

Short answer problems

a. Uncertain. The rise in the wage leads to more work through the substitution effect. If leisure is a normal good the income effect says more leisure/less work while the income effect would indicate less leisure/more work if leisure is inferior. Either case is possible, since even if it is normal, the substitution effect could outweigh the income effect.

b. True. It is not possible for both goods to be inferior. Therefore wine is a normal good, and the demand curve slopes downward for normal goods.

$$\begin{aligned} c. \quad P &= 20 - 2Q_c & \Rightarrow & \quad Q_c = 10 - \frac{1}{2}P \\ P &= 20 - Q_m & & \quad Q_m = 20 - P \end{aligned}$$

Here we do not need to be concerned about the extensive margin since there is no price such that only one consumer wants a positive amount.

So we can just add the demand curves:

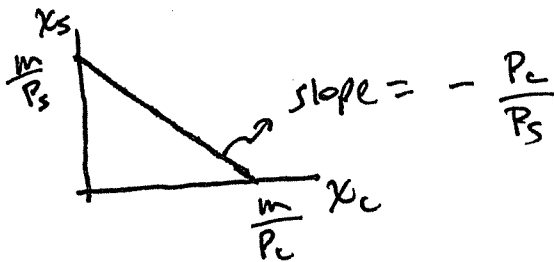
$$Q^{\text{market}} = Q_c + Q_m = 30 - \frac{3}{2}P$$

$$\epsilon = \frac{dQ^{\text{market}}}{dP} \cdot \frac{P}{Q} = -\frac{3}{2} \cdot \frac{P}{30 - \frac{3}{2}P}$$

d. Uncertain. This is like an increase in Richard's wage which has conflicting income and substitution effects.

First longer problem

a. $P_c x_c + P_s x_s \leq m$



b. At optimal bundle, two things will be true:

$$x_s = 2x_c$$

$$P_c x_c + P_s x_s = m$$

Solve these 2 equations:

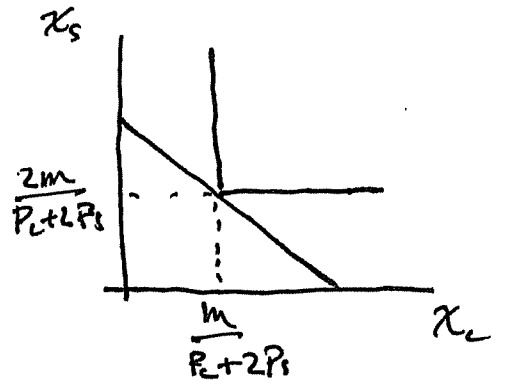
Substitute ~~for~~ for x_s in the budget

$$P_c x_c + P_s (2x_c) = m$$

$$(P_c + 2P_s) x_c = m$$

$$x_c = \frac{m}{P_c + 2P_s}$$

$$x_s = \frac{2m}{P_c + 2P_s}$$



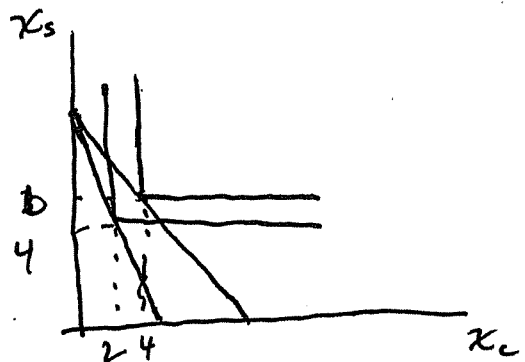
c. $x_c = \frac{300}{50 + 2(25)} = 3$

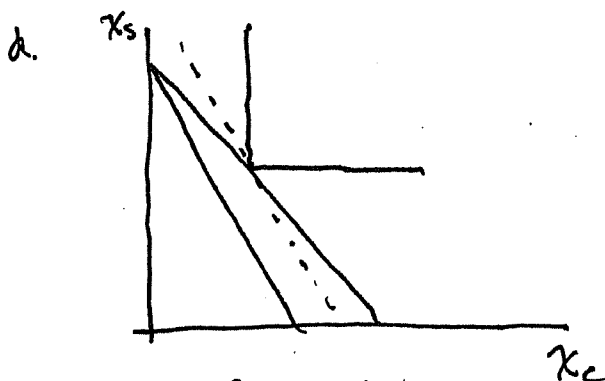
$$x_s = \frac{2(300)}{50 + 2(25)} = 6$$

Price of c ↑ to 100

$$x_c = \frac{300}{100 + 2(25)} = 2$$

$$x_s = \frac{2(300)}{100 + 2(25)} = 4$$





To find sub. and income effects, consider budget line reflecting new prices but that is just tangent to old I_c . We see from the graph that the substitution effect is zero. All of the consumption response is due to the income effect.

Second longer problem

a. ~~CRS~~ Consider increase in all inputs by proportion λ

$$\begin{aligned}
 \text{CRS } f(\lambda x_1, \lambda x_2) &= (\lambda x_1)^{3/4} (\lambda x_2)^{1/4} \\
 &= \lambda^{3/4} x_1^{3/4} \lambda^{1/4} x_2^{1/4} \\
 &= \lambda^{3/4+1/4} x_1^{3/4} x_2^{1/4} \\
 &= \lambda x_1^{3/4} x_2^{1/4} \\
 &= \lambda f(x_1, x_2)
 \end{aligned}$$

b. min $15x_1 + 5x_2$ such that $x_1^{3/4} x_2^{1/4} = y$

Two conditions

$$\begin{aligned}
 \textcircled{1} \quad -\frac{MP_1}{MP_2} &= -\frac{w_1}{w_2} \Rightarrow -\frac{\frac{3}{4} x_1^{-1/4} x_2^{1/4}}{\frac{1}{4} x_1^{3/4} x_2^{-3/4}} = -\frac{15}{5} \Rightarrow x_2 = x_1 \\
 \textcircled{2} \quad x_1^{3/4} x_2^{1/4} &= y
 \end{aligned}$$

$$\hookrightarrow x_2^{3/4} x_2^{1/4} = y$$

$x_2 = y$
 $x_1 = y$
} These are the conditional factor demand functions

$$\begin{aligned}
 c. \quad C(y) &= w_1 x_1(y) + w_2 x_2(y) \\
 &= 15y + 5y \\
 &= 20y
 \end{aligned}$$

Third longer problem

a. At optimum

$$\textcircled{1} \quad -\frac{MU_1}{MU_2} = -\frac{P_1}{P_2} \Rightarrow -\frac{\frac{1}{2}x_1^{-\frac{1}{2}}x_2^{\frac{1}{2}}}{\frac{1}{2}x_1^{\frac{1}{2}}x_2^{-\frac{1}{2}}} = -\frac{P_1}{P_2} \Rightarrow \underbrace{x_2 = \frac{P_1}{P_2}x_1}_{\text{Use in } \textcircled{2}}$$

$$\textcircled{2} \quad P_1 x_1 + P_2 x_2 = Y \quad (Y = \text{income})$$

$$\hookrightarrow P_1 x_1 + P_2 \left(\frac{P_1}{P_2} x_1 \right) = Y$$

$$2P_1 x_1 = Y$$

$$x_1 = \frac{Y}{2P_1}$$

$$\text{From } \textcircled{1} \quad x_2 = \frac{P_1}{P_2} \left(\frac{Y}{2P_1} \right) = \frac{Y}{2P_2}$$

$$b. \quad P_1 = 2, P_2 = 2, Y = 28$$

$$x_1 = \frac{28}{4} = 7$$

$$x_2 = \frac{28}{4} = 7$$

$$c. \quad P_2 = 1, P_1 = 1, Y = 28$$

$$x_1 = \frac{28}{2(1)} = 7$$

$$x_2 = \frac{28}{2(1)} = 14$$

d. Need to find choice at old utility but new prices

$$\textcircled{1} \quad x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = 7$$

$$\textcircled{2} \quad -\frac{MU_1}{MU_2} = -\frac{2}{1}$$

$$\hookrightarrow \frac{x_2}{x_1} = 2$$

$$x_2 = 2x_1 \quad \text{Use this in } \textcircled{1}$$

$$x_1^{\frac{1}{2}}(2x_1)^{\frac{1}{2}} = 7$$

$$\sqrt{2} x_1 = 7$$

$$x_1 = \frac{7}{\sqrt{2}}$$

$$x_2 = \frac{14}{\sqrt{2}}$$

Substitution effect: $\frac{14}{\sqrt{2}} - 7$

Income effect: $14 - \frac{14}{\sqrt{2}}$

