

Sample Midterm 2

The Island of Nauru is composed almost entirely of phosphate. The key question for Nauru is how quickly to extract the phosphate. Assume that the total endowment of phosphate is 80 tons. Demand in each period is the same and is given by: $p_t = 200 - q_t$. The interest is 8%.

a) (30 points) Suppose that there are only two periods. If the marginal extraction cost is \$30 per ton, what is the dynamically efficient allocation? What is the price in periods 1 and 2? What is the marginal user cost? Demonstrate mathematically that the price rise between the two periods is less than the rate of interest.

b) (30 points) Now suppose that Nauru is the sole supplier of phosphate (i.e. it has a monopoly). What is the condition that Nauru should follow to maximize profits? How much is extracted and sold in period 1? What is the price in period 1? How much is extracted and sold in period 2? What is the price in period 2? Why doesn't Nauru just extract and sell everything in period 1 and invest the earnings?

c) (20 points) Now suppose that rather than 2 periods, the world lasts for n periods (where n is some large number). Using graphs, *qualitatively* analyze the time profile of the price, marginal user cost and quantity extracted associated with a dynamically efficient allocation (where possible include quantitative information in your graphs). Based on your conclusions to part b) how would the monopoly outcome differ in this case (i.e. how would the time profiles differ)?

d) (20 points) Assume now that a synthetic phosphate substitute is also available at \$40 per ton. Discuss (with the aid of diagrams) how this is likely to affect the time paths of extraction, price and marginal user cost under a dynamically efficient allocation.