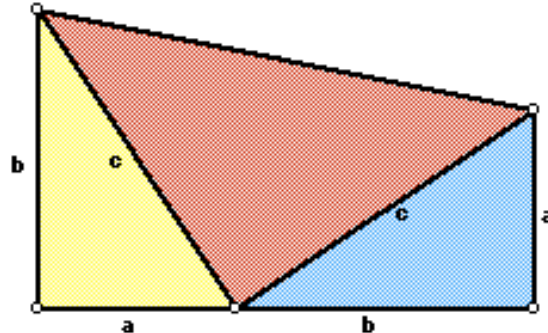


## Practice problems for the midterm, Math 128A, Fall 2011

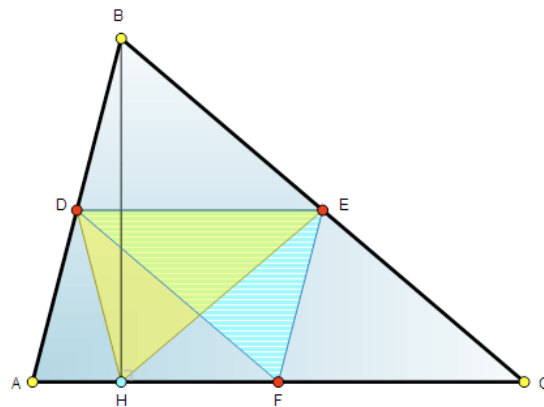
Some of these problems involve more steps than any one exam problem will, but each step is of a comparable level of difficulty and involves similar material.

1. The following figure is used in the proof of the Pythagorean Theorem discovered by James Garfield, 20th president of the US (who worked on the proof while a member of Congress, supposedly with assistance from other members):



Compute the area of this trapezoid two different ways: use this figure, and then use the decomposition of the trapezoid into a rectangle and a triangle to get the standard formula for the area of a trapezoid. Derive the Pythagorean Theorem from the fact that these two approaches must give the same answer. All three triangles are right triangles; assume that  $a \geq b$ , as indicated in the figure.

2. The figure below shows a triangle ABC. If D, E, and F are the midpoints of AB, BC, and AC, respectively, and DEH is the triangle obtained by reflecting BDE across DE, prove that the triangles DEF and DEH are congruent.



3. Are the following definitions of a rectangle equivalent:
  - (a) A quadrilateral with four right angles.
  - (b) A parallelogram with at least one right angle.

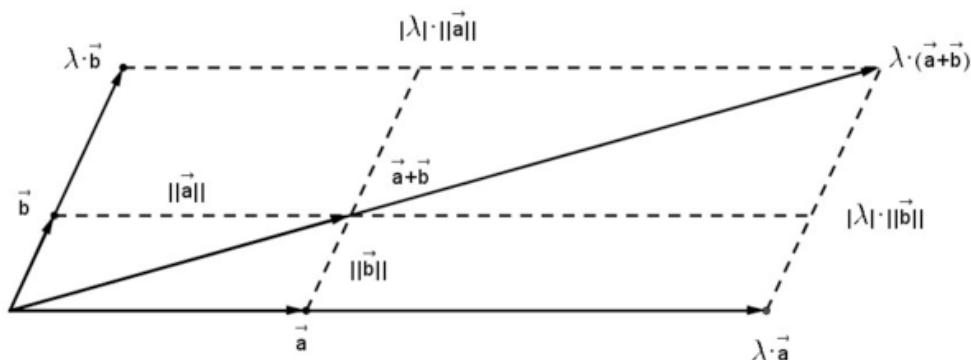
If they are, outline a proof of their equivalence; if not, show how they differ.

4. Explain the relationship between Thales' Theorem and the properties

$$\lambda(\mathbf{a} + \mathbf{b}) = \lambda\mathbf{a} + \lambda\mathbf{b} \quad \text{and} \quad |\lambda\mathbf{a}| = |\lambda| |\mathbf{a}| \quad \text{for all } \lambda \in \mathbb{R} \text{ and } \mathbf{a}, \mathbf{b} \in \mathbb{R}^2$$

of scalar multiplication in the plane.

You may use the figure below in your explanation, if you wish. You can be as formal or informal as you like in your identification of points and vectors.



5. (a) Describe how the figure below, including the dashed lines, can be constructed from the triangle  $ABC$  using a compass and straightedge. You don't have to explain the details of any constructions we've already done in class or homework.
- (b) What are the values of the ratios  $\frac{|GE|}{|BG|}$ ,  $\frac{|DG|}{|GC|}$ , and  $\frac{|GF|}{|AG|}$ ?
- (c) Where are the centroids of the triangle  $ABC$  and the parallelogram  $ACNB$ ?

