

Name (please print):

1. 2. 3. 4. 5. Total:

Midterm Math 145 Spring 2008

Instructions: Solve four of the five problems; in the list at the top of this page, circle the numbers of the problems you want graded. You may do the remaining problem for extra credit; please indicate clearly which is your extra credit problem, if any. You *must* justify your answers to receive full credit.

1. Provide an example of a map $f : \mathbb{R} \rightarrow \mathbb{R}$ with the specified properties. (Parts a, b, and c are independent; you need to give a total of three examples.)

(a) f has infinitely many periodic orbits, none of which are attracting or repelling.

(b) All points in \mathbb{R} are either fixed points or eventually fixed points of f , and 14 is an attracting fixed point.

(c) f has exactly two fixed points; one attracting, one repelling.

2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a map such that

The solutions of $f(x) = x$ are -2 and 1 . The derivative f' of f takes the values 4 and -2 at these points.

The solutions of $f(f(x)) = x$ are approximately -2 , -0.618034 , 1 , and 1.61803 . f' takes the approximate values 4 , 1.23607 , -2 , and -3.23607 at these points.

The solutions of $f(f(f(x))) = x$ are approximately -2 , -1.53209 , -1.24698 , -0.347296 , 0.445042 , 1 , 1.80194 , and 1.87939 . f' takes the approximate values 4 , 3.06418 , 2.49396 , 0.694593 , -0.890084 , -2 , -3.60388 , and -3.75877 at these points.

The solutions of $f(f(f(f(x)))) = x$ are approximately -2 , -1.86494 , -1.82709 , -1.47802 , -1.33826 , -0.891477 , -0.618034 , -0.184537 , 0.209057 , 0.547326 , 1 , 1.20527 , 1.61803 , 1.70043 , 1.9563 , and 1.96595 .

Describe as precisely as possible the fixed points of f and periodic orbits of f of period 2, 3, and 4, and their stability. (You can't determine anything about the stability of the period four orbits from the given information.)

Possibly useful: $0.694593 > \frac{2}{3}$ and $0.890084 > \frac{3}{4}$.

3. Give examples of maps $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ for which the origin is

(a) An attracting fixed point (sink).

(b) A repelling fixed point (source).

(c) A saddle point.

(d) None of the above.

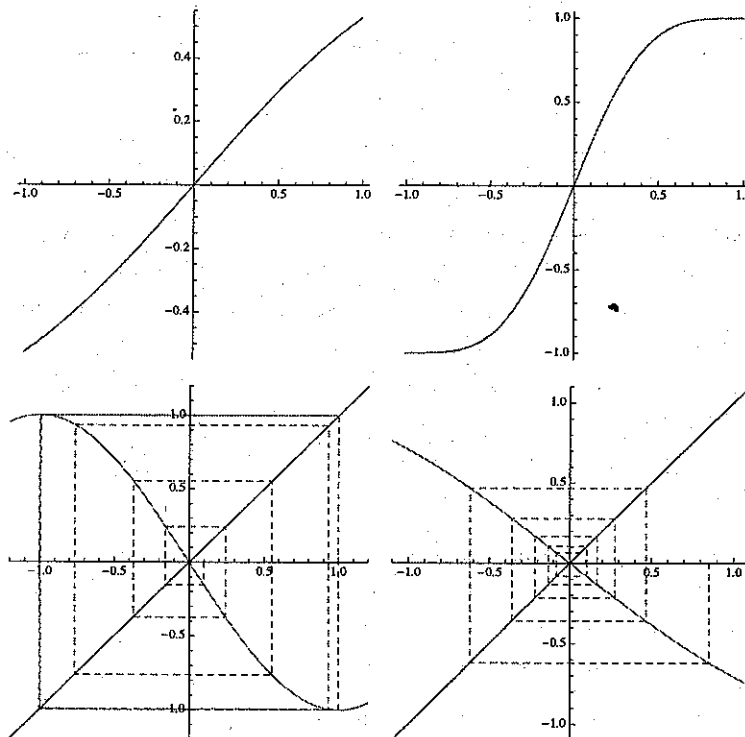
4. Consider the map $f(x) = 3x \bmod 1$ on $[0, 1)$. Assign to each point $x \in [0, 1)$ a symbol sequence $S_0 S_1 S_2 \dots$ as follows:

$$S_j := \begin{cases} L & 0 \leq f^j(x) < \frac{1}{3} \\ C & \frac{1}{3} \leq f^j(x) < \frac{2}{3} \\ R & \frac{2}{3} \leq f^j(x) < 1 \end{cases},$$

where $f^0(x) = x$.

- (a) Determine the symbol sequences for $x = \frac{1}{4}$ and $y = \frac{1}{9}$.
 (b) Determine the set of points with symbol sequences beginning with RL.
5. Let $f(x) = \sin\left(-\frac{\pi}{2}x\right)$ and $g(x) = \sin\left(-\frac{\pi}{4}x\right)$.

- (a) The first pair of plots given below are the plots of $f(f(x))$ and $g(g(x))$ (not necessarily in that order); match the plots to the maps. One of the cobweb plots shown below is generated by f , the other by g (the cobweb plots have different initial conditions); match the plots to the maps.



- (b) Find any fixed points and period two orbits of the maps f and g , and determine their stability.