

Econ 100A: Intermediate Microeconomics

Solutions to Problem Set 2

Fall 2011

Answer 1

a) Maximize the utility function:

$$U(C, E) = CE$$

such that

$$50 = 2.5C + 5E$$

To find the optimal bundle,

$$\begin{aligned} MRS &= -\frac{MU_C}{MU_E} = -\frac{p_C}{p_E} \\ -\frac{MU_C}{MU_E} &= -\frac{E}{C}, \quad -\frac{p_C}{p_E} = -\frac{2.5}{5} \\ \frac{E}{C} &= \frac{1}{2} \longrightarrow 2E = C \end{aligned}$$

We substitute $C = 2E$ into the budget constraint:

$$50 = 2.5(2E) + 5E$$

$$50 = 10E$$

$$E = 5$$

$$C = 10$$

The optimal bundle (C, E) is $(10, 5)$.

b) The price of cigarettes after tax of \$2.5 is:

$$p_c + t = 2.5 + 2.5 = 5$$

The new bundle will now be:

$$MRS = -\frac{E}{C} = -1$$

$$E = C$$

Substitute $E = C$ into the budget constraint:

$$50 = 5E + 5E$$

$$10E = 50$$

$$E = 5, C = 5$$

The optimal bundle (C, E) is $(5, 5)$.

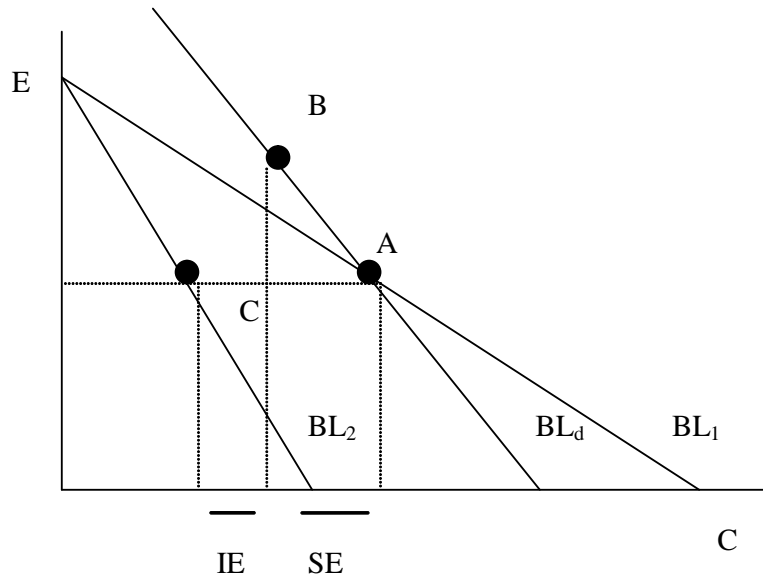
c) To calculate the substitution effect, first we need to calculate the decomposition basket. This can be calculated by plugging $C = E$ into the utility function.

The initial utility provided the bundle $(10, 5)$ is 50. Hence, if we substitute $C = E$ into the utility function, we get:

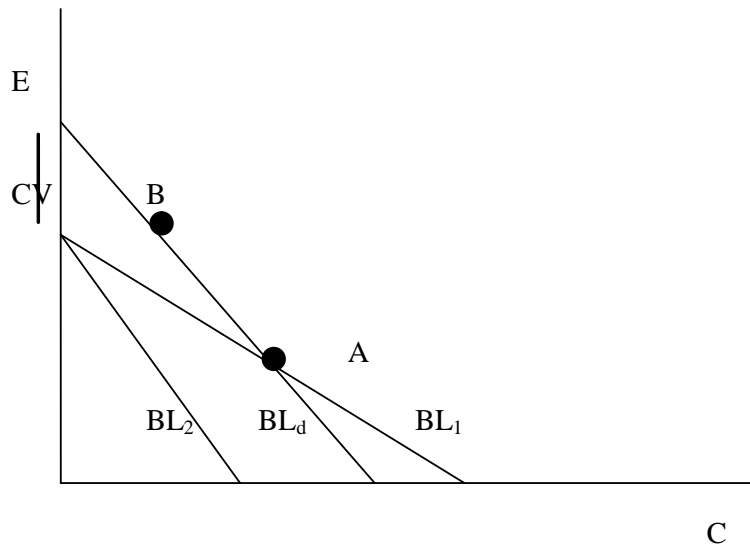
$$50 = C^2$$

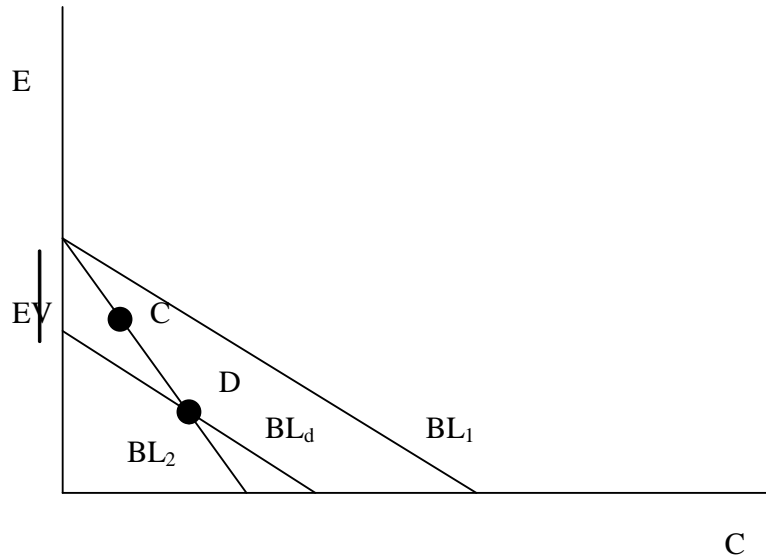
Therefore, $C = 5\sqrt{2}$ and $E = 5\sqrt{2}$

Note: The method used to solve in the TA section (Aadil's) is also correct, although the answers to the substitution and income effect may slightly vary.



d)





e) In order to calculate the compensating variation, we need to find the income at which the individual can consume at the utility level of the original bundle at the new prices.

Hence, p_C will be \$5, and p_E will be \$5. Furthermore, $C = 5\sqrt{2}$ and $E = 5\sqrt{2}$.

Our new budget line will be:

$$I' = 5(5\sqrt{2}) + 5(5\sqrt{2}) = 50\sqrt{2}$$

The compensating variation will be:

$$I' - I = 50\sqrt{2} - 50 = 50(\sqrt{2} - 1)$$

Answer 2

a) Dave's full income is $2000w$, where w is the wage rate per hour and 2000 is the amount of hours he can work. The measured income is $w(2000 - L)$, where L is the amount of hours of leisure he undertakes. The budget constraint is $pC = w(2000 - L)$ where pC is the amount of consumption goods he can purchase given the income he earns.

b) Maximize the utility function:

$$U(C, L) = \sqrt{CL}$$

such that:

$$C + wL = 2000w$$

$$MRS = -\frac{MU_C}{MU_L} = -\frac{L}{C} = -\frac{p_C}{p_L} = -\frac{1}{w}$$

$$wL = C$$

Substitute $C = wL$ into the budget constraint:

$$wL + wL = 2000w$$

$$2L = 2000$$

$$L = 1000$$

$$C = 1000w$$

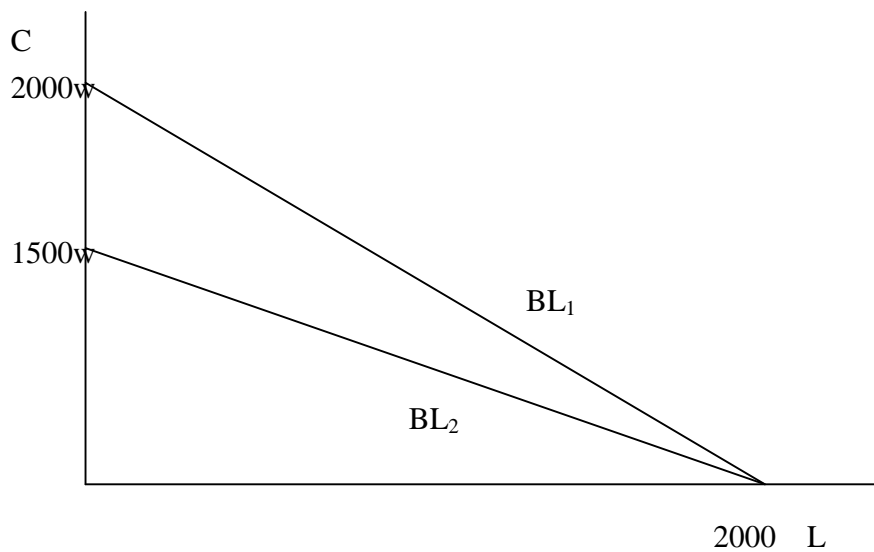
If $w = 20$, $C = 20(1000) = 20000$.

c) With payroll tax, the wage rate becomes $(1 - \tau)w$, where $\tau = 0.25$. The wage rate is $0.75w$.

The budget constraint should now read:

$$C + 0.75wL = 1500w$$

The effective wage rate is now $0.75w$, where w is the wage rate without tax.



d) The new bundle of consumption and leisure should maximize at:

$$0.75wL = C$$

such that:

$$C + 0.75wL = 1500w$$

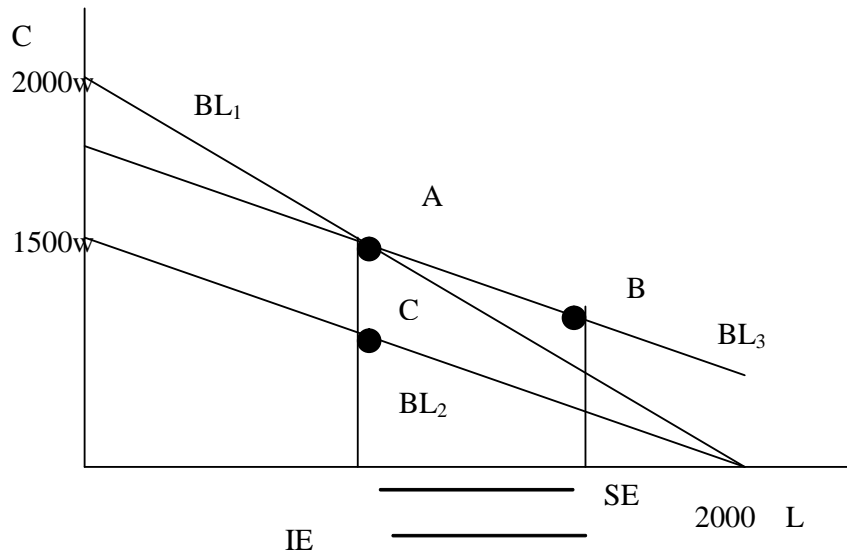
$$0.75wL + 0.75wL = 1500w$$

$$1.5wL = 1500w$$

$$L = 1000$$

$$C = 750w$$

d)



e) The substitution and income effect do not have the same sign. With a 25% tax, an individual will take home less income for every dollar earned. This will reduce the cost of leisure as for every hour of work forgone, the individual is giving up less income. Hence, the substitution effect will lead to an increase in the amount of leisure undertaken by the individual. On the other hand, as leisure is a normal good, a decrease in income will lead to less leisure. Hence, the income effect will counter the substitution effect.