

1.3 Functions

- a. Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions)
- b. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems)
- c. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems)

(Mathematics Content Standards for California Public Schools, Grade 6, Algebra and Functions: 1.0; Grade 7, Number Sense: 1.0, 2.0; Algebra and Functions: 3.0; Algebra I: 3.0-6.0, 10.0, 13.0, 15.0-18.0, 21.0-23.0; Algebra II: 1.0-4.0, 6.0-17.0, 24.0, 25.0; Trigonometry: 2.0, 4.0-8.0, 19.0; Mathematical Analysis: 6.0, 7.0; Calculus: 9.0)

1.4 Linear Algebra

- a. Understand and apply the **geometric interpretation and basic operations of vectors** in two and three dimensions, including their **scalar multiples** and **scalar (dot) and cross products**
- b. Prove the basic properties of vectors (e.g., **perpendicular vectors have zero dot product**)
- c. Understand and apply the **basic properties and operations of matrices and determinants** (e.g., to determine the solvability of linear systems of equations)

(Mathematics Content Standards for California Public Schools, Algebra I: 9.0; Algebra II: 2.0; Mathematical Analysis: 1.0; Linear Algebra: 1.0-12.0)

Domain 2. Geometry

Candidates demonstrate an understanding of the foundations of the geometry contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of geometry and its underlying structures, candidates have a deep conceptual knowledge. They demonstrate an understanding of axiomatic systems and different forms of logical arguments. Candidates understand, apply, and prove theorems relating to a variety of topics in two- and three-dimensional geometry, including coordinate, synthetic, **non-Euclidean**, and **transformational geometry**.

2.1 Parallelism

- a. Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)
- b. Know that **variants of the Parallel Postulate produce non-Euclidean geometries** (e.g., **spherical, hyperbolic**)

(Mathematics Content Standards for California Public Schools, Algebra I: 8.0, 24.0; Geometry: 1.0-3.0, 7.0, 13.0)

2.2 Plane Euclidean Geometry

- a. Prove theorems and solve problems involving similarity and congruence

- b. Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, **Triangle Inequality**, Law of Sines, **Law of Cosines**, the **Pythagorean Theorem** and its converse)
- c. Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle)
- d. Justify and perform the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular n-gons for n equal to 3, 4, 5, 6, and 8)
- e. Use techniques in coordinate geometry to prove geometric theorems

(Mathematics Content Standards for California Public Schools, Grade 6, Algebra and Functions: 2.0, 3.0; Measurement and Geometry: 2.0; Grade 7, Measurement and Geometry: 1.0-3.0; Algebra I: 8.0, 24.0; Geometry: 1.0-6.0, 8.0-16.0, 18.0-21.0; Algebra II: 16.0, 17.0; Trigonometry: 12.0-14.0, 18.0, 19.0; Mathematical Analysis: 5.0)

2.3 Three-Dimensional Geometry

- a. Demonstrate an understanding of **parallelism and perpendicularity of lines and planes** in three dimensions
- b. Understand, apply, and justify properties of three-dimensional objects from an advanced standpoint (e.g., derive the volume and surface area formulas for prisms, pyramids, cones, cylinders, and spheres)

(Mathematics Content Standards for California Public Schools, Grade 6, Measurement and Geometry: 1.0; Grade 7, Measurement and Geometry: 2.0; Algebra I: 24.0; Geometry: 2.0, 3.0, 12.0, 17.0; Mathematical Analysis: 5.0)

2.4 Transformational Geometry

- a. Demonstrate an understanding of the **basic properties of isometries** in two- and three-dimensional space (e.g., rotation, translation, reflection)
- b. Understand and prove the **basic properties of dilations** (e.g., similarity transformations or change of scale)

(Mathematics Content Standards for California Public Schools, Geometry: 11.0, 22.0)

Domain 3. Number Theory

Candidates demonstrate an understanding of the number theory and a command of the number sense contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of number theory and its underlying structures, candidates have a deep conceptual knowledge. They prove and use properties of natural numbers. They formulate conjectures about the natural numbers using inductive reasoning, and verify conjectures with proofs.

3.1 Natural Numbers

- a. Prove and use basic properties of natural numbers (e.g., properties of divisibility)
- b. Use the Principle of Mathematical Induction to prove results in number theory
- c. Know and apply the Euclidean Algorithm