SAMPLE SOLUTIONS TO PRACTICE FINAL

1. a) $r(r+3)(5r-4)=0$

$\boxed{r=0}$ OR $\boxed{r=-3}$ OR $5r-4=0$

$\boxed{r=\frac{4}{5}}$

b) $-2x^3 = 108x - 30x^2$

$2x^3 - 30x^2 + 108x = 0$

$2x(x^2-15x+54)=0$

$2x(x-6)(x-9)=0$

$\boxed{x=0}$ OR $\boxed{x=6}$ OR $\boxed{x=9}$

c) $\frac{1}{x-4} - \frac{3x}{x^2-16} = \frac{2}{x+4}$

$\frac{1}{x-4} - \frac{3x}{(x-4)(x+4)} = \frac{2}{x+4}$ LCD $= (x-4)(x+4)$

$(x-4)(x+4)\left(\frac{1}{x-4} - \frac{3x}{(x-4)(x+4)}\right) = \left(\frac{2}{x+4}\right)(x-4)(x+4)$

$x+4-3x = 2(x-4)$

$-2x+4 = 2x-8$

$-4x = -12$

$\boxed{x=3}$

BAD LIST

$(x-4)(x+4) \neq 0$

$x \neq 4 \quad x \neq -4$

$\Rightarrow$ make bad list OR CHECK!!

d) $\frac{x}{4} + \frac{5}{x} = 3$

BAD LIST : $x \neq 0$

LCD $= 4x$

$4x\left(\frac{x}{4} + \frac{5}{x}\right) = 3 \cdot 4x$

$x^2 + 20 = 12x$

$x^2 - 12x + 20 = 0$

$(x-10)(x-2)=0$

$\boxed{x=10 \text{ OR } x=2}$

e) $\sqrt{x+1} + 4 = 9$

$\sqrt{x+1} = 5$

$(\sqrt{x+1})^2 = (5)^2$

$x+1 = 25$

$x = 24$

check: $\sqrt{(24)+1} + 4 = 5 + 4$

$= 9$ ✓

$\boxed{x = 24}$

f) $\sqrt{x-3} + \sqrt{x+2} = 5$

$\sqrt{x-3} = 5 - \sqrt{x+2}$

$(\sqrt{x-3})^2 = (5 - \sqrt{x+2})^2$

$x-3 = 25 - 10\sqrt{x+2} + (x+2)$

$x-3 = 27 + x - 10\sqrt{x+2}$

$10\sqrt{x+2} = 30$

$(\sqrt{x+2})^2 = (3)^2$

$x+2 = 9$

$\boxed{x = 7}$ check! ✓

2. a) $\dfrac{a^2b}{a^2-b^2} \cdot \dfrac{a+b}{4a^3b} = \dfrac{\cancel{a^2b}}{(a-b)\cancel{(a+b)}} \dfrac{\cancel{(a+b)}}{4a^3\cancel{b}}$

$= \boxed{\dfrac{1}{4a(a-b)}}$

b) $\dfrac{5x-15}{3-x} \cdot \dfrac{x+2}{10x+20} \div \dfrac{x^2-x-6}{x^2-9}$

$= \dfrac{\cancel{5(x-3)}}{-1\cancel{(x-3)}} \cdot \dfrac{\cancel{(x+2)}}{\cancel{10}^{2}\cancel{(x+2)}} \cdot \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x+2)}$

$= \boxed{-\dfrac{x+3}{2(x+2)}}$

c) $\dfrac{2}{a^2-2a+1} + \dfrac{3}{a^2-1}$

$= \dfrac{2}{(a-1)^2} + \dfrac{3}{(a-1)(a+1)}$ $\quad LCD = (a-1)^2(a+1)$

$= \dfrac{2}{(a-1)^2}\cdot\dfrac{(a+1)}{(a+1)} + \dfrac{3}{(a-1)(a+1)}\cdot\dfrac{(a-1)}{(a-1)}$

$= \dfrac{2a+2+3a-3}{(a-1)^2(a+1)}$

$= \boxed{\dfrac{5a-1}{(a-1)^2(a+1)}}$

d) $\dfrac{3}{2x}\left(\dfrac{2}{x+1} - \dfrac{2}{x-3}\right)$ $\quad$ LCD of $\left(\dfrac{2}{x+1} - \dfrac{2}{x-3}\right) \Rightarrow (x+1)(x-3)$

$= \dfrac{3}{2x}\left(\dfrac{2}{x+1}\cdot\left(\dfrac{x-3}{x-3}\right) - \dfrac{2}{x-3}\left(\dfrac{x+1}{x+1}\right)\right)$

$= \dfrac{3}{2x}\left(\dfrac{2(x-3)-2(x+1)}{(x+1)(x-3)}\right)$

$= \dfrac{3}{2x}\left(\dfrac{2x-6-2x-2}{(x+1)(x-3)}\right)$

$= \dfrac{3}{2x}\left(\dfrac{-8}{(x+1)(x-3)}\right)$

$= \dfrac{-24}{2x(x+1)(x-3)}$

$= \boxed{\dfrac{-12}{x(x+1)(x-3)}}$

3. $(3x^3+12x-4) \div (x-2)$

$$\begin{array}{r} 3x^2+6x+24+\frac{44}{x-2} \\ x-2\overline{)\,3x^3+0x^2+12x-4} \\ -3x^3+6x^2 \\ \hline 6x^2+12x \\ -6x^2+12x \\ \hline 24x-4 \\ -24x+48 \\ \hline 44 \end{array}$$

$$3x^2+6x+24+\frac{44}{x-2}$$

4. $(x^5-1) \div (x-1)$ Synthetic

1	1	0	0	0	0	−1
		1	1	1	1	1
	1	1	1	1	1	0

$$x^4+x^3+x^2+x+1$$

5. a) $\sqrt{\frac{12}{25}} = \frac{\sqrt{12}}{\sqrt{25}} = \frac{2\sqrt{3}}{5}$

b) $\sqrt[3]{\frac{8x^4}{27y^3}} = \frac{\sqrt[3]{8x^4}}{\sqrt[3]{27y^3}} = \frac{2x\sqrt[3]{x}}{3y}$

c) $\sqrt{18}\sqrt{50} = 3\sqrt{2} \cdot 5\sqrt{2}$

$= 15 \cdot 2$

$= 30$

d) $\sqrt{20x^4y^2}$

$= \boxed{2\sqrt{5}\,x^2y}$

e) $-\sqrt{75} + \sqrt{12} - 3\sqrt{3} = -5\sqrt{3} + 2\sqrt{3} - 3\sqrt{3}$

$= \boxed{-6\sqrt{3}}$

f) $(\sqrt{x} - y)(\sqrt{x} + y) = (\sqrt{x})^2 - y^2$

$= \boxed{x - y^2}$

g) $\dfrac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}} \cdot \left(\dfrac{x^2y^2}{x^2y^2}\right)$ LCD $= x^2y^2$

$= \dfrac{\frac{x^2y^2}{x} - \frac{x^2y^2}{y}}{\frac{x^2y^2}{x^2} - \frac{x^2y^2}{y^2}}$

$= \dfrac{xy^2 - x^2y}{y^2 - x^2}$

$= \dfrac{xy(\cancel{y-x})}{\cancel{(y-x)}(y+x)}$

$= \boxed{\dfrac{xy}{y+x}}$

6 a) $\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{6}}{2}}$

b) $\frac{2}{\sqrt[3]{3x^2}} = \frac{2\sqrt[3]{3}}{\sqrt[3]{3x^2}\sqrt[3]{3}} = \frac{2}{\sqrt[3]{3x^2}} \cdot \frac{\sqrt[3]{9x}}{\sqrt[3]{9x}} = \frac{2\sqrt[3]{9x}}{\sqrt[3]{27x^6}} = \boxed{\frac{2\sqrt[3]{9x}}{3x^2}}$

c) $\frac{x}{\sqrt{x}-\sqrt{y}} = \frac{x}{\sqrt{x}-\sqrt{y}} \cdot \frac{(\sqrt{x}+\sqrt{y})}{(\sqrt{x}+\sqrt{y})}$

$= \frac{x+\sqrt{xy}}{(\sqrt{x})^2-(\sqrt{y})^2}$

$= \boxed{\frac{x+\sqrt{xy}}{x-y}}$

7 a) $2x^2+7x=4$

$x^2+\frac{7}{2}x=2$

$\left(\frac{7}{2}\cdot\frac{1}{2}\right)^2 = \left(\frac{7}{4}\right)^2 = \frac{49}{16}$

$x^2+\frac{7}{2}+\left(\frac{7}{4}\right)^2 = 2+\frac{49}{16}$

$\left(x+\frac{7}{4}\right)^2 = \frac{32+49}{16}$

$\left(x+\frac{7}{4}\right)^2 = \frac{81}{16}$

$\sqrt{\left(x+\frac{7}{4}\right)^2} = \pm\sqrt{\frac{81}{16}}$

$x+\frac{7}{4} = \pm\frac{9}{4}$

$x = -\frac{7}{4} \pm \frac{9}{4}$

$x = -\frac{7}{4}+\frac{9}{4} = \frac{2}{4} = \frac{1}{2}$

OR

$x = -\frac{7}{4}-\frac{9}{4} = -\frac{16}{4} = 4$

$\boxed{x=\frac{1}{2}}$

$\boxed{x=4}$

b) $x^2+6x+2=0$

$x^2+6x=-2$ $\qquad \left(\frac{6}{2}\right)^2=3^2=9$

$x^2-6x-9=-2+9$

$(x-3)^2=7$

$\sqrt{(x-3)^2}=\pm\sqrt{7}$

$x+3=\pm\sqrt{7}$

$x=\boxed{-3\pm\sqrt{7}}$

c) $x^2-4x+8=0$

$x^2-4x=-8$ $\qquad \left(\frac{4}{2}\right)^2=2^2=4$

$x^2-4x+4=-8+4$

$(x-2)^2=-4$

$\sqrt{(x-2)^2}=\pm\sqrt{-4}$

$x-2=\pm 2i$

$x=\boxed{2\pm 2i}$

d) $x^2+5x=-7$ $\qquad \left(\frac{5}{2}\right)^2=\frac{25}{4}$

$x^2+5x+\frac{25}{4}=-7+\frac{25}{4}$

$\left(x+\frac{5}{2}\right)^2=-\frac{28}{4}+\frac{25}{4}$

$\sqrt{\left(x+\frac{5}{2}\right)^2}=\pm\sqrt{\frac{3}{4}}$

$x+\frac{5}{2}=\pm\frac{\sqrt{3}}{2}$

$x=\boxed{-\frac{5}{2}\pm\frac{\sqrt{3}}{2}}$

8. a) $(x+5)(x-1) = 2$

$x^2 + x - 5 - 2 = 0$

$x^2 + x - 7 = 0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = 1$
$b = 1$
$c = -7$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-7)}}{2(1)}$

$= \dfrac{-1 \pm \sqrt{29}}{2}$

$= \dfrac{-4 \pm \sqrt{44}}{2} = \dfrac{-4 \pm 2\sqrt{11}}{2} = \boxed{-2 \pm \sqrt{11}}$

b) $5x^2 - 3 = 14x$

$5x^2 - 14x - 3 = 0$

$a = 5$
$b = -14$
$c = -3$

$x = \dfrac{-(-14) \pm \sqrt{(-14)^2 - 4(5)(-3)}}{2(5)}$

$= \dfrac{14 \pm \sqrt{256}}{10} = \dfrac{14 \pm 16}{10} = \dfrac{7 \pm 8}{5}$

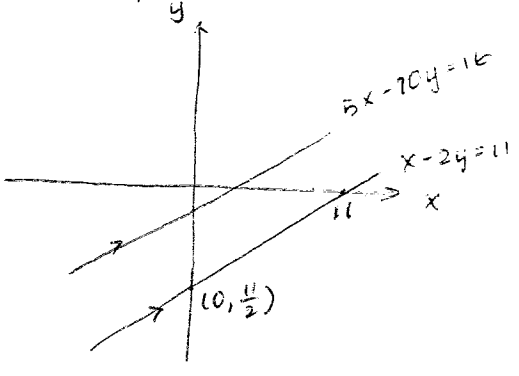
$x = \dfrac{7-8}{5} = \dfrac{-1}{5}$

$x = \dfrac{7+8}{5} = \dfrac{15}{5} = \boxed{3}$

9. a) p239

(3) $\begin{cases} 2x - 4y = 22 \\ 5x - 10y = 16 \end{cases} \Rightarrow \begin{cases} x - 2y = 11 \\ 5x - 10y = 16 \end{cases}$

1) graphing



$x - 2y = 11$
$2y = x - 11$
$y = \frac{1}{2}x - \frac{11}{2}$

$5x - 10y = 16$
$10y = 5x - 16$
$y = \frac{1}{2}x - \frac{8}{5}$

$\boxed{\phi}$

2) substitution

$x = 2y + 11$

$5(2y+11) - 10y = 16$

$10y + 55 - 10y = 16$

$55 = 16$

$\emptyset$

3) elimination

$-5x + 10y = -55$

$5x - 10y = 16$

$0 = -39$

$\emptyset$

4) $3x - 6y = 12 \Rightarrow x - 2y = 4$

$2y = x - 4 \Rightarrow 2y = x - 4$

1)

$x - 2y = 4$

$2y = x - 4$

$y = \frac{1}{2}x - 2$

$2y = x - 4$

$y = \frac{1}{2}x - 2$

$\{(x,y) \mid y = \frac{1}{2}x - 2\}$

2) $x - 2y = 4$

$2y = x - 4$

$x = 2y + 4$

$2y = (2y+4) - 4$

$0 = 0$ always true $\{(x,y) \mid y = \frac{1}{2}x - 2\}$

3) $x - 2y = 4$

$2y = x - 4$

$x - 2y = 4$

$-x + 2y = -4$

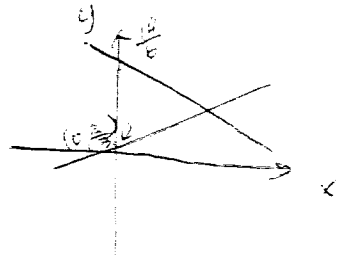
$0 = 0$ always true $\{(x,y) \mid y = \frac{1}{2}x - 2\}$

5) $\frac{1}{2}x - \frac{3}{4}y = -\frac{1}{2}$

$\frac{1}{8}x + \frac{3}{4}y = \frac{19}{8}$ $\Rightarrow$ $2x - 3y = -2$

$x + 6y = 19$

1)



$2x - 3y = -2$

$3y = 2x + 2$

$y = \frac{2}{3}x + \frac{2}{3}$

$x + 6y = 19$

$6y = -x + 19$

$y = -\frac{1}{6}x + \frac{19}{6}$

2) $\begin{cases} 2x-3y=-2 \\ x+6y=19 \end{cases} \Rightarrow x=-6y+19$

$2(-6y+19)-3y=-2$

$-12y+38-3y=-2$

$-15y=-40$

$y=\frac{-40}{-15}=\frac{8}{3}$

$x=-6\left(\frac{8}{3}\right)+19$

$x=-16+19=3$

$\left(3,\frac{8}{3}\right)$

$2x-3y=-2$

$-2x-2y=-38$

$-5y=-40$

$y=\frac{-40}{-15}=\frac{8}{3}$

$x+2\left(\frac{8}{3}\right)=19$

$x+\frac{16}{3}=19$

$x=3$

$\left(3,\frac{8}{3}\right)$

when $x=4$:

$y=2(4)-4=4$

when $x=1$

$y=2(1)-4=-2$

$(4,4)\ (1,-2)$

(73) $\begin{cases} x^2+y^2=10 \\ 9x^2+y^2=18 \end{cases}$ (-1)

$x^2+y^2=10$

$-9x^2-y^2=-18$

$-8x^2=-8$

$x^2=1$

$x=\pm 1$

Let $x=1$

$1^2+y^2=10$

$y^2=9$

$y=\pm 3$

Let $x=-1$

$(-1)^2+y^2=10$

$1+y^2=10$

$y^2=9$

$y=\pm 3$

$(1,3)(1,-3)$
$(-1,3)(-1,-3)$

(74) $\begin{cases} x^2+2y=9 \\ 5x-2y=5 \end{cases}$

$x^2+5x=14$

$x^2+5x-14=0$

$(x+7)(x-2)=0$

$x=-7$

$x=2$

when $x=-7$

$5(-7)-2y=5$

$-35-2y=5$

$-40=2y$

$y=-20$

$(-7,-20)$

when $x=2$

$5(2)-2y=5$

$10-2y=5$

$5=2y$

$y=\frac{5}{2}$

$(2,\frac{5}{2})$

(76) $\begin{cases} x^2-3y^2=1 \\ 4x^2+5y^2=21 \end{cases}$

$x^2=3y^2+1$

$4(3y^2+1)+5y^2=21$

$12y^2+4+5y^2=21$

$17y^2=17$

$y^2=1$

$y=\pm 1$

$x^2=3(1)+1$

$x^2=4$

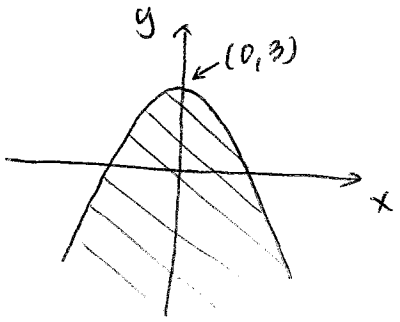
$x=\pm 2$

$(2,1)\ (2,-1)\ (-2,1)\ (-2,-1)$

10.

(79) $y \leq -x^2 + 3$

Let $y = -x^2 + 3$

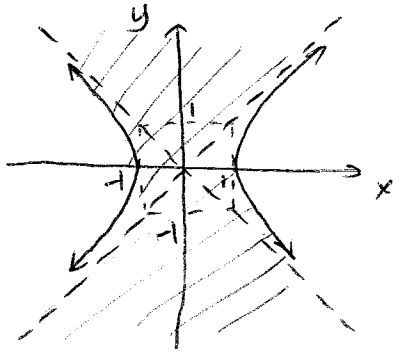


test $(0,0)$: $0 \overset{?}{\leq} -0^2 + 3$

$0 \leq 3$ ✓ true

(81) $x^2 - y^2 < 1$

Let $x^2 - y^2 = 1$

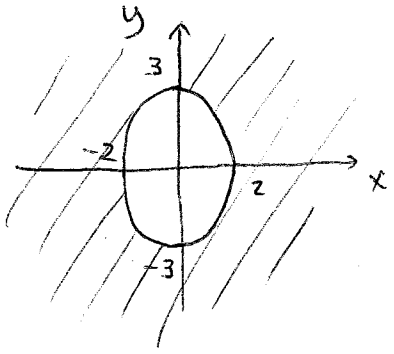


test $(0,0)$: $0^2 - 0^2 \overset{?}{<} 1$

$0 < 1$ ✓ true

(82) $\frac{x^2}{4} + \frac{y^2}{9} \geq 1$

Let $\frac{x^2}{4} + \frac{y^2}{9} = 1$



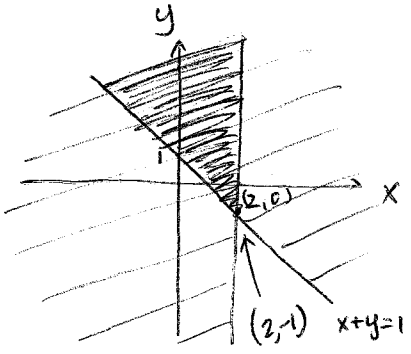
test $(0,0)$: $\frac{0^2}{4} + \frac{0^2}{9} \overset{?}{=} 1$

$0 = 1$ False.

(83) $\begin{cases} 2x \le 4 \\ x+y \ge 1 \end{cases}$

Let $2x = 4$, $x = 2$

Let $x+y=1$, $y = -x+1$



TEST: $2x \le 4$ @ $(0,0)$

$0 \le 4$ TRUE

$x+y \ge 1$ @ $(0,0)$

$0+0 \ge 1$

$0 \ge 1$ FALSE

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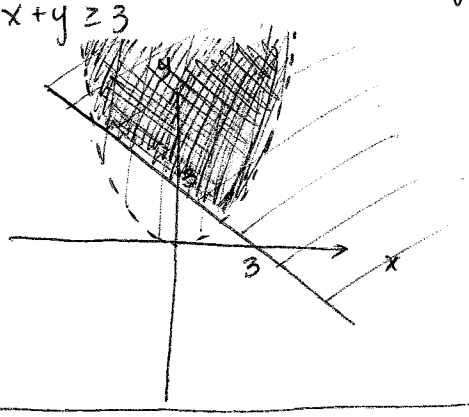
(85) $\begin{cases} y > x^2 \\ x+y \ge 3 \end{cases}$

Let $y = x^2$

Let $x+y=3$, $y = -x+3$



Test: $y > x^2$ @ $(0,5)$

$5 \stackrel{?}{>} 0^2$ TRUE

$x+y \ge 3$ @ $(0,0)$

$0+0 \stackrel{?}{\ge} 3$

$0 \ge 3$ FALSE

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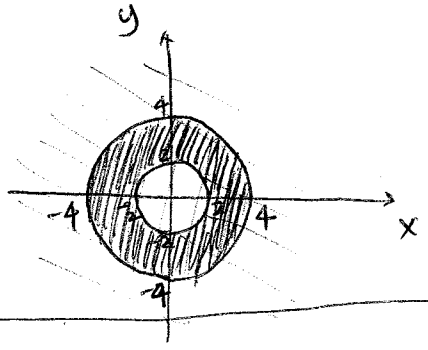
(86) $\begin{cases} x^2+y^2 \le 16 \Rightarrow x^2+y^2 \le 4^2 \\ x^2+y^2 \ge 4 \Rightarrow x^2+y^2 \ge 2^2 \end{cases}$



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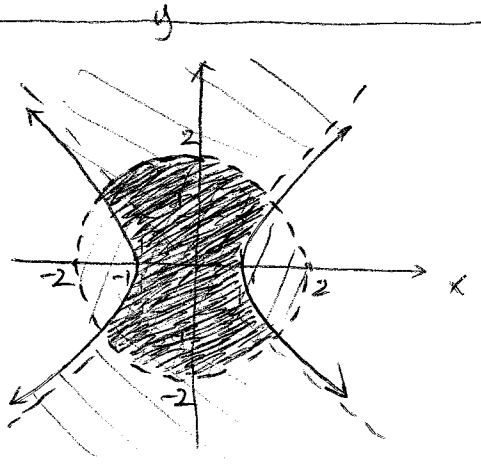
(87) $\begin{cases} x^2+y^2 < 4 \Rightarrow x^2+y^2 < 2^2 \\ x^2-y^2 \le 1 \end{cases}$

Test: $x^2-y^2 \le 1$ @ $(0,0)$

$0-0 \le 1$

$0 \le 1$ TRUE



12. a) $f(x) = 2x^2 - 4x + 5$ ← parabola ∪

vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$-\frac{b}{2a} = \frac{-(-4)}{2 \cdot 2} = \frac{4}{4} = 1$

$f(1) = 2 - 4 + 5 = 3$ vertex $(1, 3)$

Range $= [3, \infty)$

b) $g(x) = -x^2 + 6x + 1$ ← parabola ∩

$-\frac{b}{2a} = \frac{-6}{2(-1)} = \frac{-6}{-2} = 3$

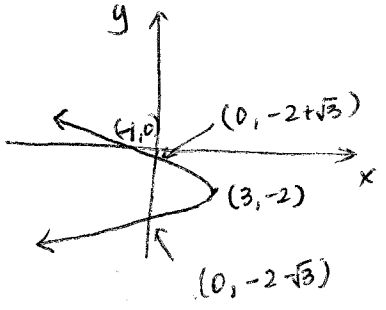
$g(3) = -9 + 6 \cdot 3 + 1 = -9 + 18 + 1 = 10$ vertex $(3, 10)$

Range $= (-\infty, 10]$

13.

(23) $x = -(y+2)^2 + 3$

vertex $(3, -2)$



y-int: $x = 0$

$0 = -(y+2)^2 + 3$

$(y+2)^2 = 3$

$y + 2 = \pm\sqrt{3}$

$y = -2 \pm \sqrt{3}$

x-int: $y = 0$

$x = -(0+2)^2 + 3$

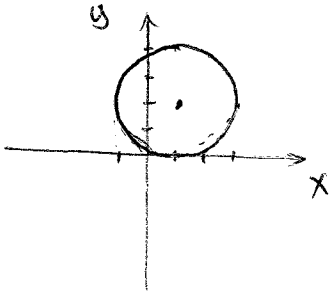
$= -4 + 3$

$= -1$

(24) $(x-1)^2 + (y-2)^2 = 4$

center $(1, 2)$

radius $= 2$



(31) $4x^2+4y^2+16x+8y=1$

$4x^2+16x+4y^2+8y=1$

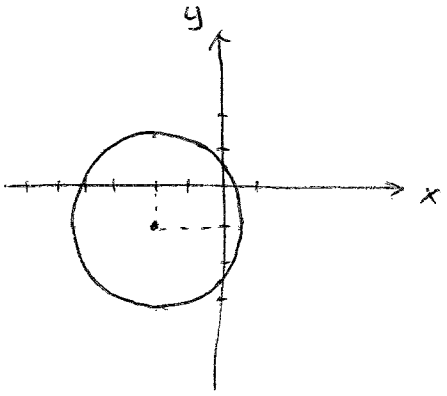
$(x^2+4x)+(y^2+2y)=\frac{1}{4}$

$(x^2+4x+\underset{m}{4})+(y^2+2y+\underset{m}{1})=\frac{1}{4}+4+1$

$(x+2)^2+(y+1)^2=\frac{21}{4}$

center $(-2,-1)$

radius $=\frac{\sqrt{21}}{2}$



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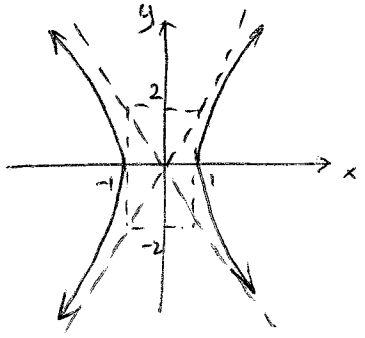
(37) $x^2-\frac{y^2}{4}=1$

$\frac{x^2}{1^2}-\frac{y^2}{2^2}=1$



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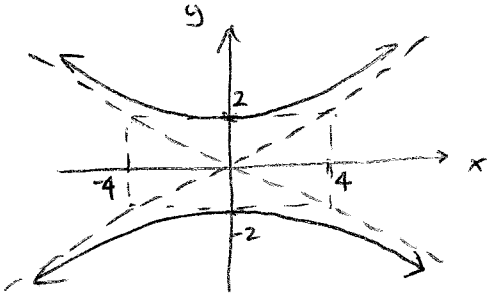
(38) $\frac{y^2}{4}-\frac{x^2}{16}=1$

$\frac{y^2}{2^2}-\frac{x^2}{4^2}=1$



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(59) $\frac{6(x-2)^2}{36}+\frac{9(y+5)^2}{36}=\frac{36}{36}$ → ellipse

$\frac{(x-2)^2}{6}+\frac{(y+5)^2}{4}=1$

center $(2,-5)$