

1. Solve for x:  $e^{2x+1} = 5$

$$2x + 1 = \ln 5$$

$$2x = \ln 5 - 1$$

$$x = \frac{\ln 5 - 1}{2}$$

2. Solve for x:  $\ln(x - 1) = 3$

$$x - 1 = e^3$$

$$x = 1 + e^3$$

3. Find the range of  $f(x) = x^2 - 1$ ,  $-2 < x \leq 1$ .

Vertex is  $(0, -1)$ ;  $f(-2) = 3$  and  $f(1) = 0$  so the range is the interval  $[-1, 3)$ .

4. Let  $f(x) = x^2 - x$  and  $g(x) = x - 1$ . Find  $(f \circ g)(x)$  and  $(g \circ f)(x)$ .

$$(f \circ g)(x) = f[g(x)] = f(x - 1) = (x - 1)^2 - (x - 1) = x^2 - 2x + 1 - x + 1 = x^2 - 3x + 2$$

$$(g \circ f)(x) = g[f(x)] = g(x^2 - x) = x^2 - x - 1$$

5. 
$$\lim_{x \rightarrow \infty} \frac{x^2}{2 - x^2} = \lim_{x \rightarrow \infty} \frac{1}{2/x^2 - 1} = \frac{\lim_{x \rightarrow \infty} 1}{2 \lim_{x \rightarrow \infty} \frac{1}{x^2} - \lim_{x \rightarrow \infty} 1} = \frac{1}{0 - 1} = -1$$

6. 
$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 4x + 3} = \lim_{x \rightarrow 3} \frac{(x - 3)(x + 3)}{(x - 3)(x - 1)} = \lim_{x \rightarrow 3} \frac{(x + 3)}{(x - 1)} = \frac{3 + 3}{3 - 1} = 3$$

by continuity of  $\frac{(x + 3)}{(x - 1)}$  at  $x = 3$ .

7. 
$$\lim_{x \rightarrow 0} \frac{\sqrt{3-x} - \sqrt{3}}{x} = \lim_{x \rightarrow 0} \frac{(\sqrt{3-x} - \sqrt{3})(\sqrt{3-x} + \sqrt{3})}{x(\sqrt{3-x} + \sqrt{3})} = \lim_{x \rightarrow 0} \frac{3 - x - 3}{x(\sqrt{3-x} + \sqrt{3})} =$$

$$\lim_{x \rightarrow 0} \frac{3 - x - 3}{x(\sqrt{3-x} + \sqrt{3})} = \lim_{x \rightarrow 0} \frac{-x}{x(\sqrt{3-x} + \sqrt{3})} = \lim_{x \rightarrow 0} \frac{-1}{(\sqrt{3-x} + \sqrt{3})} = \frac{-1}{\sqrt{3} + \sqrt{3}} = \frac{-1}{2\sqrt{3}}$$

by the continuity of  $\frac{-1}{(\sqrt{3-x} + \sqrt{3})}$  at  $x=0$

$$8. \lim_{x \rightarrow 0} \frac{e^{2x} + e^{-x}}{2 + e^x} = \frac{e^0 + e^0}{2 + e^0} = \frac{2}{3} \text{ by the continuity of } \frac{e^{2x} + e^{-x}}{2 + e^x} \text{ for all real } x$$

$$9. \lim_{x \rightarrow \infty} \frac{e^x}{2 - e^x} = \lim_{x \rightarrow \infty} \frac{e^x e^{-x}}{(2 - e^x) e^{-x}} = \lim_{x \rightarrow \infty} \frac{1}{2e^{-x} - 1} = \frac{\lim_{x \rightarrow \infty} 1}{2 \lim_{x \rightarrow \infty} e^{-x} - \lim_{x \rightarrow \infty} 1} = \frac{1}{2 \cdot 0 - 1} = -1$$

$$10. \lim_{x \rightarrow 0} \frac{\sin^2 x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \lim_{x \rightarrow 0} \sin x = 1 \cdot 0 = 0 \text{ by the continuity of } \sin x \text{ at } x=0$$

$$11. \lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{3x} = \lim_{x \rightarrow 0} \frac{2(1 - \cos(2x))}{3 \cdot 2x} = \frac{2}{3} \lim_{x \rightarrow 0} \frac{(1 - \cos(2x))}{2x} = \frac{2}{3} \cdot 0 = 0$$

12. Let  $f(x) = \begin{cases} \frac{x^2 + x - 2}{x - 1} & \text{if } x \neq 1 \\ a & \text{if } x = 1 \end{cases}$ . Which value must you assign to  $a$  so that  $f(x)$  is continuous at 1?

We must have  $\lim_{x \rightarrow 1} f(x) = f(1)$  therefore let

$$a = \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - 1)(x + 2)}{x - 1} = \lim_{x \rightarrow 1} x + 2 = 3$$

by the continuity of  $x + 2$

13. Let  $f(x) = x^3 - 2x + 3$ ,  $-3 \leq x \leq -1$ . Use the Intermediate Value Theorem to show that  $x^3 - 2x + 3 = 0$  has a solution in the interval  $(-3, -1)$ .

$f(x) = x^3 - 2x + 3$  is a polynomial hence continuous for all real numbers.

$f(-3) = -18$  and  $f(-1) = 4$  since  $f(-3) < 0 < f(-1)$  the Intermediate Value Theorem implies that there must be at least one number  $c \in (-3, -1)$  with  $f(c) = 0$ .