Solution possibility scribbles

1) These are just possibilities:

a) $f(x) = -x$.
All pts are period 2, except 0, which is a fixed pt. The distances between orbits never changes.

b) $f(x) = 14$.
14 is a fixed pt; all other pts are eventually fixed, arriving at 14 after one application of f.

c) The logistic map $f(x) = ax(1-x)$ has fixed pts at 0 and $1-\frac{1}{a}$; if $1 < a < 3$, 0 is repelling and $1-\frac{1}{a}$ is attracting.

2) The sols -2 and 1 of $f(x) = x$ are the fixed pts; since $|f'(x)| > 1$ at both pts, both are sources.

The sols $-.618..$ and $1.618..$ of $f(f(x)) = x$ that are not sols of $f(x) = x$ form the unique period 2 orbit; since $|f'(-.618..)\, f'(1.618)| > 1$, this orbit is repelling.

The six non-integer sols of $f^3(x) = x$ form two period 3 orbits; we can't tell from the given info which pts belong to which orbit, or the order in which they are visited. Since the three smallest magnitude derivatives at these six pts are $.694.. > \frac{2}{3}$ and $.890.. > 3/4$, the product of magnitudes of derivatives along the orbits must be greater than $\frac{2}{3} \times \frac{3}{4} \times 2 = 1$; hence both orbits are repelling.

Dropping the two fixed pts + one period 2 orbit from the sols. of $f^4(x) = x$, we have 12 pts, which form 3 period 4 orbits. We don't know how to group these points, or anything about the stability of the orbits.

3) These are just some possibilities:

a) A linear map with eigenvalues of magnitude less than 1, e.g. $f(x,y) = (\frac{x}{2}, \frac{y}{3})$.

b) A linear map with eigenvalues of magnitude greater than 1, e.g. $f(x,y) = (2x, 3y)$.

c) A linear map with ~~ee~~ one eigenvalue of magnitude > 1, one of mag < 1, e.g. $f(x,y) = (\frac{x}{2}, 3y)$.

d) A linear map with at least one eigenvalue ~~eq~~ of mag. equal to 1, e.g. $f(\underset{\sim}{x}) = \underset{\sim}{x}$, or a rotation about the origin.

4) a) ~~The orbit of~~ $1/4$ is a period 2 pt, with orbit $\{1/4, 3/4, 1/4, \ldots\}$; since $0 < 1/4 < 1/3$ and $2/3 < 3/4 < 1$, the symbol sequence is $LRLR\ldots\ldots$ (LR repeats forever)

~~The~~ $1/9$ is an eventually fixed pt with orbit $\{1/9, 1/3, 0, 0, \ldots\}$; the symbol sequence is $LCLL\ldots\ldots$ (L repeats forever)

b) $S_0 = R \Rightarrow x \in [2/3, 0)$, and hence ~~so~~ if we ~~write~~ set $\delta := x - 2/3$, $\delta \in [0, 1/3)$, and $f(x) = 3x \mod 1$

$= 3(\frac{2}{3} + \delta) \mod 1$

$= 3\delta$.

Thus $S_1 = L \Rightarrow f(x) \in [0, 1/3)$

$\Rightarrow \delta \in [0, 1/9)$.

$\Rightarrow$ the set of pts with symbol sequences beginning with RL is the interval $[2/3, 2/3 + 1/9) = [2/3, 7/9)$.

5) a) $f(\pm 1) = \mp 1$, so $f(f(\pm 1)) = \pm 1$,

since the upper right hand plot shows (approximately) 1 being mapped to 1 + -1 being mapped to -1, while the upper left hand plot shows these pts being mapped to approximately $\pm 1/2$, the plot on the right must be that of $f \circ f$, + the one on the left must be that of $g \circ g$.

For the cobweb plots, the simplest way to see which is which is to look at the fcn graphs: the one on the left contains (approximately) the pts $(-1,1)$ and $(1,-1)$, which are on the graph of f, but nowhere near the graph of g. Hence the plot on the left is a cobweb plot for f + the one on the right is the cobweb plot for g.

b) Since the domains of f + g are $[-1,1]$, you can read what you need to know about fixed pts + period 2 orbits off the graphs in a): f has a fixed pt at 0 + the period 2 orbit $(1,-1)$; g has a fixed pt at 0 + no period 2 orbits. (The graphs aren't near the diagonal $y = x$ anywhere else.)

From the cobweb plots you could guess that 0 is a source for f + a sink for g, + the period 2 orbit is attracting. Using the "magnitude of derivative" test, we see that in fact $f'(0) = -\pi/2 < -1$, so 0 is a source for f, while $f'(\pm 1) = 0$, so the period 2 orbit is attracting; $g'(0) = -\pi/4$ has mag. less than 1, so 0 is a sink for g.