

Practice Problems from Chapter 5

1. Let $f(x) = \frac{1}{1+x^2}$ for $x \in [-1,1]$. Show that there must be a number $c \in (-1,1)$ with $f'(c) = 0$ and find the value for c .
2. Show $f(x) = |x|$ has a local minimum at $x = 0$ but $f(x)$ is not differentiable at $x = 0$.
3. Let $y = \frac{1}{x+1}$, $x \neq -1$. Determine where the function is increasing, decreasing, concave up, and concave down.
4. Let $y = x^2(1+x)$, $x \in \mathbb{R}$. Find the local maxima and minima. Also find the intervals where the function is increasing and decreasing.
5. Determine all inflection points for $f(x) = \sqrt[3]{x}$.
6. Let $y = |x^2 - 9|$, $x \in [-4,5]$. Find the local maxima and minima. Also find the absolute maxima and minima. Find intervals where the function is increasing, decreasing, concave up, and concave down.
7. Let $y = e^{-x^2/4}$, $x \in \mathbb{R}$. Find the local maxima and minima. Also find the absolute maxima and minima. Find intervals where the function is increasing, decreasing, concave up, and concave down.
8. Find the largest possible area of a right triangle whose hypotenuse is 4 cm long.
9. Use l'Hospital's Rule to find the limits:

a) $\lim_{x \rightarrow \pi/2} \frac{\sin(\pi/2 - x)}{\cos x}$

b) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x - \frac{x^2}{2}}{x^3}$