① $\displaystyle\int_{-2}^{1} \frac{x}{x+3}\,dx = \lim_{\|P\|\to 0} \sum_{k=1}^{n} \frac{c_k}{c_k+3}\,\Delta x_k$

WHERE $P = \{-2, x_1, x_2, \dots, x_{n-1}, 1\}$ IS A PARTITION OF $[-2,1]$ $(-2 < x_1 < x_2 < \dots < 1)$

$\|P\| = \max\{x_k - x_{k-1} \mid k = 1, 2, \dots, n\}$

$c_k \in [x_{k-1}, x_k]$ AND $\Delta x_k = x_k - x_{k-1}$

② $\displaystyle\int_{1}^{3} \frac{x-1}{x}\,dx \approx \left(\frac{1.5-1}{1.5} + \frac{2-1}{2} + \frac{2.5-1}{2.5} + \frac{3-1}{3}\right) .5$

$$= \boxed{\frac{21}{20}}$$

③ $\displaystyle y = \int_{\sin x}^{5} \frac{1}{u^3}\,du = -\int_{5}^{\sin x} \frac{1}{u^3}\,du$

$$= \int_{5}^{\sin x} \frac{1}{u^3}\,du$$

LET $w = \sin x$ SO $\displaystyle y = \int_{5}^{w} \frac{1}{u^3}\,du$

$\dfrac{dy}{dw} = \dfrac{1}{w^3}$ $\qquad \dfrac{dy}{dx} = \dfrac{dy}{dw} \cdot \dfrac{dw}{dx}$

$\dfrac{dy}{dx} = \dfrac{1}{w^3} \cdot \cos x = \boxed{\dfrac{\cos x}{\sin^3 x}}$

④ USE AREA $\displaystyle\int_{3}^{8} f(x)\,dx = \boxed{7.5}$

$f_{AVG} = \dfrac{7.5}{8-3} = \boxed{1.5}$

⑤ (a) $f(x) = \frac{x^2+2x-1}{x^2} = 1 + 2x^{-1} - x^{-2}$

$F(x) = x + 2\ln|x| + x^{-1} + C$

(b) $g(\theta) = \sec\theta\tan\theta + \sin\theta$

$G(\theta) = \sec\theta - \cos\theta + C$

(c) $g(x) = \frac{1}{1+9x^2} = \frac{1}{1+(3x)^2}$

$G(x) = \arctan(3x) \cdot \frac{1}{3} + C$

or $= \frac{1}{3}\tan^{-1}(3x) + C$

⑥ (a) $\int_1^4 \frac{1+\sqrt{x}}{\sqrt{x}}\,dx = \int_1^4 (x^{-1/2}+1)\,dx$

$= (2x^{1/2} + x)\Big|_1^4 = \boxed{5}$

(b) $\int_1^e \frac{3}{x}\,dx = (3\ln x)\Big|_1^e = 3\ln e - 3\ln 1$

$= \boxed{3}$

(c) $\int_{\pi/6}^{\pi/2} \sin x\,dx = -\cos x\Big|_{\pi/6}^{\pi/2} = \cos x\Big|_{\pi/2}^{\pi/6}$

$= \cos\frac{\pi}{6} - \cos\frac{\pi}{2} = \frac{\sqrt{3}}{2} - 0 = \boxed{\frac{\sqrt{3}}{2}}$

7. NET CHANGE $N(6)-N(1)=\int_1^6 2e^{-5t}\,dt$

$$= -\tfrac{2}{5}e^{-5t}\Big|_1^6 = -\tfrac{2}{5}\left(e^{-5/6}-e^{-1/6}\right)$$

$$= \boxed{\tfrac{2}{5}\left(e^{-1/6}-e^{-5/6}\right)}$$

$$N(t)=\int_0^t 2e^{-5u}\,du + N(0)$$

$$N(t)= -\tfrac{2}{5}\left(e^{-5t}-e^{0}\right)+100$$

$$= -\tfrac{2}{5}e^{-5t}+\tfrac{2}{5}+100$$

$$\boxed{N(t)=\frac{502}{5}-\frac{2}{5}e^{-5t}}$$

8. $4x=x^3 \quad x^3-4x=0 \quad (x+2)(x-2)x=0$

$$\text{AREA}=\int_{-2}^{0}(x^3-4x)\,dx+\int_0^2(4x-x^3)\,dx$$

$$=\left(\frac{x^4}{4}-2x^2\right)\Big|_{-2}^{0}+\left(2x^2-\frac{x^4}{4}\right)\Big|_0^2=4+4=\boxed{8}$$

9. $\boxed{\text{SAVE FOR 2ND MIDTERM}}$

$$L=\int_0^1\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\int_0^1\sqrt{\tfrac{27}{8}x+1}\,dx$$

$$=\left(\tfrac{27}{8}x+1\right)^{3/2}\cdot\tfrac{2}{3}\cdot\tfrac{8}{27}\Big|_0^1=\frac{16}{81}\left(\left(\frac{35}{8}\right)^{3/2}-\left(\frac{27}{8}\right)^{3/2}\right)$$