Problem set 4 solutions

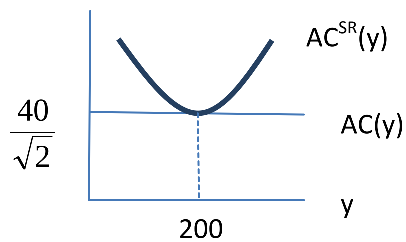
1. a. To find the conditional factor demand functions, minimize the firm's costs subject to producing at least y units of output. Two conditions will hold at the cost minimizing input choice: MRTS = -w/r and the constraint that $y = K^{1/2}L^{1/2}$.

   MRTS = -K/L, so these two conditions are 1. –K/L = -2 and 2. $y = K^{1/2}L^{1/2}$.

   Solve these two equations to obtain $L = \frac{y}{\sqrt{2}}$ and $K = \frac{2y}{\sqrt{2}}$

   b. $$C(y) = 20L(y) + 10K(y) = \frac{20y}{\sqrt{2}} + \frac{20y}{\sqrt{2}} = \frac{40y}{\sqrt{2}}.$$

   $$AC(y) = \frac{C(y)}{y} = \frac{40}{\sqrt{2}}$$



   c. If y=200, $K = \frac{40}{\sqrt{2}}$ When K is fixed at this level, then we can see how output depends on labor: $y = \left(\frac{400}{\sqrt{2}}\right)^{1/2} L^{1/2}$. In the short-run, labor depends on output in the following way:

   $$L = \left(\frac{\sqrt{2}}{400}\right) y^2$$

   d. The short run cost function is $C^{SR} = 10\left(\frac{400}{\sqrt{2}}\right) + 20\left(\frac{\sqrt{2}}{400}\right)y^2 = \frac{4000}{\sqrt{2}} + \frac{\sqrt{2}}{200}y^2$

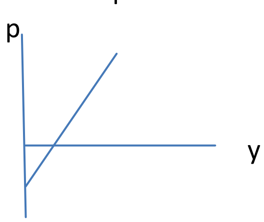
   See above for the graph of the short-run average cost function.

2. a. MC = 4y – 4 AC = 2y – 4 + 8/y AFC = 8/y AVC = 2y – 4

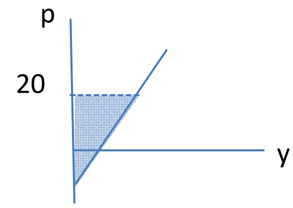
   b. π = py – C(y) = py-$(2y^2 – 4y + 8)$

   To find the output level where profits are maximized, take the derivative of the profit function and set it equal to zero. Here, p= MC(y), or p = 4y – 4. This is the firm's inverse supply function.

   c.



d. Plugging y = 6 into the inverse supply curve, p = 20.



3.

Output	Total Cost	AC	MC
0	0		
1	10	10	10
2	16	8	6
3	21	7	5
4	32	8	11
5	45	9	13
6	66	11	21
7	91	13	25
8	112	14	21

a. The long-run equilibrium price is the minimum of average cost. Here we see that AC reaches a minimum of 7.
b. If the price is seven, each firm in the industry will produce 3 units. (At three units, the marginal cost of producing the 4^{th} unit is 11, which is greater than the marginal revenue of p=7)
c. If the price is seven, quantity demanded by consumers is 60 – 2(7) = 46.
d. Industry production is 46, each firm produces 3, so there will be 15.3 firms in the market.