

## Very disorganized Henon stuff

$$\text{henon}[\{\mathbf{x}_-, \mathbf{y}_-\}] := \{\mathbf{a} - \mathbf{x}^2 + \mathbf{b} \mathbf{y}, \mathbf{x}\}$$

$$\text{Solve}[\text{henon}[\{\mathbf{x}, \mathbf{y}\}] = \{\mathbf{x}, \mathbf{y}\}, \{\mathbf{x}, \mathbf{y}\}]$$

$$\left\{ \left\{ \mathbf{y} \rightarrow \frac{1}{2} \left( -1 + \mathbf{b} - \sqrt{1 + 4 \mathbf{a} - 2 \mathbf{b} + \mathbf{b}^2} \right), \mathbf{x} \rightarrow \frac{1}{2} \left( -1 + \mathbf{b} - \sqrt{1 + 4 \mathbf{a} - 2 \mathbf{b} + \mathbf{b}^2} \right) \right\}, \right. \\ \left. \left\{ \mathbf{y} \rightarrow -\frac{1}{2} + \frac{\mathbf{b}}{2} + \frac{1}{2} \sqrt{1 + 4 \mathbf{a} - 2 \mathbf{b} + \mathbf{b}^2}, \mathbf{x} \rightarrow \frac{1}{2} \left( -1 + \mathbf{b} + \sqrt{1 + 4 \mathbf{a} - 2 \mathbf{b} + \mathbf{b}^2} \right) \right\} \right\}$$

$$\text{henon}[\{\mathbf{x}, \mathbf{x}\}]$$

$$\{\mathbf{a} + \mathbf{b} \mathbf{x} - \mathbf{x}^2, \mathbf{x}\}$$

$$\text{Solve}[\text{First}[\%] = \mathbf{x}, \mathbf{x}]$$

$$\left\{ \left\{ \mathbf{x} \rightarrow \frac{1}{2} \left( -1 + \mathbf{b} - \sqrt{1 + 4 \mathbf{a} - 2 \mathbf{b} + \mathbf{b}^2} \right) \right\}, \left\{ \mathbf{x} \rightarrow \frac{1}{2} \left( -1 + \mathbf{b} + \sqrt{1 + 4 \mathbf{a} - 2 \mathbf{b} + \mathbf{b}^2} \right) \right\} \right\}$$

$$\text{Factor}[\text{First}[\%] - \mathbf{x}]$$

$$\mathbf{a} - \mathbf{x} + \mathbf{b} \mathbf{x} - \mathbf{x}^2$$

$$\text{Solve}[\text{henon}[\text{henon}[\{\mathbf{x}, \mathbf{y}\}]] = \{\mathbf{x}, \mathbf{y}\}, \{\mathbf{x}, \mathbf{y}\}]$$

$$\left\{ \left\{ \mathbf{y} \rightarrow \frac{-1 + 2 \mathbf{b} - \mathbf{b}^2 + \sqrt{-3 + 4 \mathbf{a} + 6 \mathbf{b} - 3 \mathbf{b}^2} - \mathbf{b} \sqrt{-3 + 4 \mathbf{a} + 6 \mathbf{b} - 3 \mathbf{b}^2}}{2 (-1 + \mathbf{b})}, \mathbf{x} \rightarrow \frac{1}{2} \left( 1 - \mathbf{b} + \sqrt{-3 + 4 \mathbf{a} + 6 \mathbf{b} - 3 \mathbf{b}^2} \right) \right\}, \right. \\ \left\{ \mathbf{y} \rightarrow \frac{-1 + 2 \mathbf{b} - \mathbf{b}^2 - \sqrt{-3 + 4 \mathbf{a} + 6 \mathbf{b} - 3 \mathbf{b}^2} + \mathbf{b} \sqrt{-3 + 4 \mathbf{a} + 6 \mathbf{b} - 3 \mathbf{b}^2}}{2 (-1 + \mathbf{b})}, \mathbf{x} \rightarrow \frac{1}{2} \left( 1 - \mathbf{b} - \sqrt{-3 + 4 \mathbf{a} + 6 \mathbf{b} - 3 \mathbf{b}^2} \right) \right\}, \\ \left\{ \mathbf{y} \rightarrow \frac{1 - 2 \mathbf{b} + \mathbf{b}^2 + \sqrt{1 + 4 \mathbf{a} - 2 \mathbf{b} + \mathbf{b}^2} - \mathbf{b} \sqrt{1 + 4 \mathbf{a} - 2 \mathbf{b} + \mathbf{b}^2}}{2 (-1 + \mathbf{b})}, \mathbf{x} \rightarrow \frac{1}{2} \left( -1 + \mathbf{b} - \sqrt{1 + 4 \mathbf{a} - 2 \mathbf{b} + \mathbf{b}^2} \right) \right\}, \\ \left. \left\{ \mathbf{y} \rightarrow \frac{\frac{1}{2} - \mathbf{b} + \frac{\mathbf{b}^2}{2} - \frac{1}{2} \sqrt{1 + 4 \mathbf{a} - 2 \mathbf{b} + \mathbf{b}^2} + \frac{1}{2} \mathbf{b} \sqrt{1 + 4 \mathbf{a} - 2 \mathbf{b} + \mathbf{b}^2}}{-1 + \mathbf{b}}, \mathbf{x} \rightarrow \frac{1}{2} \left( -1 + \mathbf{b} + \sqrt{1 + 4 \mathbf{a} - 2 \mathbf{b} + \mathbf{b}^2} \right) \right\} \right\}$$

$$\text{Factor}[\text{henon}[\text{henon}[\{\mathbf{x}, \mathbf{y}\}]] - \{\mathbf{x}, \mathbf{y}\}]$$

$$\{\mathbf{a} - \mathbf{a}^2 - \mathbf{x} + \mathbf{b} \mathbf{x} + 2 \mathbf{a} \mathbf{x}^2 - \mathbf{x}^4 - 2 \mathbf{a} \mathbf{b} \mathbf{y} + 2 \mathbf{b} \mathbf{x}^2 \mathbf{y} - \mathbf{b}^2 \mathbf{y}^2, \mathbf{a} - \mathbf{x}^2 - \mathbf{y} + \mathbf{b} \mathbf{y}\}$$

$$\text{Solve}[\text{Last}[\%] = 0, \mathbf{y}]$$

$$\left\{ \left\{ \mathbf{y} \rightarrow \frac{-\mathbf{a} + \mathbf{x}^2}{-1 + \mathbf{b}} \right\} \right\}$$

$$\text{Factor}[\text{First}[\%] /. \mathbf{y} \rightarrow \frac{-\mathbf{a} + \mathbf{x}^2}{-1 + \mathbf{b}}]$$

$$-\frac{(-1 + \mathbf{a} + 2 \mathbf{b} - \mathbf{b}^2 + \mathbf{x} - \mathbf{b} \mathbf{x} - \mathbf{x}^2) (\mathbf{a} - \mathbf{x} + \mathbf{b} \mathbf{x} - \mathbf{x}^2)}{(-1 + \mathbf{b})^2}$$

$$\text{Solve}[-1 + \mathbf{a} + 2 \mathbf{b} - \mathbf{b}^2 + \mathbf{x} - \mathbf{b} \mathbf{x} - \mathbf{x}^2 = 0, \mathbf{x}]$$

$$\left\{ \left\{ \mathbf{x} \rightarrow \frac{1}{2} \left( 1 - \mathbf{b} - \sqrt{-3 + 4 \mathbf{a} + 6 \mathbf{b} - 3 \mathbf{b}^2} \right) \right\}, \left\{ \mathbf{x} \rightarrow \frac{1}{2} \left( 1 - \mathbf{b} + \sqrt{-3 + 4 \mathbf{a} + 6 \mathbf{b} - 3 \mathbf{b}^2} \right) \right\} \right\}$$

**Solve** $[-1 + a + 2 b - b^2 + x - b x - x^2 == 0, x]$

**Factor** $[\text{henon}[\{x, x\}] - \{x, x\}]$

$\{a - x + b x - x^2, 0\}$

**Factor** $[\% /. a \rightarrow 3 / 4 (b - 1)^2]$

$$\frac{(-3 + 3 b - 2 x) (-1 + b + 2 x)^3}{16 (-1 + b)^2}$$

**Solve** $[\% == 0, x]$

$\left\{ \left\{ x \rightarrow \frac{1-b}{2} \right\}, \left\{ x \rightarrow \frac{1-b}{2} \right\}, \left\{ x \rightarrow \frac{1-b}{2} \right\}, \left\{ x \rightarrow \frac{3}{2} (-1+b) \right\} \right\}$

**Solve** $[-1 + a + 2 b - b^2 + x - b x - x^2 == 0, x]$

$\left\{ \left\{ x \rightarrow \frac{1}{2} \left( 1 - b - \sqrt{-3 + 4 a + 6 b - 3 b^2} \right) \right\}, \left\{ x \rightarrow \frac{1}{2} \left( 1 - b + \sqrt{-3 + 4 a + 6 b - 3 b^2} \right) \right\} \right\}$

**Collect** $[-1 + a + 2 b - b^2 + x - b x - x^2, x]$

$-1 + a + 2 b - b^2 + (1 - b) x - x^2$

**Factor** $[-3 + 6 b - 3 b^2]$

$-3 (-1 + b)^2$

**Transpose** $[D[\text{henon}[\{x, y\}], \#] \& /@ \{x, y\}]$

$\{\{-2 x, b\}, \{1, 0\}\}$

**Eigenvalues** $[\%]$

$\{-x - \sqrt{b + x^2}, -x + \sqrt{b + x^2}\}$

**Solve** $[\text{henon}[\{x, x\}] == \{x, x\}, x]$

$\left\{ \left\{ x \rightarrow \frac{1}{2} \left( -1 + b - \sqrt{1 + 4 a - 2 b + b^2} \right) \right\}, \left\{ x \rightarrow \frac{1}{2} \left( -1 + b + \sqrt{1 + 4 a - 2 b + b^2} \right) \right\} \right\}$

**Factor** $[\% /. \%]$

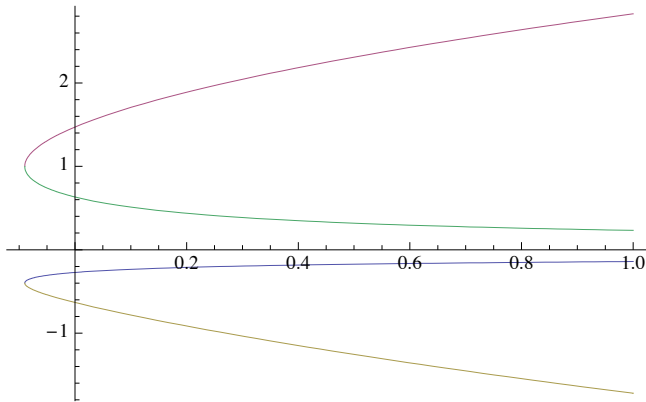
$$\left\{ \frac{1}{2} \left( 1 - b + \sqrt{1 + 4 a - 2 b + b^2} - \sqrt{2} \sqrt{1 + 2 a + b^2 + \sqrt{1 + 4 a - 2 b + b^2} - b \sqrt{1 + 4 a - 2 b + b^2}} \right), \right.$$

$$\left. \frac{1}{2} \left( 1 - b + \sqrt{1 + 4 a - 2 b + b^2} + \sqrt{2} \sqrt{1 + 2 a + b^2 + \sqrt{1 + 4 a - 2 b + b^2} - b \sqrt{1 + 4 a - 2 b + b^2}} \right), \right.$$

$$\left\{ \frac{1}{2} \left( 1 - b - \sqrt{1 + 4 a - 2 b + b^2} - \sqrt{2} \sqrt{1 + 2 a + b^2 - \sqrt{1 + 4 a - 2 b + b^2} + b \sqrt{1 + 4 a - 2 b + b^2}} \right), \right.$$

$$\left. \frac{1}{2} \left( 1 - b - \sqrt{1 + 4 a - 2 b + b^2} + \sqrt{2} \sqrt{1 + 2 a + b^2 - \sqrt{1 + 4 a - 2 b + b^2} + b \sqrt{1 + 4 a - 2 b + b^2}} \right) \right\}$$

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Plot[Evaluate[% /. b -> .4], {a, -.1, 1}]
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Factor[henon[henon[{x, x}]] - {x, x}]
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{-(a - x + b x - x^2) (-1 + a + x + b x - x^2), a - x + b x - x^2}
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