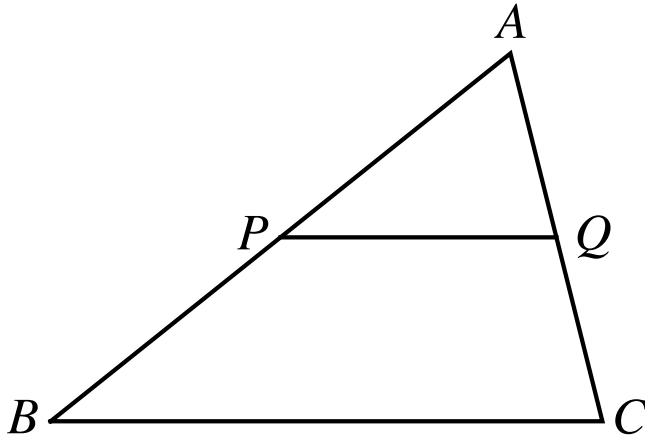


Out[115]=

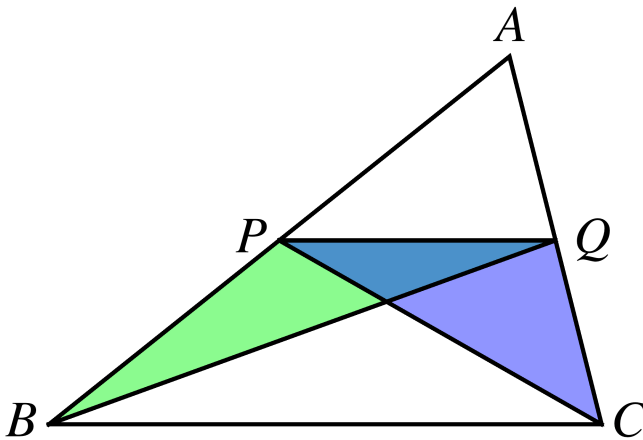


Thales theorem :  $\frac{|AQ|}{|QC|} = \frac{|AP|}{|PB|}$

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Step 1: construct a pair of overlapping triangles of equal area

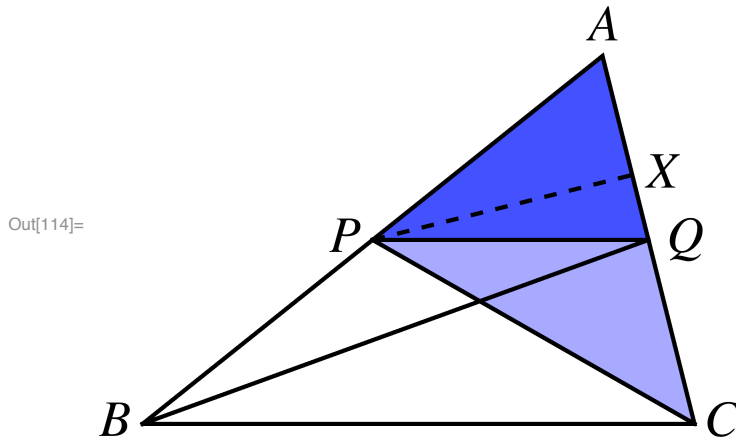
Out[46]=



Common base  $PQ$  and common height implies  
area  $BPQ = \text{area } CPQ$

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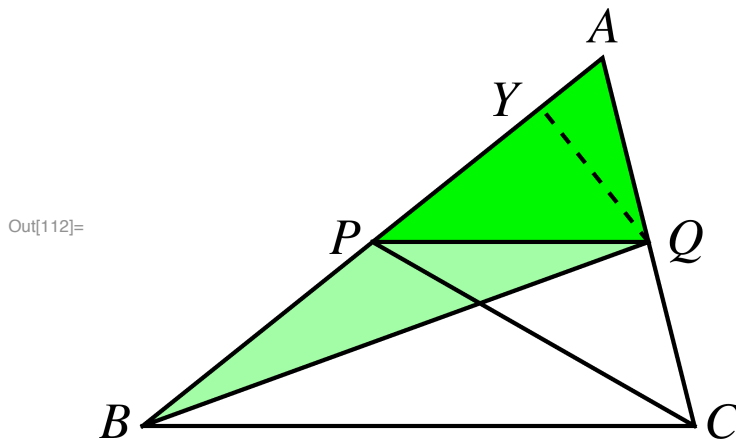
Step 2: relate the area of CPQ to that of APQ



$$\frac{\text{area APQ}}{\text{area CPQ}} = \frac{\frac{1}{2}|AQ||PX|}{\frac{1}{2}|QC||PX|} = \frac{|AQ|}{|QC|}$$

---

Step 3: relate the area of BPQ to that of APQ



$$\frac{\text{area APQ}}{\text{area BPQ}} = \frac{\frac{1}{2}|AP||YQ|}{\frac{1}{2}|PB||YQ|} = \frac{|AP|}{|PB|}$$

---

Step 4: using  $\text{area BPQ} = \text{area CPQ}$ , link these equalities

$$\frac{|AQ|}{|QC|} = \frac{\text{area APQ}}{\text{area CPQ}} = \frac{\text{area APQ}}{\text{area BPQ}} = \frac{|AP|}{|PB|}$$

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Code