Midterm Examples Solutions

October 14, 2011

Problem 1 Short Problems

a. Suppose there are two goods consumed by Simone, cigarettes and coffee, with prices p_{ci} and p_{co}. Simones income is m. Draw her budget constraint, carefully labeling the slope and intercepts. Show how a quantity tax t on cigarettes affects her budget constraint, and draw this on the graph. Show what has happened to her budget set.

See graph page.

b. True/False/Uncertain. Jacks inverse demand for a good is given by $P = 10 - \frac{1}{2}Q_{jack}$ while Jills demand for the same good is $P = 15 - Q_{jill}$. Jack has elastic demand while Jill has inelastic demand.

Uncertain.

Price elasticity of demand for a linear demand function is

$$\epsilon_{Q,P} = -b\frac{P}{Q}$$

where $\frac{1}{b}$ is equal to the coefficient on the inverse demand curve. Jack's and Jill's respective elasticities of demand are:

$$\epsilon_{jack} = -2\frac{P}{Q}$$

$$\epsilon_{jill} = -\frac{P}{Q}$$

At a price of zero, both curves are perfectly inelastic. At a price of 10, Jack's demand is perfectly elastic and Jill's is elastic. Whether the curves are elastic or inelastic depends on market price.

c. Justins demand for toothpaste and candy bars is as follows: $x_t = \frac{p_c}{p_t}$ and $x_c = \frac{m-p_c}{p_t}$. Suppose the price of both candy bars and toothpaste is 1. Draw the income consumption curve and the Engel curves for both toothpaste and candy.

See graph page.

d. Utility over x and y is given by $U = 4\ln(x) + 2\ln(y)$. Suppose consumption of x and y is given by $(x = 5, y = 10)$. If one unit of x is taken away, approximately how many extra y would this individual need to be given to keep utility constant?

The definition of marginal rate of substitution is the rate at which the consumer will give up one good to get more of another holding utility constant.

$$MRS_{x,y} = \frac{MU_x}{MU_y}$$

$$MU_x = \frac{\delta U}{\delta x} = \frac{4}{x}$$

$$MU_y = \frac{\delta U}{\delta y} = \frac{2}{y}$$

$$MRS_{x,y}(x, y) = \left(\frac{4}{x}\right)\left(\frac{y}{2}\right) = \frac{2y}{x}$$

For $x = 5$ and $y = 10$, this is

$$MRS_{x,y}(5, 10) = \frac{2(10)}{5} = 4$$

e. Suppose Sean and Justin both live in Santa Cruz and both consume tacos and burritos at Taco Bell. They each face the same prices for tacos and burritos, but Sean likes tacos a lot more than Justin, who prefers burritos. True/False/Uncertain. At their optimal consumption levels, the marginal rate of substitution between tacos and burritos will be greater for Sean than Justin.

Uncertain, it depends on prices.

Recall that an optimal point on the budget line is the point that the individual's highest utility indifference curve is tangent to the budget line. Recall also that one interpretation of the MRS is the slope of the tangent line of a utility curve. It then follows that at an optimum the MRS of two goods should equal the slope of the budget line times (-1), which remember is $\frac{p_{tacos}}{p_{burritos}}$. If prices are equal, $MRS = 1$ for both individuals. If prices are not equal, generally their respective MRS are not equal to one. But there exists at least one price ratio at which Sean's marginal rate of subtitution is not greater than Justin's.

Problem 2 Multi-part problem

Whitney is from Texas and therefore consumes only two goods, gasoline and shotgun shells, represented by x_g and x_s. Her preferences for x_g and x_s are given by $U = \ln(x_g) + \ln(x_s)$. Prices for x_g and x_s are given initially by $p_g = 2$ and $p_s = 2$. Her income to allocate across these two goods is given by \$30.

a. Using the marginal rate of substitution and the prices of gasoline and shotgun shells, explain why the bundle $x_g = 5$ and $x_s = 10$, though satisfying her budget constraint, do not maximize Whitneys utility.

$$MRS_{g,s} = \frac{1}{x_g}\frac{x_s}{1} = \frac{x_s}{x_g}$$

For $x_g = 5$ and $x_s = 10$, $MRS = 2$. This means that she is willing to trade up to two gallons of gas for one shotgun shell. Since prices are equal, we know that she would be able to trade one for one in the market. This means that she could buy incrementally more gas and less shotgun ammo and she would value the gas more highly than the loss sustained from having less ammo. So she must not have been optimizing utility.

b. At the optimum, the $MRS = \frac{p_g}{p_s} = 1$ at $p_g = p_s$. So,

$$\frac{x_s}{x_g} = 1 \Longrightarrow x_s = x_g$$

From our budget equation:

$$2x_s + 2x_g = 30 \Longrightarrow 4x_g = 30$$

So $x_g = \frac{30}{4}$. From the symmetry, we can see that $x_s = x_g = \frac{30}{4}$.

c. If the price of shotgun shells increased to 4, what is her new level of consumption of x_g and x_s.

$$MRS = \frac{p_g}{p_s} = \frac{1}{2}$$

$$\frac{x_s}{x_g} = \frac{1}{2} \Longrightarrow x_g = 2x_s$$

Into budget equation:

$$4x_s + 2x_g = 30$$

$$4x_s + 2(2x_s) = 30 \Longrightarrow x_s = \frac{30}{8}$$

So she buys $\frac{30}{8}$ shotgun shells. Plug this back into the budget constraint to find that this implies she buys $\frac{30}{4}$ gallons of gas.

Problem 3 Multi-part Problem

Suppose that two goods are purchased by a consumer, cd players and speakers. These goods are perfect complements, and two speakers are consumed for every cd player. Prices are given by P_c and P_s, and income is given m.

a. What is the inequality describing the budget constraint faced by the consumer? Graph the budget constraint, labeling the axes and the slope of the budget line.

See graph page.

b. Solve the consumer's optimization problem, finding the demand functions $X_c(P_c, P_s, m)$ and $X_s(P_c, P_s, m)$. Graphically represent the indifference curve and budget line corresponding to the optimal bundle.

The utility function associated with the given preferences is:

$$U(C, S) = \min(C, 2S)$$

With utility of this form, we know that an optimizing consumer will choose a quantity of cd players and speakers such that they spend all their money and have exactly one cd player for every two speakers, so $C = 2S$. Their budget constraint is

$$P_cC + P_sS = m$$

Since $C = 2S$, we can solve for the two demand equations:

$$C = \frac{m}{P_c + 2P_s}$$

$$S = \frac{2m}{P_c + 2P_s}$$

c. In this example, are cd players a normal good? Are the ordinary or Giffen? Holding prices

constant, if income increases then C increases, so they are a normal good. If the price of C increasing, holding everything else constant then C decreases. This means that cd players are an ordinary good, not Giffen.

d. Suppose income is given by $m = 300$. The price of a cd player is initially 50 while the price of a speaker is 25. What is the optimal bundle based on what you found in part b?

$$C = \frac{300}{50 + 2(25)} = 3$$

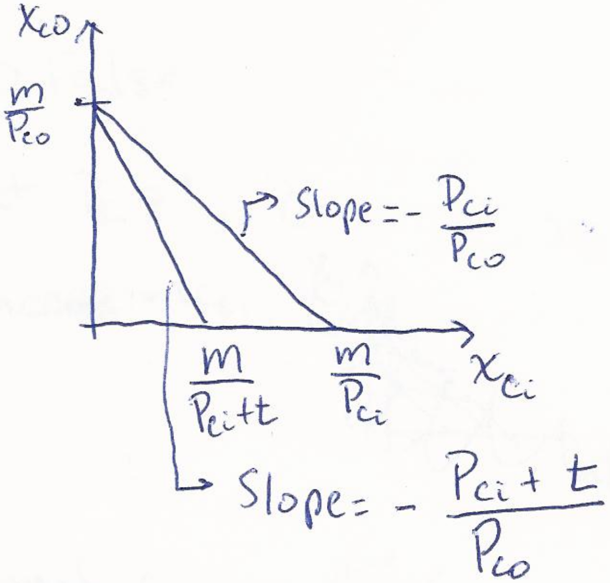
$$S = \frac{2 * 300}{50 + 2(25)} = 6$$

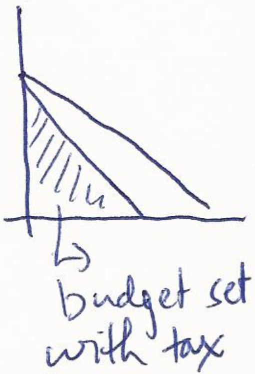
1. Short answer 25 pts (5 points each)

a. $P_{ci} X_{ci} + P_{co} X_{co} \le m$

with quantity tax:

$(P_{ci} + t) X_{ci} + P_{co} X_{co} \le m$





b.

$P_c = P_t = 1$

$X_t = \frac{P_c}{P_t}$ $\quad X_c = \frac{m - P_c}{P_t}$

Income = {10, 20, 30}

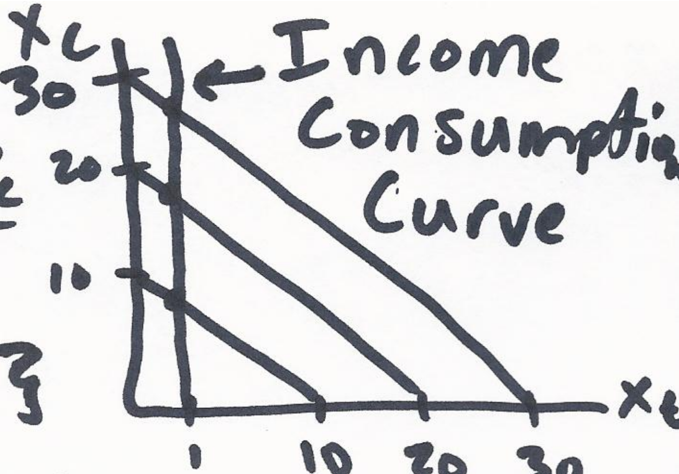
$I = 10 \quad x_t = 1 \quad X_c = \frac{10-1}{1} = 9$

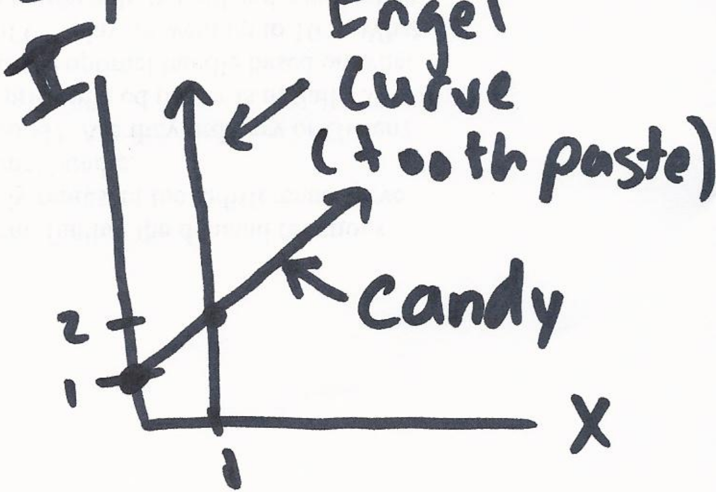
$I = 20 \quad X_t = 1 \quad X_c = 19$

$I = 30 \quad X_t = 1 \quad X_c = 29$

X_t Constant as I ↑

X_c increases as I ↑





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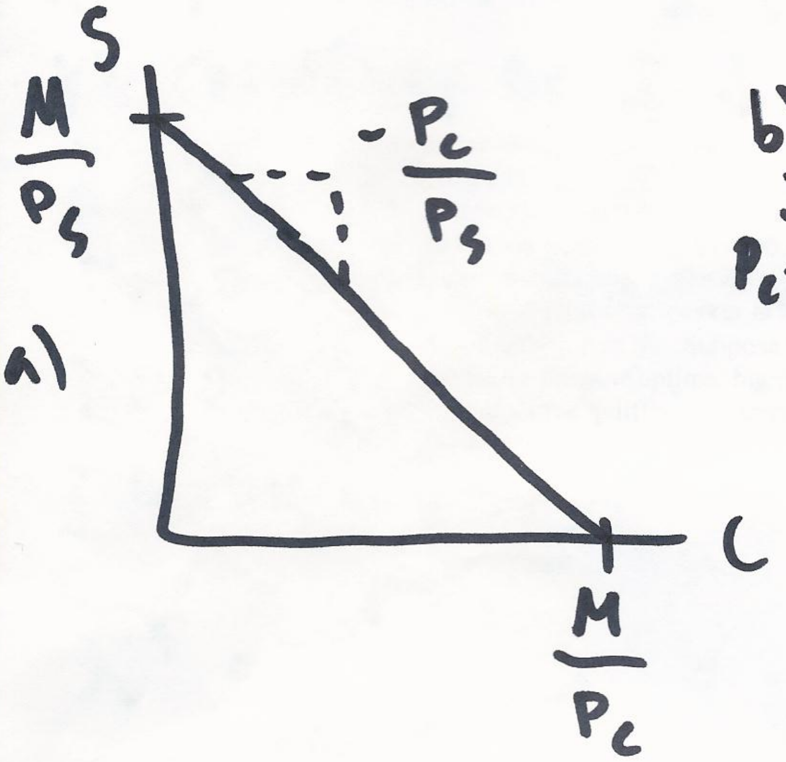

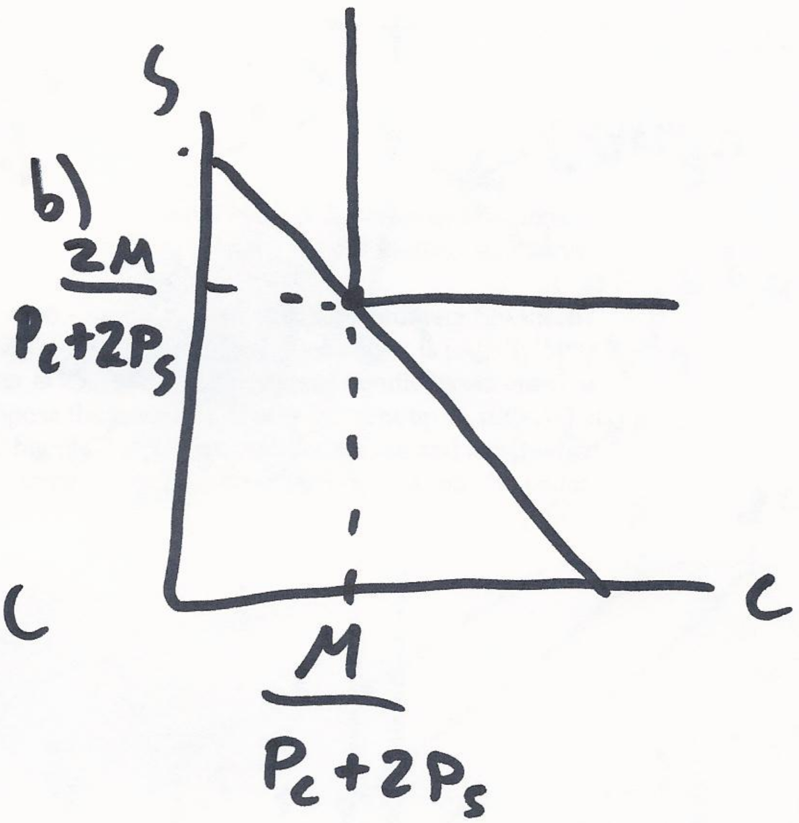

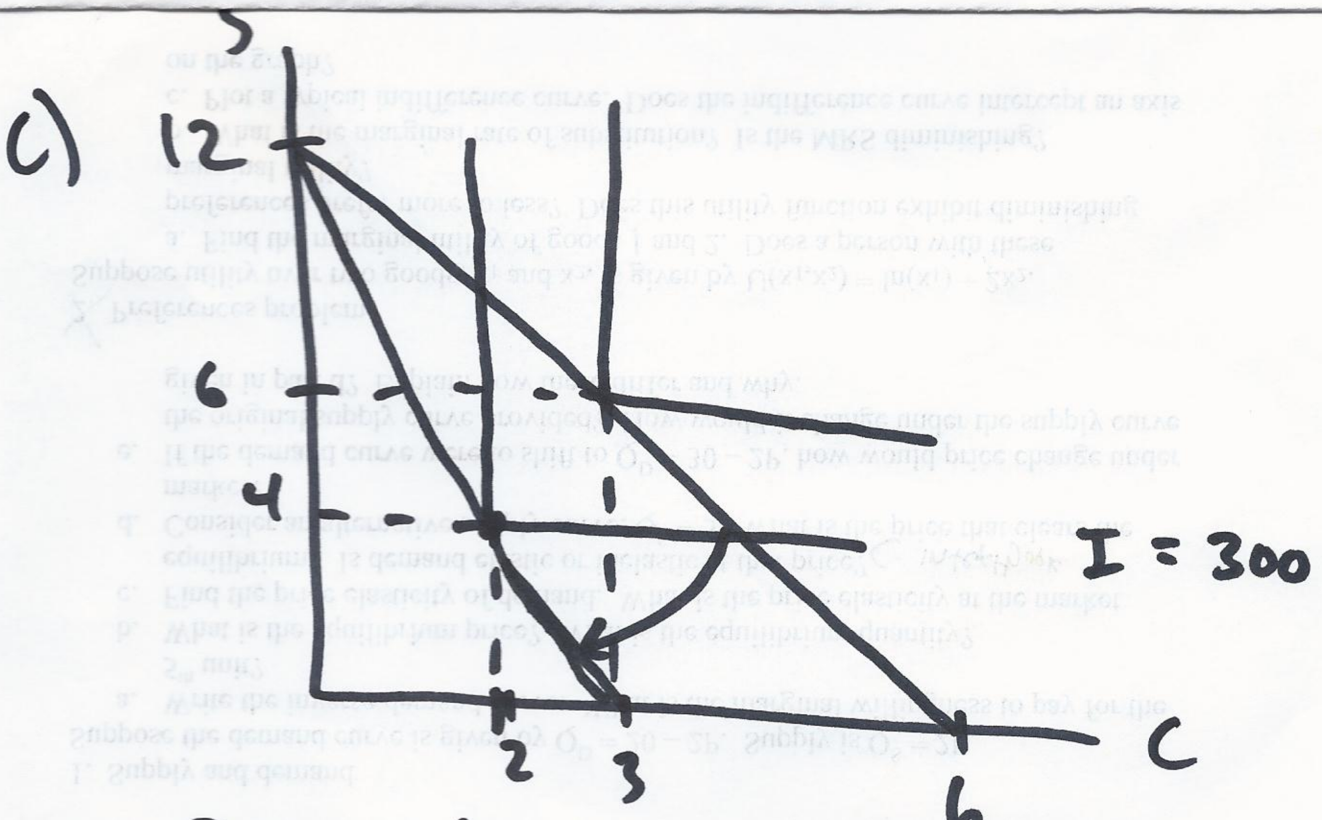

Price ↑ ⇒ Move to lower utility I.C.