

Some very simple linear algebra examples

```
In[15]:= plotellipse[matrix_, range_] := ParametricPlot[matrix.{Cos[s], Sin[s]},  
  {s, 0, 2 π}, PlotRange → range {{-1, 1}, {-1, 1}}, AspectRatio → 1]
```

First example

```
In[41]:= a = {{1, 2}, {3, 1}}
```

```
Out[41]= {{1, 2}, {3, 1}}
```

```
In[42]:= Eigensystem[a]
```

```
Out[42]= {{{1 + √6, 1 - √6}}, {{-1/3 + 1/3 (1 + √6), 1}, {-1/3 + 1/3 (1 - √6), 1}}}
```

```
In[43]:= N[%]
```

```
Out[43]= {{3.44949, -1.44949}, {{0.816497, 1.}, {-0.816497, 1.}}}
```

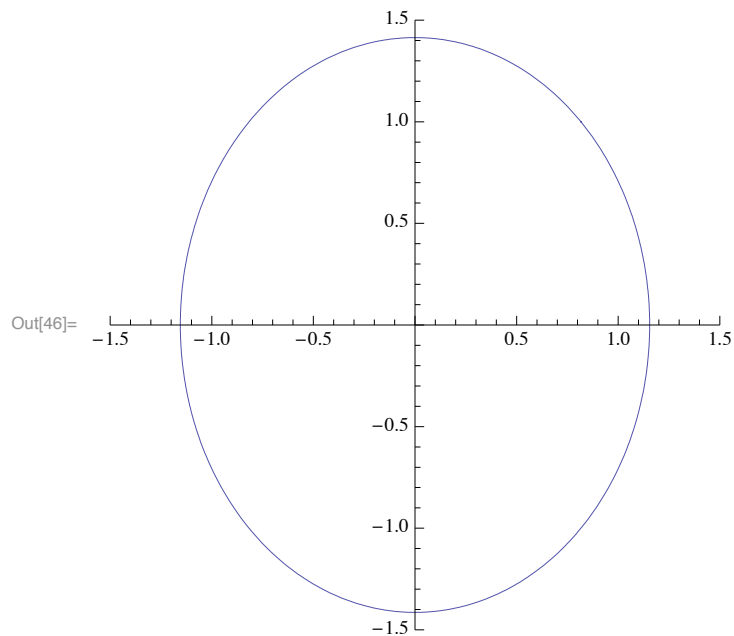
```
In[44]:= u = Transpose[Last[%]]
```

```
Out[44]= {{0.816497, -0.816497}, {1., 1.}}
```

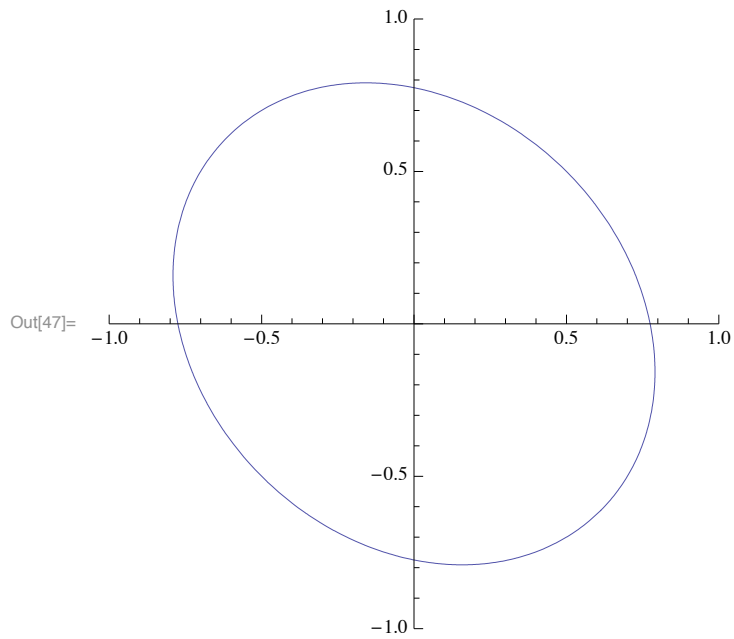
```
In[45]:= Inverse[u].a.u
```

```
Out[45]= {{3.44949, 0.}, {0., -1.44949}}
```

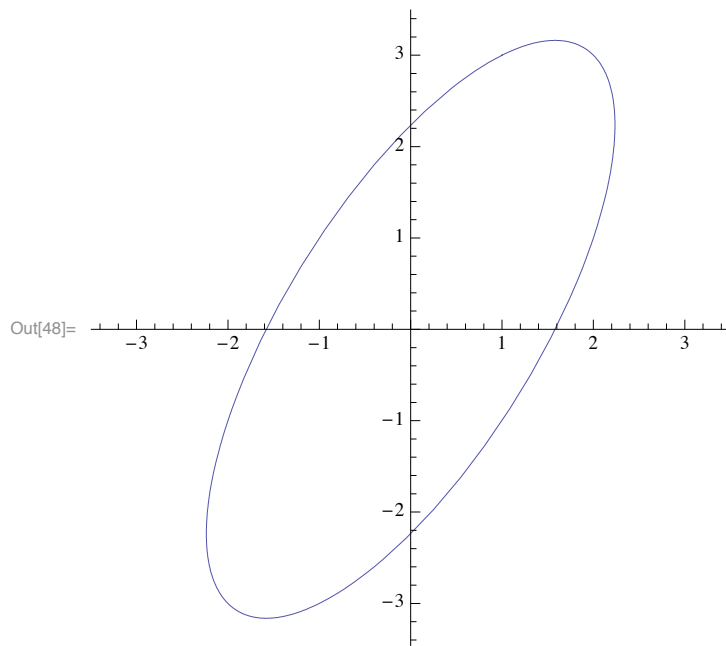
```
In[46]:= plotellipse[u, 1.5]
```



In[47]:= `plotellipse[Inverse[u], 1]`



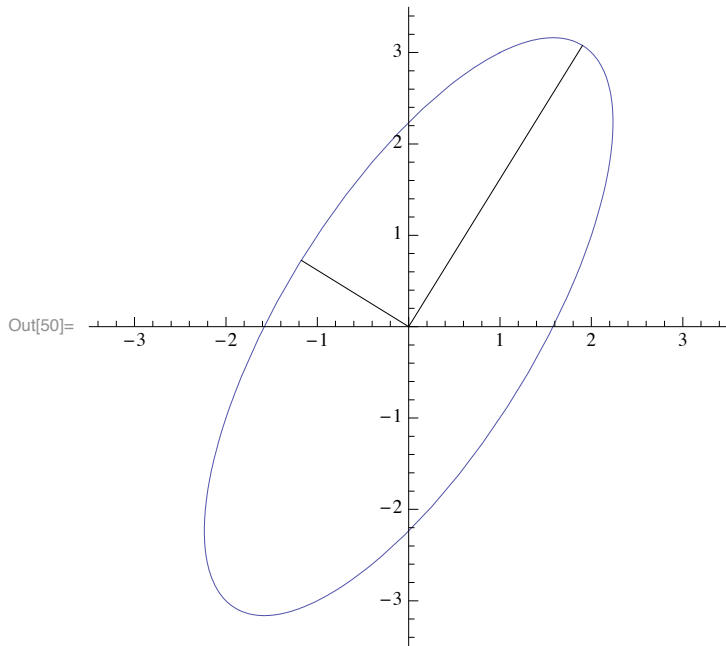
In[48]:= `plotellipse[a, 3.5]`



In[49]:= `Eigensystem[a.Transpose[a]]`

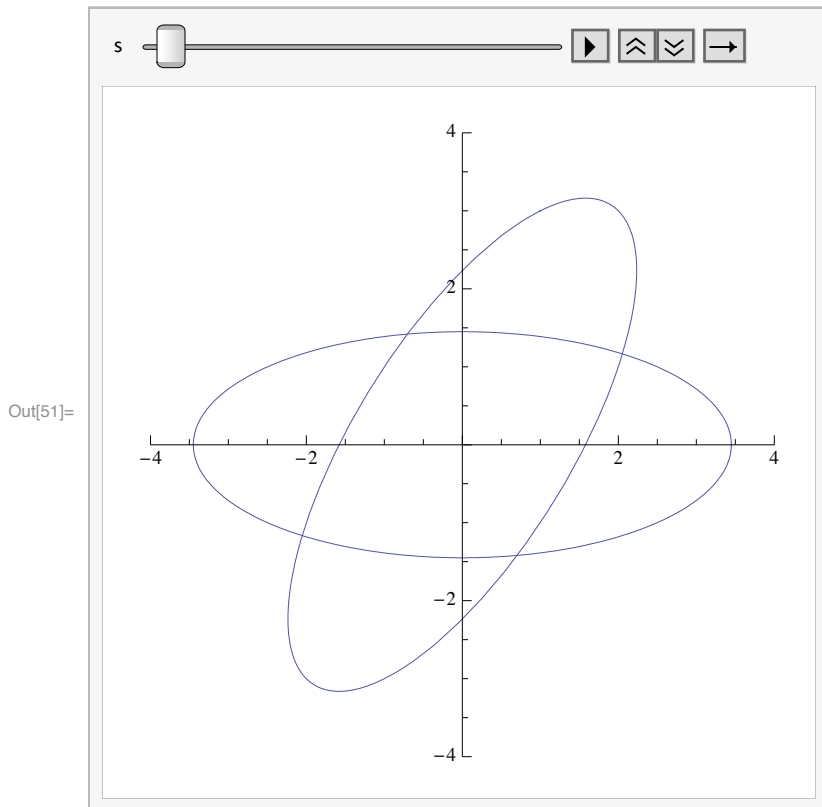
Out[49]= $\left\{ \left\{ \frac{1}{2} (15 + 5\sqrt{5}), \frac{1}{2} (15 - 5\sqrt{5}) \right\}, \left\{ \left\{ -2 + \frac{1}{10} (15 + 5\sqrt{5}), 1 \right\}, \left\{ -2 + \frac{1}{10} (15 - 5\sqrt{5}), 1 \right\} \right\} \right\}$

```
In[50]:= Show[{{%, Graphics[Table[Line[{{0, 0},  $\sqrt{\text{First}[\%][j]}$   $\frac{\#}{\sqrt{\#.\#}}$  &[Last[\%][j]]}], {j, 2}]]]}
```



Is the image of the circle under a just a rotation of its image under $u^{-1} a u$? Check :

```
In[51]:= Animate[Show[plotellipse[Inverse[u].a.u, 4], plotellipse[{{Cos[s], Sin[s]}, {-Sin[s], Cos[s]}.a, 4}], {s, 0,  $\pi/3$ }]
```



No.

```
In[52]:= N[ $\sqrt{\text{Eigenvalues}[\mathbf{a}.\text{Transpose}[\mathbf{a}] ]}$ ]
```

```
Out[52]:= {3.61803, 1.38197}
```

```
In[53]:= N[ $\sqrt{\text{Eigenvalues}[\#\text{.Transpose}[\#] ]}$ ] &[Inverse[u].a.u]
```

```
Out[53]:= {3.44949, 1.44949}
```

Second example

```
In[54]:= b = {{1, -2}, {3, 1}}
```

```
Out[54]:= {{1, -2}, {3, 1}}
```

```
In[55]:= Eigensystem[b]
```

```
Out[55]:=  $\left\{ \left\{ 1 + i\sqrt{6}, 1 - i\sqrt{6} \right\}, \left\{ \left\{ -\frac{1}{3} + \frac{1}{3}(1 + i\sqrt{6}), 1 \right\}, \left\{ -\frac{1}{3} + \frac{1}{3}(1 - i\sqrt{6}), 1 \right\} \right\} \right\}$ 
```

```
In[56]:= N[%]
```

```
Out[56]:= {{1. + 2.44949 i, 1. - 2.44949 i}, {{0. + 0.816497 i, 1.}, {0. - 0.816497 i, 1.}}}
```

```
In[57]:= v = Transpose[{Re[%], Im[%]}] &[%[[2, 1]]]
```

```
Out[57]:= {{0., 0.816497}, {1., 0}}
```

```
In[58]:= Inverse[v].b.v
```

```
Out[58]:= {{1., 2.44949}, {-2.44949, 1.}}
```

```
In[59]:=  $\sqrt{\#\#}$  &[First[%]]
```

```
Out[59]:= 2.64575
```

```
In[60]:=  $\frac{\%\#}{\%}$ 
```

```
Out[60]:= {{0.377964, 0.92582}, {-0.92582, 0.377964}}
```

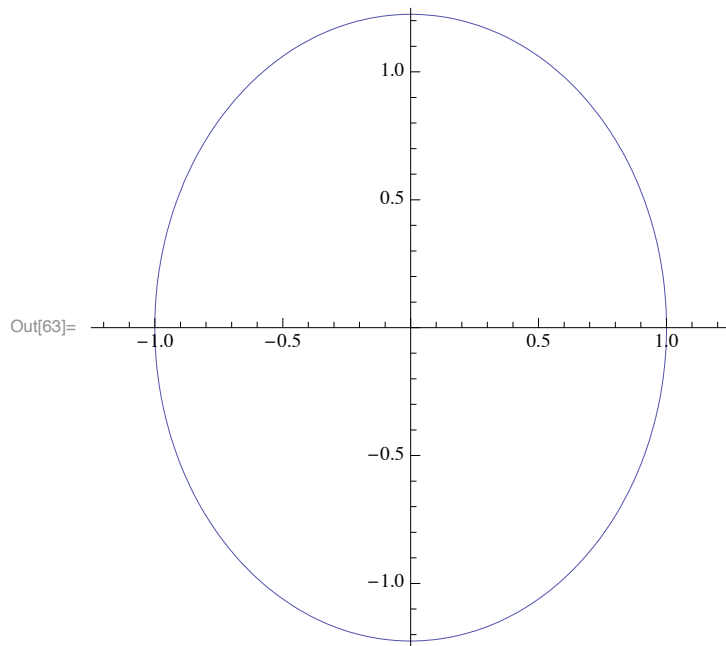
```
In[61]:= ArcCos[%[[1, 1]], ArcSin[%[[2, 1]]]
```

```
Out[61]:= {1.1832, -1.1832}
```

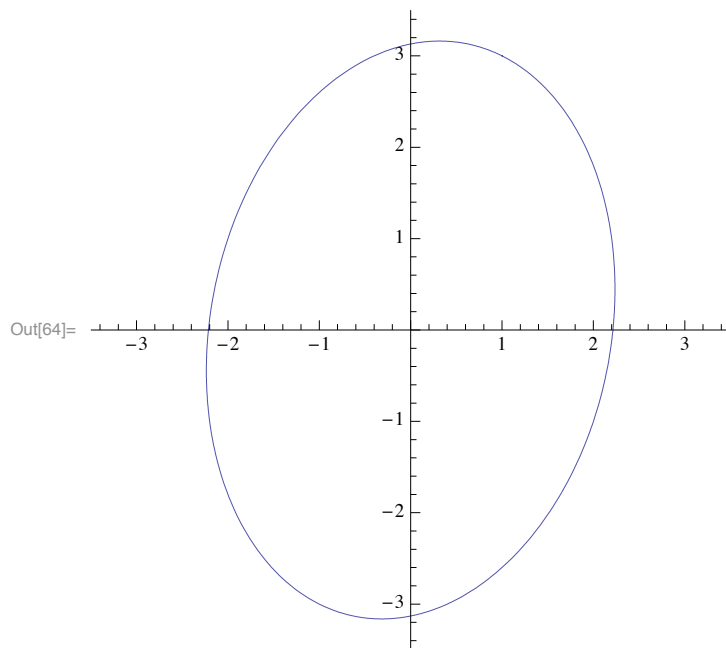
```
In[62]:= {{Cos[%], -Sin[%]}, {Sin[%], Cos[%]}} &[Last[%]]
```

```
Out[62]:= {{0.377964, 0.92582}, {-0.92582, 0.377964}}
```

In[63]:= **plotellipse[Inverse[v], 1.25]**



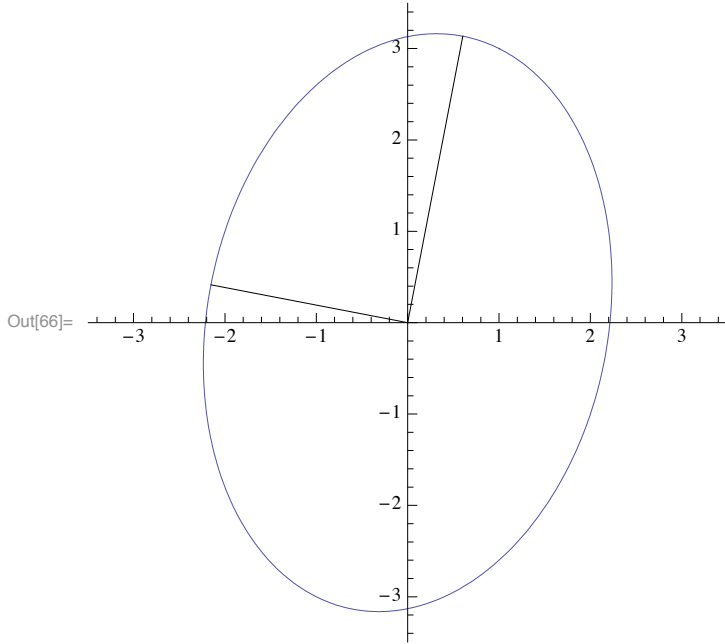
In[64]:= **plotellipse[b, 3.5]**



In[65]:= **Eigensystem[b.Transpose[b]]**

Out[65]= $\left\{ \left\{ \frac{1}{2} (15 + \sqrt{29}), \frac{1}{2} (15 - \sqrt{29}) \right\}, \left\{ \left\{ -10 + \frac{1}{2} (15 + \sqrt{29}), 1 \right\}, \left\{ -10 + \frac{1}{2} (15 - \sqrt{29}), 1 \right\} \right\} \right\}$

```
In[66]:= Show[{{%, Graphics[Table[Line[{{0, 0},  $\sqrt{\text{First}[\%][[j]] \frac{\#}{\sqrt{\#.\#}}$  &[Last[\%][[j]]}], {j, 2}]]}]}
```



```
In[67]:= plotellipse[Inverse[v].b.v, 3.5]
```

