# Summary of Series Convergence and Divergence Tests

| Test | Series | Convergence or Divergence | Comments |
|---|---|---|---|
| **Divergence Test** | $\sum a_n$ | Diverges if $\lim_{n\to\infty} a_n \neq 0$ | Inconclusive if $\lim_{n\to\infty} a_n = 0$ |
| **Geometric Series** | $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=0}^{\infty} ar^n$ | Converges, $s = \frac{a}{1-r}$ if $\lvert r\rvert < 1$<br>Diverges if $\lvert r\rvert \geq 1$ | Useful for doing comparison tests on series with general term *similar* to $ar^{n-1}$. |
| ***p*-series** | $\sum_{n=1}^{\infty} \frac{1}{n^p}$ | Converges if $p > 1$<br>Diverges if $p \leq 1$ | Useful for doing comparison tests on series with general term *similar* to $1/n^p$. |
| **Integral Test** | $\sum_{n=1}^{\infty} a_n$, $f(n) = a_n$ | Converges if $\int_1^{\infty} f(x)dx$ converges,<br>Diverges if $\int_1^{\infty} f(x)dx$ diverges. | The function $f(x)$ **must** be continuous and (eventually) positive and decreasing… and something you don't mind integrating. |
| **Direct Comparison Test** | $\sum_{n=1}^{\infty} a_n$ "like" $\sum_{n=1}^{\infty} b_n$ (both series have positive terms … eventually) | If $\sum b_n$ converges and $a_n \leq b_n$ (eventually) then $\sum a_n$ converges.<br>If $\sum b_n$ diverges and $a_n \geq b_n$ (eventually) then $\sum a_n$ diverges. | To decide which series $\sum b_n$ to compare to, consider terms of $\sum a_n$ that have the greatest effect. |
| **Limit Comparison Test** | $\sum_{n=1}^{\infty} a_n$ "like" $\sum_{n=1}^{\infty} b_n$ (both series have positive terms … eventually) | If $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, with $L>0$, then both series converge or both diverge. If $L= 0$ and $\sum b_n$ converges, so does $\sum a_n$.<br>If $L=\infty$, and $\sum b_n$ diverges, so does $\sum a_n$. | If the inequality needed for direct comparison doesn't work, try limit comparison. |
| **Alternating Series Test** | $\sum_{n=1}^{\infty} (-1)^n b_n$, $b_n>0$ | Converges if:<br>$b_n$ is eventually <u>decreasing</u> and<br>$\lim_{n\to\infty} b_n = 0$ | Only applies to alt. series! It's ok if the exponent on the -1 is $n$+1 or $n$-1. |
| **Ratio Test** | $\sum_{n=1}^{\infty} a_n$ | If $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L$ (or $\infty$) the series:<br>converges **absolutely** if $L<1$<br>diverges if $L>1$ (or is $\infty$) | This test gives no information if $L=1$.<br>This test works well if factorials or lots of products are involved. |
| **Root Test** | $\sum_{n=1}^{\infty} a_n$ | If $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L$ (or $\infty$) the series:<br>converges **absolutely** if $L<1$<br>diverges if $L>1$ (or is $\infty$) | This test gives no information if $L=1$.<br>This test works well if $n$th powers are involved. |
| **Absolute Convergence Test** | $\sum_{n=1}^{\infty} a_n$ | If $\sum_{n=1}^{\infty} \lvert a_n\rvert$ then $\sum_{n=1}^{\infty} a_n$ converges | Useful for series that have both positive and negative terms, but aren't necessarily alternating. |