Problem 13 Solution Hints

November 6, 2007

This problem asks you to show that the following relationship holds true in all inertial refernce frames:

$$\int \vec{F} \cdot d\vec{l} = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 \tag{1}$$

In other words, is this equation still true in some other, non-accelerating reference frame?

If a reference frame isn't accelerating (e.g. a car that you're riding in), it must be moving with a constant velocity. Let's call that velocity $\vec{u}$.

We'll call $\vec{v}$ the velocity of the biker, as measured by your (stationary) buddy on the sidewalk.

We'll call $\vec{v'}$ the velocity of the biker, as measured by you. Remember, you're riding in a car with constant velocity $\vec{u}$.

From here out, any variable with the prime symbol (') refers to stuff measured in your moving car frame.

So, let's check: is the following equation true?

$$\int \vec{F'} \cdot d\vec{l'} = \frac{1}{2}mv_i'^2 - \frac{1}{2}mv_f'^2 \tag{2}$$

You'll need to use the following relations:

$$\vec{v'} = \vec{v} - \vec{u} \tag{3}$$

and

$$\vec{l'} = \vec{l} + \vec{u}t \tag{4}$$

On the problem set, you will need to show *why* both of these relationships are true. Don't just quote them verbatim! If you're not sure where to start, try plugging in some sample numbers for l, v, t, and u.

From (4), we can find

$$\frac{d\vec{l'}}{dt} = \frac{d\vec{l}}{dt} + u \tag{5}$$

Multiply through by dt to get:

$$d\vec{l'} = d\vec{l} + \vec{u}dt \tag{6}$$

We know that $\vec{F'} = \vec{F}$, from our discussion during office hours. Finally, plug (3) and (6) into (2), and show that the equation is true: i.e., that you end up with the same expression on both sides of the equal sign.

To do this, it may be helpful to take a look at page 176 of your textbook.

Happy calculating!