; \nu, \alpha) - c_{2}(k+l)t_{s} \right] = 0.$$
 (37)

Thus

$$t_s = \frac{1}{(k+l)\delta} \log \left(\frac{pR}{c_2} \right). \tag{38}$$

(If t_s , given by Equation (38) is greater than the remaining length of the season (T-t) then t_s is set equal to T-t.)

Our two-period dynamic programming problem is herefore

$$J = \max_{0 \le k \le N} \left\{ \frac{p\nu}{\alpha\delta} \left(1 - e^{-k\delta t} \right) - (c_1 + c_2 t)k + \sum_{n=0}^{\infty} P_n(k,t) \max_{-k \le l \le N-k} \left[pH(n, k, l, t_s; \nu, \alpha) - \tilde{H}(l)c_1 - c_2(k+l)t_s \right] \right\}.$$
(39)

Here $\hat{H}(l) = l$ if $l \ge 0$, 0 if l < 0.

The solution of Equation (39) is the AFOAOFA strategy. In the EMFH strategy, all N vessels go out in the first period. Total catch is assumed to be known to each fisherman, who then updates λ as above. The vessels leave the fishery at time t_s given by Equation (38) with k + l = N.

Results of some sample calculations based on the above formulas are given in Tables 2 and 3. The base-level parameter values used were: $\bar{\lambda}=6\cdot67; \nu=8\cdot0;$ $\delta=0\cdot5; T=1\cdot0; t=0\cdot15; p=\$2000; c_1=\$4000;$ $c_2=\$2000;$ and N=6.

Table 2. Single-ground model: the effect of uncertainty on k^* = optimal number of vessels in period 1, and on the US \$ values of the AFOAOFA and EMFH strategies. (CV = coefficient of variation = $1/\sqrt{\nu}$.)

ν	CV	k*	Value in US \$		
,			AFOAOFA	EMFH	
8	0.35	3	7 234	2 737	
5	0.45	4	8 324	5 974	
3	0.58	5	11 119	10 765	

Table 2, obtained by varying the value of ν from its base level, shows the influence of uncertainty on the values (net returns) for the AFAOAFA and EMFH strategies, as well as the optimal amount of searching (k^*) in period 1.

Why does uncertainty increase the value of the fishery, given that the average recruitment $\bar{\lambda}$ remains constant? The answer again lies in the process of updating estimates of abundance λ , and adjusting subsequent fishing policy accordingly. This procedure is most profitable when the year-to-year fluctuations in abundance are high. Both strategies profit from updating, and in fact the competitive (EMFH) strategy does so relatively more than the AFOAOFA strategy, because searching by vessels is appropriate at high levels of uncertainty. In fact, when $k^* = N$, the two strategies are equally profitable (for the parameter values used here).

Table 3 shows the effect of the total fleet size N (here the value $\delta=0.25$ was used). The EMFH strategy performs progressively worse as N increases. In fact, for N>13 the EMFH strategy yields a negative return. This reflects the well-known economic phenomenon of rent dissipation in the competitive fishery.

Extensions of the models

The models used in the previous sections were the simplest ones possible, chosen to illustrate the key points of search theory in fisheries operations. In this section, we consider some extensions, that will add realism (and complexity) to the models. Some of these factors are: 1, non-uniform school size; 2, correlations on λ between the two fishing grounds; 3, imperfect information; and 4, set times, false detections, and other operational factors.

Table 3. Single-ground model: the effect of fleet size N on the US γ value of the AFOAOFA and EMFH strategies.

N	k*	Value in US \$		
		AFOAOFA	EMFH	
2	2	10 252	10 252	
4	4	14 022	14 022	
6	5	14 076	13 736	
8	5	14 076	11 146	
10	5	14 076	7 451	

The simplest assumption on school size is that schools come in two sizes: big and small. Since the actual school size is not observed by the fishermen (only the catch is observed) this assumption may not be as unreasonable as it first sounds. Thus, let us suppose that the probability that a randomly chosen school is large, is q and that the prior density on q is denoted by $f_0(q)$. If schools are encountered randomly, i.e. school size does not affect detectability (again a reasonable assumption for some fisheries (R. Allen, personal communication)), then the probability that m of n schools encountered will be large is $\binom{n}{m} q^m (1-q)^{n-m}$. Hence, as our conjugate prior we choose the beta distribution

$$f_0(q) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} q^{a-1} (1-q)^{b-1}.$$
 (40)

A simple calculation shows that if n schools are encountered and m of them are large, the posterior density on q is

$$f_1(q) = \frac{\Gamma(n+a+b)}{\Gamma(m+a)\Gamma(n-m+b)} q^{m+a-1} (1-q)^{n-m+b-1}$$
(41)

i.e. another beta distribution. With these choices, it is easy to incorporate school size into our stochastic dynamic programming equations.

Another approach, following the work of Cozzolino (1972), would be to assume that detectability is proportional to school size and to introduce a utility structure on school size. We hope to report on the result of such calculations in a later paper.

In the analysis of the two-ground problem, we assumed that the densities on the two grounds fluctuated independently. A more realistic assumption would allow for correlations between the densities on the two grounds. These correlations could be either positive (i.e., λ_1 high implies that λ_2 is high and vice versa) or negative (i.e., λ_1 high implies that λ_2 is low and vice versa). One way of treating this problem would be to assume that from historical records it is known that $\lambda_1/\lambda_2 = \bar{c}$ (positive correlation case) where \bar{c} is a random variable. For example, we might assume that $\log \bar{c}$ is $N(0,\sigma_c^2)$. Another approach would be to work with a given joint density $f(\lambda_1,\lambda_2)$ rather than assuming independence (in which case $f(\lambda_1,\lambda_2) = f_1(\lambda_1)f_2(\lambda_2)$).

In our analysis, we assumed that the information was perfect, but in a highly stochastic system such as a fishery the chance of imperfect information is high. One source of imperfect information is that the vessels may not be allocated properly, i.e. there is a certain probability p_n that a vessel will go to the wrong ground. This can be included in our dynamic program by adding another expectation.

A second source of imperfect information is that schools may be fished more than once. Such a practice is common, for example, in the purse-seine fishery for yellowfin tuna where a school will be fished and then followed as it regroups and then fished again. In a case

such as this, the imperfect information is of two types. First, the search is highly non-random, so that exponentially distributed times (e.g. Models (1) or (18)), which correspond to random encounters, are no longer valid. Second, the count on number of schools encountered must be modified. One way of doing this is to introduce another expectation as follows. Let p_f be the probability that a school will be fished twice and assume that no school is fished more than twice. Then

 $Pr(N_0 \text{ observations of schools})$

$$= \sum_{j=0}^{N_0-1} \Pr(N_{0-j} \text{ schools were really observed})$$

and
$$j$$
 schools were fished twice). (42)

Let $P_0(k)$ be the probability of observing k schools, calculated from Equations (12) or (22). Then, assuming independence between observations and double fishing,

 $Pr(N_0 \text{ observations of schools})$

$$= \sum_{j=0}^{N_0-1} P_0(N_0-j) \begin{bmatrix} N_0-j \\ j \end{bmatrix} [p_f^j (1-p_f)^{N_0-j}]. \tag{43}$$

Equation (43) can be modified in an obvious way to include the chance of fishing a school 3,4,..., times. Equation (43), or one of its extensions can be included in our dynamic programming equation in a simple way.

Finally, we consider the inclusion of some other operational factors. One of these is set time; our analysis has assumed that set time can be ignored, but if it is not ignored the problem becomes much harder to solve (Mangel, 1982; Neyman, 1949). Our dynamic programming equations would then be changed in a fundamental way. The same holds true for the inclusion of false detections, in which time is lost pursuing a signal that turns out to be false.

Discussions with fishermen and analysts familiar with fishermen indicate that not missing a good year may be as important as maximizing profit. One way to include this factor in the formulation of the problem is as follows. Let p_{λ^*} be the prior probability that $\lambda > \lambda^*$. For our models using the gamma density, we have that

$$p_{\lambda^{\bullet}} = \int_{\lambda^{\bullet}}^{\infty} \frac{e^{-a\lambda} \lambda^{\nu-1} \alpha^{\nu}}{\Gamma(\nu)} d\lambda. \tag{44}$$

Let p'_{λ} be the posterior probability that $\lambda > \lambda^*$, given one period of searching. With no searching $p'_{\lambda} = p_{\lambda}$. To our objective function involving profit, we add the term $c_b[(p_{\lambda^*}/p_{\lambda^*})-1]$, where c_b is constant. This term is large if the posterior chance of a good year is large and is zero if no searching is done. In this way, we can include another operational factor.

A third operational factor is that our choice of net expected gain as the optimization criterion may not ac-

curately describe the behaviour of fishermen. For example, a recent study of the behaviour of herring fishermen (Swierzbinski, 1981) shows that they are more likely to be fishing for a quota than maximizing profit. We plan in a later paper to discuss the difference between the effects of profit maximization and quotas on fisherman behaviour in detail.

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