

Determine whether each of the following SEQUENCES converges or diverges. If it converges, find the limit.

1. $a_n = \frac{n!}{(n-2)!}$

2. $a_n = \frac{4n^3 - 2n}{3n^3 + 5}$

For #3 - #7: Determine whether each of the following SERIES **converges** or **diverges**. If the series is **GEOMETRIC** in form, give the **sum** of the series. In each case state the conclusion and the test used.

3. $\sum_{n=1}^{\infty} \frac{(\ln n)^n}{(1+n)^n}$

4. $\sum_{n=1}^{\infty} \frac{3 \cdot 2^n}{5^{n+1}}$

5. $\sum_{n=1}^{\infty} \frac{\sin^2 n}{1+n^3}$

6. $\sum_{n=1}^{\infty} \frac{(n+1) \cdot 5^n}{2^{2n+1}}$

7. $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

Determine whether each of the following series **converges absolutely**, **converges conditionally**, or **diverges**.

8. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n! 10^n}{3^n}$

9. $\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{5n^2 + 1}$

1. Given each of the following power series, find the radius of convergence “R” and the interval of convergence “I”. Check your endpoints!!!

a. $\sum_{n=1}^{\infty} \frac{(x+4)^n}{n \cdot 2^{n+1}}$

b. $\sum_{n=0}^{\infty} \frac{\pi^n (x-1)^{2n}}{(2n+1)!}$

2. Using a known power series, find the series representation for $f(x) = \ln(2x+1)$. Please give the radius of convergence.

3. a. Use a known Maclaurin series to get a Maclaurin series for $f(x) = \tan^{-1}(3x^2)$.

b. Use your answer from part (a) to find a series for $\int \tan^{-1}(3x^2) dx$.

4. Find a Taylor series for $f(x) = e^{-3x}$ centered at $a = 2$. Use the definition of a Taylor series.

5. Use known Maclaurin series (or use the definition of a Maclaurin series) to find the first four non-zero terms of the Maclaurin series for $f(x) = e^{2x} \cos x$.

TRICKY PROBLEM: Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+2} \pi^{2n}}{2^{2n-1} (2n+1)!}$.

1. What is a sequence?
2. How do we define sequence convergence and divergence?
3. What is the squeeze theorem and how can it be used to find the limit of a sequence?
4. What do we know about the convergence or divergence of the sequence $\{r^n\}$?
5. You should be able to determine whether a sequence converges or diverges and, if it converges, find the limit. You may need to use L'Hospital's rule or the squeeze theorem.
6. What is a series? How is a series different from a sequence?
7. What is the sequence of partial sums and how is it used to define series convergence?
8. What is the harmonic series? Does it converge or diverge?
9. What is a geometric series? Under what conditions does it converge? What is the limit of a convergent geometric series?
10. State the test for divergence.
11. What is a telescoping series? You should be able to find the sum of a convergent telescoping series.
12. What does the integral test say? Under what conditions does it apply? You should be able to test series for convergence and divergence using the integral test.
13. What is a "p-series" and under what conditions does a p-series converge?
14. What is the difference between the "direct comparison" and "limit comparison" tests?
15. Must **all** terms of your series be positive for these tests to apply? Must they be eventually positive?
16. Suppose the limit you get when you apply the limit comparison test is 3. What does this tell you about the series in question?
17. You should be able to test series using direct and limit comparison tests.
18. What is an alternating series? How do you know you're looking at one?
19. What does the alternating series test say? You should be able to test alternating series for convergence or divergence using the alternating series test.
20. Why is it necessary for the absolute value of the series terms to be eventually decreasing? Why must the absolute value of the series terms be headed for 0?
21. Give an example of an alternating series that diverges.
22. What does it mean for a series to converge absolutely? Converge conditionally?
23. Do all convergent series converge absolutely?
24. Do all absolutely convergent series converge?
25. Is it possible for a divergent series to converge absolutely?
26. State the ratio test and its conclusions.
27. What do you do if the limit you take during the ratio test is 1?
28. State the root test and its conclusions.
29. What do you do if the limit you take during the root test is 1?

30. What is a power series?
31. How do you know where the "center" of the power series is?
32. Which convergence tests are useful in finding the radius and interval of convergence of a power series?
33. You should be able to find the radius and interval of convergence of a power series - remember to **check endpoints**.
34. What is the power series representation of the function $f(x)=1/(1-x)$? On what interval is this series convergent? You should be able to use this series to find power series representations for some other functions. Derivatives and integrals may be involved. Watch the radius and interval of convergence!
35. How do you differentiate and integrate power series?
36. What is the definition of a Taylor series? A Maclaurin series?
37. Memorize the well-know Maclaurin series and their intervals of convergence in your book (pg. 743). Be able to use them to find series for other functions or to find the sum of a series.
38. You should be able to do indefinite integrals by finding a series for the integrand and integrating term by term.
39. Explain how to find the first few non-zero terms of the product of two series. How do you find the radius of convergence for the new series?
40. You should be able to use the **definition** of a Taylor or Maclaurin series to find a series for a function. This means actually make the chart and find a pattern for the coefficients.
41. You should be able to find the nth order Taylor polynomial of a function centered at $x=a$.