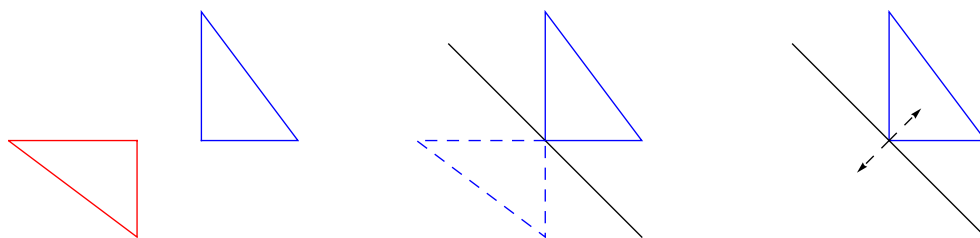


A glide reflection is a combination of a reflection across a line \mathcal{L} and a translation ‘in the direction of \mathcal{L} ’, i.e. mapping \mathcal{L} onto itself. We will call an orientation-reversing isometry of \mathbb{R}^2 a reflection if it fixes all of the points in some line—the line of reflection—and reserve the name glide reflection for an orientation-reversing isometry without fixed points. (Some references consider reflections to be a special case of glide reflections.)

One way to decompose a glide reflection mapping a figure S_1 onto a congruent figure S_2 into a reflection followed by a translation in the appropriate direction is to first identify the family of parallel lines such that reflection across a member of that family maps S_1 onto a figure S'_1 that can be mapped onto S_2 by a translation. You can do this working with whatever member of the family is most convenient (e.g. determining the easiest to visualize reflection of S_1). In the figure below, the left most image is Wei’s example: we need to map the blue triangle onto the red one by a glide reflection. In this example, we need to map a horizontal line segment onto a vertical one, and vice versa; reflection across a ‘downward diagonal’, i.e. a line with slope -1 , maps the blue triangle onto one in the correction position modulo translation (see the middle image below). We now need to translate that diagonal line—I will figure out the appropriate translation algebraically; to reduce the number of variables to track, I’ll only translate in the direction perpendicular to the diagonal, since motion in the direction of the line doesn’t effect the position of the line itself (see the right hand image below).



I’ve chosen coordinates in which the vertices of the blue triangle are at $(0,0)$, $(p,0)$, and $(0,q)$, while those of the red triangle are $(r,0)$, $(r,-p)$, and $(r-q,0)$. Reflection across the downward diagonal through the origin is given by $\rho_0(x,y) := (-y,-x)$. Translations in the direction of the upward diagonal have the form $\tau_a(x,y) := (x+a,y+a)$, $a \in \mathbb{R}$; translations in the direction of the downward diagonal have the form $\tilde{\tau}_b(x,y) := (x+b,y-b)$, $b \in \mathbb{R}$. Reflection across a line with slope -1 is given by $\rho_a := \tau_a \circ \rho_0 \circ \tau_{-a} = \tau_{2a} \circ \rho_0$ for some $a \in \mathbb{R}$. We’re looking for a and b such that the glide reflection $\tilde{\tau}_b \circ \rho_a$ maps the blue triangle onto the red one. We have $\tilde{\tau}_b(\rho_a(x,y)) = (2a+b-y, 2a-b-x)$, so a and b need to satisfy $2a+b=r$ and $2a-b=0$ to map $(0,0)$ onto $(r,0)$; the solution is $a = \frac{r}{4}$ and $b = \frac{r}{2}$, so $\tilde{\tau}_b(\rho_a(x,y)) = (r-y,-x)$. (Obviously, this can also be seen as the reflection ρ_0 followed by horizontal translation by r .) The reflection and translation are shown below.

