

Chapter 9

Interest Rates

Concept Questions

1. Short-term rates have ranged between zero and 14 percent. Long-term rates have fluctuated between about two and 13 percent. Long-term rates, which are less volatile, have historically been in the four-to-five percent range (the 1960 - 1980 experience is the exception). Short-term rates have about the same typical values, but more volatility (and lower rates in the unusual 1930 - 1960 period).
2. A pure discount security is a financial instrument that promises a single fixed payment (the face value) in the future with no other payments in between. Such a security sells at a discount relative to its face value, hence the name. Treasury bills and commercial paper are two examples.
3. The Fed funds rate is set in a very active market by banks borrowing and lending from each other. The discount rate is set by the Fed at whatever level the Fed feels is appropriate. The Fed funds rate changes all the time; the discount rate only changes when the Fed decides; the Fed funds rate is therefore much more volatile. The Fed funds market is much more active. Banks usually borrow from the Fed only as a last resort, which is the primary reason for the Fed's discount rate-based lending.
4. Both are pure discount money market instruments. T-bills, of course, are issued by the government; while commercial paper is issued by corporations. The primary difference is that commercial paper has default risk, so it offers a higher interest rate.
5. LIBOR is the London Interbank Offered Rate. It is the interest rate offered by major London banks for dollar-denominated deposits. Interest rates on loans are often quoted on a LIBOR-plus basis, so the LIBOR is an important, fundamental rate in business lending, among other things.
6. Such rates are much easier to compute by hand; they predate (by hundreds of years or more) computing machinery.
7. They are coupon interest, note principal, and bond principal, respectively. Recalling that each STRIPS represents a particular piece of a Treasury note or bond, these designations tell us which piece is which. A "ci" is one of the many coupon payments on a note or bond; an "np" is the final principal payment on a Treasury note; and a "bp" is the final principal payment on a Treasury bond.
8. We observe nominal rates almost exclusively. Which one is more relevant actually depends on the investor and, more particularly, what the proceeds from the investment will be used for. If the proceeds are needed to make payments that are fixed in nominal terms (like a loan repayment, perhaps), then nominal rates are more important. If the proceeds are needed to purchase real goods (like groceries) and services, then real rates are more important.
9. Trick question! It depends. Municipals have a significant tax advantage, but they also have default risk. Low risk municipals usually have lower rates; higher risk municipals can (and often do) have

higher rates.

B-44 SOLUTIONS

10. AIMR suggested answer:

a. The pure expectations theory states the term structure of interest rates is explained entirely by interest rate expectations. The theory assumes that forward rates of interest embodied in the term structure are unbiased estimates of expected future spot rates of interest. Thus, the pure expectations theory would account for a declining yield curve by arguing that interest rates are expected to fall in the future rather than rise. Investors are indifferent to holding (1) a short-term bond at a higher rate to be rolled over at a lower expected future short-term rate, and (2) a longer term

bond at a rate between the higher short-term rate and the lower expected future short-term rate.

b. Liquidity preference theory (Maturity preference) states that the term structure is a combination of

future interest rate expectations and an uncertainty “risk” or uncertainty yield “premium.” The longer the maturity of a bond, the greater the perceived risk (in terms of fluctuations of value) to the investor, who accordingly prefers to lend short term and thus requires a premium to lend longer term. This yield “premium” is added to the longer-term interest rates to compensate investors for their additional risk. Theoretically, liquidity preference could account for a downward slope if future expected rates were lower than current rates by an amount greater than their respective term risk premium. Liquidity preference theory is consistent with any shape of the term structure but suggests an upward bias or “tilt” to any term structure shape given by unbiased expectations.

c. Market segmentation theory states that the term structure results from different market participants

establishing different yield equilibriums between buyers and seller of funds at different maturity preferences. Market segmentation theory can account for any term structure shape because of the different supply/demand conditions posted at maturity ranges. Borrowers and lenders have preferred maturity ranges, based largely on institutional characteristics, and the yield curve is the average of these different suppliers’ and demanders’ maturity preferences. These maturity preferences are essentially fixed; that is, the participants do not tend to move between or among maturity ranges, so different supply and demand conditions exist across the maturity spectrum. In each maturity range, a higher demand for funds (supply of bonds) relative to the supply of funds will drive bond prices down, and rates up, in that maturity range. A downward sloping yield curve, in the context of market segmentation, indicates that a larger supply of short-term debt relative to demand has led to lower short-term bond prices and/or a small supply of long-term debt relative to demand has led to higher long-term bond prices. Either set of supply/demand conditions works to drive long-term rates lower and short-term rates higher.

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple

steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is

found without rounding during any step in the problem.

Core Question

1. Price = $\$100 / (1 + .056/2)^{2(7)} = \$68.8674 = \$68.8674/\$100 = 68.8674\%$ or 68:28

2. Price = $\$100,000 / (1 + .069/2)^{2(7.5)} = \$60,123.27 = \$60,123.27/\$100,000 = 60.1233\%$ or 60:04

3. YTM = $2 \times [(100 / 78.75)^{1/(2 \times 6)} - 1] = .0402$

CHAPTER 9 B-45

4. YTM = $2 \times [(100 / 54.84375)^{1/(2 \times 10)} - 1] = .0610$

5. $12.2\% - 4.3\% = 7.9\%$

6. $11.4\% - 5.8\% = 5.6\%$

7. $d = .047 = [(\$1M - P)/\$1M](360/84)$; $P = \$989,033.33$

8. $y = [365(.047)]/[360 - (84)(.047)] = 4.818\%$

9. $d = .0385 = [(\$1M - P)/\$1M](360/27)$; $P = \$997,112.50$

10. $y = [365(.0385)]/[360 - (27)(.0385)] = 3.915\%$

Intermediate Questions

11. $99.43 = 100 \times [1 - (35/360) \times DY]$; discount yield = .05863

bond equivalent yield = $[365(.05863)]/[360 - (35)(.05863)] = .05978$

EAR = $[1 + .05978/(365/35)]^{365/35} - 1 = .06143$

12. $d = .0468 = [(\$100 - P)/\$100](360/49)$; $P = 99.36\%$ of par

$y = [366(.0468)]/[360 - (49)(.0468)] = .04789\%$

Note, 2008 is a leap year so there are 366 days used in the calculation of the bond equivalent yield.

13. $1.067 = [1 + (APR)(120/365)]^{365/120}$; APR = bond equivalent yield = 6.555%

discount yield = $[360(.06555)]/[365 + (120)(.06555)] = 6.329\%$

14. Recall that the prices are given as a percentage of par value, and the units after the colon are 32nds

of 1 percent by convention.

Feb07 STRIP: $95.65625 = 100/[1 + (y/2)]^2$; $y = 4.491\%$

Feb08 STRIP: $90.84375 = 100/[1 + (y/2)]^4$; $y = 4.860\%$

Feb09 STRIP: $85.75000 = 100/[1 + (y/2)]^6$; $y = 5.191\%$

Feb10 STRIP: $81.31250 = 100/[1 + (y/2)]^8$; $y = 5.239\%$

Feb11 STRIP: $76.15625 = 100/[1 + (y/2)]^{10}$; $y = 5.523\%$

Feb12 STRIP: $71.56250 = 100/[1 + (y/2)]^{12}$; $y = 5.655\%$

Note that the term structure is upward sloping; the expectations hypothesis then implies that this reflects market expectations of rising interest rates in the future.

15. EAR = $[1 + (.04860/2)]^2 - 1 = 4.919\%$

B-46 SOLUTIONS

16. $[1 + (.04860/2)]^4 = [1 + (.04491/2)]^2 (1 + f_{1,1})$; $f_{1,1} = 5.298\% = \text{EAR}$

$f_{1,1} = 100/[1.05268] = 94.9690\%$ of par = 94:31 rounded to the nearest 32nd.

Note that this price can be found directly from the relationship $f_{t,k} = 100P_{t,k}/P_t$; where the first subscript refers to the time when the forward rate/price begins, the second subscript refers to the length of the forward rate/price, and P represents current or spot prices of various maturities.

Similarly, $f_{t,k} = [(P_t/P_{t+k})]^{1/n} - 1$. Thus:

$f_{1,1} = 100(90.84375/95.65625) = 94.9690$; $f_{1,1} = (95.65625/90.84375) - 1 = 5.298\%$

The implied 1-year forward rate is larger than the current 1-year spot rate, reflecting the expectation

that interest rates will go up in the future. Hence, for upward-sloping term structures, the implied forward rate curve lies above the spot rate curve.

17. $f_{1,5} = 100(71.5625/95.65625) = 74.8122\%$ of par = 74:26 rounded to the nearest 32nd.

$74.8122 = 100/(1 + f_{1,5})^5$; $f_{1,5} = 5.976\%$

$f_{3,2} = 100(76.15625/85.75) = 88.8120\%$ of par = 88:26 rounded to the nearest 32nd.

$88.8120 = 100/(1 + f_{3,2})^2$; $f_{3,2} = 6.112\%$

18. $[1 + (.04860/2)]^4 = [1 + (.04491/2)]^2 (1 + f_{1,1} + .0030)$; $f_{1,1} = 4.998\%$

$f_{1,1} = 100/(1.04998) = 95.2403\%$ of par = 95:08 rounded to the nearest 32nd.

Intuitively, the maturity premium on 2-year investments makes the future 1-year STRIP more valuable; hence, the forward price is greater and the forward rate lower. Alternatively, verify that if

the forward rate and 1-year spot rate stayed the same as before, the spot 2-year price would become

90.5168% of par and the corresponding yield would be 5.108%; i.e., the longer maturity investment would be less valuable.

19. Feb07 STRIPS: $P^* = 100/[1 + (.0491 + .0025) / 2]_2 = 95.4228\%$ of par

$$\Delta\%P = (95.4228 - 95.6563)/95.6563 = -0.244\%$$

Feb09 STRIPS: $P^* = 100/[1 + (.05191 + .0025) / 2]_6 = 85.1258\%$ of par

$$\Delta\%P = (85.1258 - 85.7500)/85.7500 = -0.728\%$$

Feb12 STRIPS: $P^* = 100/[1 + (.05655 + .0025) / 2]_{12} = 70.5268\%$ of par

$$\Delta\%P = (70.5268 - 71.5625)/71.5625 = -1.447\%$$

For equal changes in yield, the longer the maturity, the greater the percentage price change.

Hence,

for parallel yield curve shifts, the price volatility is greater for longer-term instruments.

CHAPTER 9 B-47

Feb07 STRIPS: $95.6563 - .50 = 100/[1 + (y^*/2)]_2$; $y^* = 5.027\%$

$$\Delta y = 5.027 - 4.491 = +0.536\% ; \Delta\% = .536/4.491 = 11.95\%$$

Feb09 STRIPS: $85.7500 - .50 = 100/[1 + (y^*/2)]_6$; $y^* = 5.391\%$

$$\Delta y = 5.391 - 5.191 = +0.500\% ; \Delta\% = .500/5.191 = 3.85\%$$

Feb12 STRIPS: $71.5625 - .50 = 100/[1 + (y^*/2)]_{12}$; $y^* = 5.775\%$

$$\Delta y = 5.775 - 5.655 = +0.120\% ; \Delta\% = .120/5.655 = 2.13\%$$

For equal changes in price, the absolute yield volatility is greater the shorter the maturity; the effect

is magnified for percentage yield volatility when the yield curve is upward sloping, because yields

(the divisor) are smaller for short maturities. Because of this, note that for sharply downward sloping

yield curves, it's possible for shorter maturity instruments to have less percentage yield volatility, but greater absolute yield volatility, than slightly longer maturity instruments.

20. Approximate real rate = $4.24\% - 3.50\% = 0.74\%$

Real interest rates are not observable because they do not correspond to any traded asset (at least not

until very recently in the U.S.); hence, they must be inferred from nominal interest rates (which do

correspond to traded assets), and from estimated inflation data. Real interest rate estimates are therefore only as good as (1) the inflation estimates used in the Fisher relation and (2) the degree to

which the Fisher relation itself actually describes the behavior of economic agents.

$$\mathbf{21.} f_{1,1} = (1.0572/1.049)^{1/1} - 1 = 6.51\%$$

$$f_{1,2} = (1.0643/1.049)^{1/2} - 1 = 7.16\%$$

$$f_{1,3} = (1.0714/1.049)^{1/3} - 1 = 7.84\%$$

$$\mathbf{22.} f_{2,1} = 1.0643/1.0572 - 1 = 7.81\%$$

$$f_{3,1} = 1.0714/1.0643 - 1 = 9.23\%$$

$$\mathbf{23.} I_1 = r_1 - 2\% = 4.90\% - 2\% = 2.90\%$$

$$I_2 = f_{1,1} - 2\% = 6.51\% - 2\% = 4.51\%$$

$$I_3 = f_{2,1} - 2\% = 7.81\% - 2\% = 5.81\%$$

$$I_4 = f_{3,1} - 2\% = 9.23\% - 2\% = 7.23\%$$

B-48 SOLUTIONS

Spreadsheet Problems

Chapter 11

Diversification and Risky Asset Allocation

Concept Questions

1. Based on market history, the average annual standard deviation of return for a single, randomly chosen stock is about 50 percent. The average annual standard deviation for an equally-weighted portfolio of many stocks is about 20 percent.
2. If the returns on two stocks are highly correlated, they have a strong tendency to move up and down together. If they have no correlation, there is no particular connection between the two. If they are negatively correlated, they tend to move in opposite directions.
3. An efficient portfolio is one that has the highest return for its level of risk.
4. True. Remember, portfolio return is a weighted average of individual returns.
5. False. Remember the principle of diversification.
6. Because of the effects of diversification, an investor will never receive the highest return possible from a single asset. However, the investor will also never receive the lowest return. More importantly, even though an investor does give up the potential “home run” investment, the reduction in return is more than offset by the reduction in risk. In other words, you give up a little return for a lot less risk.
7. You know your current portfolio is the minimum variance portfolio (or below). Below and to the right of the minimum variance portfolio, as you add more of the lower risk asset, the standard deviation of your portfolio increases and the expected return decreases.
8. The importance of the minimum variance portfolio is that it determines the lower bound of the efficient frontier. While there are portfolios on the investment opportunity set to the right and below the minimum variance portfolio, they are inefficient. That is, there is a portfolio with the same level of risk and a higher return. No rational investor would ever invest in a portfolio below the minimum variance portfolio.
9. False. Individual assets can lie on the efficient frontier depending on its expected return, standard deviation, and correlation with all other assets.
10. If two assets have zero correlation and the same standard deviation, then evaluating the general expression for the minimum variance portfolio shows that $x = 1/2$; in other words, an equally weighted portfolio is minimum variance.

B-58 SOLUTIONS

Solutions to Questions and Problems

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Core Questions

1. $.2(-.10) + .6(.14) + .2(.27) = 11.80\%$
2. $.2(-.10 - .1180)^2 + .4(.14 - .1180)^2 + .2(.27 - .1180)^2 = .01442$; $\sigma = 12.01\%$
3. $(1/3)(-.10) + (1/3)(.14) + (1/3)(.27) = 10.33\%$
 $(1/3)(-.10 - .1033)^2 + (1/3)(.14 - .1033)^2 + (1/3)(.27 - .1033)^2 = .02349$; $\sigma = 15.33\%$
- 4.

Calculating Expected Returns

Roll Ross

(1)

State of
Economy

(2)

Probability of State
of Economy

(3)

Return if
State

Occurs

(4)

Product

(2) × (3)

(5)

Return if
State

Occurs

(6)

Product

(2) × (5)

Bust .40 -10% -.0400 21% .0840

Boom .60 28% .1680 8% .0480

$E(R) = 12.80\%$ $E(R) = 13.20\%$

5.

(1)

State
of Economy

(2)

Probability of
State of

Economy

(3)

Return Deviation
from Expected

Return

(4)

Squared

Return

Deviation

(5)

Product

(2) × (4)

Roll

Bust .40 -.2280 .0520 .0208

Boom .60 .1520 .0231 .0139

$\sigma^2 = .0347$

Ross

Bust .40 .0780 .0061 .0024

Boom .60 -.0520 .0027 .0016

$$\sigma_2 = .0041$$

Taking square roots, the standard deviations are 18.62% and 6.37%.

CHAPTER 11 B-59

6.

Expected Portfolio Return

(1)

State of

Economy

(2)

Probability of

State of

Economy

(3)

Portfolio Return if State Occurs

(4)

Product

(2) \times (3)

Bust

.40

$$.40(-10\%) + .60(21\%) = 9\%$$

.0344

Boom

.60

$$.40(28\%) + .60(8\%) = 16\%$$

.0960

$E(R_p) =$

13.04%

7.

Calculating Portfolio Variance

(1)

State of

Economy

(2)

Probability of

State of Economy

(3)

Portfolio Return

if State Occurs

(4)

Squared Deviation from

Expected Return

(5)

Product

(2) \times (4)

Bust

.40

-.01

.0186

.00742

Boom

.60

.22
.0082
.00495

σ_2

$\rho =$

.01237

$\sigma_P =$

11.12%

8. $E[R_A] = .15(.04) + .70(.09) + .15(.12) = 8.70\%$

$E[R_B] = .15(-.20) + .70(.13) + .15(.33) = 11.05\%$

σ_A

$\sigma_A^2 = .15(.04 - .0870)^2 + .70(.09 - .0870)^2 + .15(.12 - .0870)^2 = .000501$; $\sigma_A = [.000501]^{1/2} = .0224$

σ_B

$\sigma_B^2 = .15(-.2 - .1105)^2 + .70(.13 - .1105)^2 + .15(.33 - .1105)^2 = .021955$; $\sigma_B = [.021955]^{1/2} = .1482$

9. a. boom: $E[R_p] = .25(.30) + .50(.45) + .25(.33) = .3825$

good: $E[R_p] = .25(.12) + .50(.10) + .25(.15) = .1175$

poor: $E[R_p] = .25(.01) + .50(-.15) + .25(-.05) = -.0850$

bust: $E[R_p] = .25(-.06) + .50(-.30) + .25(-.09) = -.1875$

$E[R_p] = .20(.3825) + .40(.1175) + .30(-.0850) + .10(-.1875) = .0793$

b. σ_p

$\sigma_p^2 = .20(.3825 - .0793)^2 + .40(.1175 - .0793)^2 + .30(-.0850 - .0793)^2 + .10(-.1875 - .0793)^2$

σ_p

$\sigma_p = .03419$; $\sigma_p = [.03419]^{1/2} = .1849$

B-60 SOLUTIONS

10. Notice that we have historical information here, so we calculate the sample average and sample

standard deviation (using $n - 1$) just like we did in Chapter 1. Notice also that the portfolio has less

risk than either asset.

Annual Returns on Stocks A and B

Year

Stock A

Stock B Portfolio AB

2001

18%

50%

40.40%

2002

40

-30

-9.00

2003

-15

45

27.00

2004

20

2

7.40

2005

4

20

15.20

Avg return 13.40 17.40 16.20

Std deviation 20.42 32.85 18.81

Intermediate Questions

11. Boom: $.20(15\%) + .35(18\%) + .45(20\%) = 18.30\%$

Bust: $.20(10\%) + .35(0\%) + .45(-10\%) = -2.50\%$

$E(R_P) = .70(.1830) + .30(-.0250) = 12.06\%$

$\sigma^2 = .70(.1830 - .1206)^2 + .30(-.0250 - .1206)^2 = .00909$; $\sigma_P = 9.53\%$

12. $E(R_P) = .40(.15) + .60(.10) = 12.00\%$

$\sigma^2 = .40_2(.50_2) + .60_2(.38_2) + 2(.40)(.60)(.50)(.38)(.15) = .10566$; $\sigma_P = 32.51\%$

13. ₂

$\sigma^2 = .40_2(.50_2) + .60_2(.38_2) + 2(.40)(.60)(.50)(.38)(1.0) = .18318$; $\sigma_P = 42.80\%$

$\sigma^2 = .40_2(.50_2) + .60_2(.38_2) + 2(.40)(.60)(.50)(.38)(0.0) = .09198$; $\sigma_P = 30.33\%$

$\sigma^2 = .40_2(.50_2) + .60_2(.38_2) + 2(.40)(.60)(.50)(.38)(-1.0) = .00078$; $\sigma_P = 2.80\%$

As the correlation becomes smaller, the standard deviation of the portfolio decreases. In the extreme

with a correlation of -1 , this means that as one asset has a higher than expected return, the other asset has a lower than expected return. The extra returns, whether positive or negative, will offset each other resulting in smoother portfolio return with less variance.

14. $w_{3 \text{ Doors}} =$

$.50 \ .38 - 2 \ .50 \ .38 \ .20$

$.38 - .60 \ .38 \ .20$

_{2 2}

₂

$+ \times \times \times$

$\times \times$

.

$= 0.3435$; $w_{\text{Doors}} = (1 - .3435) = .6565$

$E(R_P) = .3435(.15) + .6565(.10) = 11.72\%$

₂

$\sigma^2 = .3435_2(.50_2) + .6565_2(.38_2) + 2(.3435)(.6565)(.50)(.38)(.20) = .10459$

$\sigma_P = 32.34\%$

CHAPTER 11 B-61

15.

Risk and Return with Stocks and Bonds

Portfolio Weights

Expected

Standard

Stocks

Bonds

Return

Deviation

1.00

0.00 12.00% 21.00%

0.80

0.20 10.80% 17.16%

0.60

0.40 9.60% 13.61%

0.40
 0.60 8.40% 10.65%
 0.20
 0.80 7.20% 8.86%
 0.00
 1.00 6.00% 9.00%

16. $w_D =$

$$\frac{.54 \cdot .78 - 2 \cdot .54 \cdot .78 \cdot .10}{.78^2 - .54 \cdot .78 \cdot .10 + .10^2}$$

$= 0.6609$; $w_1 = (1 - .6609) = .3391$
17. $E(R_p) = .6609(.13) + .3391(.16) = 14.02\%$

$$\sigma_p = .6609^2(.54^2) + .3391^2(.78^2) + 2(.6609)(.3391)(.54)(.78)(-.10) = .17845$$

 $\sigma_p = 42.24\%$

18. $w_K =$

$$\frac{.53 \cdot .10 - 2 \cdot .53 \cdot .10 \cdot .02}{.10^2 - .53 \cdot .10 \cdot .02 + .02^2}$$

$= 0.0310$; $w_L = (1 - .0310) = .9690$
 $E(R_p) = .0310(.15) + .9690(.06) = 6.28\%$

$$\sigma_p = .0310^2(.53^2) + .9690^2(.10^2) + 2(.0310)(.9690)(.53)(.10)(.02) = .00972$$

 $\sigma_p = 9.86\%$

19. $w_{Bruin} =$

$$\frac{.43 \cdot .52 - 2 \cdot .43 \cdot .52 \cdot .25}{.52^2 - .43 \cdot .52 \cdot .25 + .25^2}$$

$= 0.6245$; $w_{Wildcat} = (1 - .6245) = .3755$
 $E(R_p) = .6245(.17) + .3755(.15) = 16.25\%$

$$\sigma_p = .6245^2(.43^2) + .3755^2(.52^2) + 2(.6245)(.3755)(.43)(.52)(.25) = .13645$$

 $\sigma_p = 36.94\%$

20. $E(R) = .30(12\%) + .50(16\%) + .20(13\%) = 14.20\%$

$$\sigma_p = .30^2(.41^2) + .50^2(.58^2) + .20^2(.48^2) + 2(.30)(.50)(.53)(.41)(.58) + 2(.30)(.20)(.41)(.48)(.20) + 2(.50)(.20)(.58)(.48)(.05) = 0.13735$$

 $\sigma_p = 37.06\%$

B-62 SOLUTIONS

21. $w_J =$

$$\frac{.58 \cdot .24 - 2 \cdot .58 \cdot .24 \cdot .60}{.24^2 - .58 \cdot .24 \cdot .60 + .60^2}$$

$$+ x \times x$$

$$x \times$$

$$= -0.1142; w_S = (1 - (-.1142)) = 1.1142$$

$$\sigma_P^2 = (-.1142)^2(.58^2) + (1.1142)^2(.24^2) + 2(.58)(.24)(-.1142)(1.1142)(.60) = .05464$$

$$\sigma_P = (.05464)^{1/2} = 23.38\%$$

$$E(R_P) = -.1142(.15) + 1.1142(.06) = 8.31\%$$

Even though it is possible to mathematically calculate the standard deviation and expected return of

a portfolio with a negative weight, an explicit assumption is that no asset can have a negative weight.

The reason this portfolio has a negative weight in one asset is the relatively high correlation between

the two assets. If you look at the investment opportunity sets in the chapter, you will notice that as the correlation decreases, the investment opportunity set bends further backwards. However, for a portfolio with a correlation of +1, there is no minimum variance portfolio with a variance lower than

the lowest variance asset. This implies there is some necessary level of correlation to make the minimum variance portfolio have a variance lower than the lowest variance asset. The formula to determine if there is a minimum variance portfolio with a variance less than the lowest variance asset

is:

$$\frac{\max}{\min}$$

$$\sigma$$

$$\sigma$$

> ρ . In this case,

$$.58$$

.24 = .414 < .60 so there is no minimum variance portfolio with a variance lower than the lowest variance asset assuming non-negative asset weights.

22. Look at 2

$$\sigma_P^2 :$$

$$2$$

$$\sigma_P^2 = (x_A \times \sigma_A + x_B \times \sigma_B)^2$$

$$= x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B \sigma_A \sigma_B \rho$$

$$2$$

$$B$$

$$2$$

$$B$$

$$2$$

$$A$$

$$2$$

$x_A \times \sigma_A + x_B \times \sigma_B + 2 x_A x_B \sigma_A \sigma_B \rho$, which is precisely the expression for the

variance on a two-asset portfolio when the correlation is +1.

23. Look at 2

$$\sigma_P^2 :$$

$$2$$

$$\sigma_P^2 = (x_A \times \sigma_A - x_B \times \sigma_B)^2$$

$$= x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 - 2 x_A x_B \sigma_A \sigma_B (-1)$$

$$2$$

$$B$$

$$2$$

$$B$$

$$2$$

$$A$$

$$2$$

$x_A \times \sigma_A + x_B \times \sigma_B + 2 \times x_A \times x_B \times \sigma_A \times \sigma_B \times \rho$, which is precisely the expression for the variance on a two-asset portfolio when the correlation is -1 .

24. From the previous question, with a correlation of -1 :

$$\sigma_p = x_A \times \sigma_A - x_B \times \sigma_B$$

$$= x \times \sigma_A - (1 - x) \times \sigma_B$$

Set this to equal zero and solve for x to get:

$$0 = x \times \sigma_A - (1 - x) \times \sigma_B$$

$$x = \sigma_B / (\sigma_A + \sigma_B)$$

This is the weight on the first asset.

CHAPTER 11 B-63

25. Let ρ stand for the correlation, then:

$$\sigma_p^2 = x^2 \times \sigma_A^2 + x^2 \times \sigma_B^2 + 2 \times x_A \times x_B \times \sigma_A \times \sigma_B \times \rho$$

2

B

2

B

2

A

2

A

$$= x^2 \times \sigma_A^2$$

$$+ (1 - x)^2 \times \sigma_B^2$$

$$+ 2 \times x \times (1 - x) \times \sigma_A \times \sigma_B \times \rho$$

Take the derivative with respect to x and set equal to zero:

$$d\sigma_p$$

$$2/dx = 2 \times x \times \sigma_A$$

$$- 2 \times (1 - x) \times \sigma_B$$

$$+ 2 \times \sigma_A \times \sigma_B \times \rho - 4 \times x \times \sigma_A \times \sigma_B \times \rho = 0$$

Solve for x to get the expression in the text.

Chapter 12

Return, Risk, and the Security Market Line

Concept Questions

1. Some of the risk in holding any asset is unique to the asset in question. By investing in a variety of assets, this unique portion of the total risk can be almost completely eliminated at little cost. On the

other hand, there are some risks that affect all investments. This portion of the total risk of an asset

cannot be costlessly eliminated. In other words, systematic risk can be controlled, but only by a costly reduction in expected returns.

2. If the market expected the growth rate in the coming year to be 2 percent, then there would be no

change in security prices if this expectation had been fully anticipated and priced. However, if the market had been expecting a growth rate different than 2 percent and the expectation was incorporated into security prices, then the government's announcement would most likely cause security prices in general to change; prices would drop if the anticipated growth rate had been more

than 2 percent, and prices would rise if the anticipated growth rate had been less than 2 percent.

3. a. systematic

b. unsystematic

c. both; probably mostly systematic

d. unsystematic

e. unsystematic

f. systematic

4. a. An unexpected, systematic event occurred; market prices in general will most likely decline.

b. No unexpected event occurred; company price will most likely stay constant.

c. No unexpected, systematic event occurred; market prices in general will most likely stay constant.

d. An unexpected, unsystematic event occurred; company price will most likely decline.

e. No unexpected, systematic event occurred unless the outcome was a surprise; market prices in general will most likely stay constant.

5. False. Expected returns depend on systematic risk, not total risk.

6. Earnings contain information about recent sales and costs. This information is useful for projecting

future growth rates and cash flows. Thus, unexpectedly low earnings lead market participants to reduce estimates of future growth rates and cash flows; price drops are the result. The reverse is often true for unexpectedly high earnings.

7. Yes. It is possible, in theory, for a risky asset to have a beta of zero. Such an asset's return is simply

uncorrelated with the overall market. Based on the CAPM, this asset's expected return would be equal to the risk-free rate. It is also possible to have a negative beta; the return would be less than the

risk-free rate. A negative beta asset would carry a negative risk premium because of its value as a diversification instrument. A negative beta asset can be created by shorting an asset with a positive

beta. A portfolio with a zero beta can always be created by combining long and short positions.

CHAPTER 12 B-65

8. The rule is always "buy low, sell high." In this case, we buy the undervalued asset and sell (short)

the overvalued one. It does not matter whether the two securities are misvalued with regard to some

third security; all that matters is their relative value. In other words, the trade will be profitable as long as the relative misvaluation disappears; however, there is no guarantee that the relative misvaluation will disappear, so the profits are not certain.

9. If every asset has the same reward-to-risk ratio, the implication is that every asset provides the same

risk premium for each unit of risk. In other words, the only way to increase your return (reward) is to

accept more risk. Investors will only take more risk if the reward is higher, and a constant reward-to-risk

ratio ensures this will happen. We would expect every asset in a liquid, well-functioning market to have the same reward-to-risk ratio due to competition and investor risk aversion. If an asset has a reward-to-

risk ratio that is lower than all other assets, investors will avoid that asset, thereby driving the price down, increasing the expected return and the reward-to-risk ratio. Similarly, if an asset has a reward-to-risk ratio that is higher than other assets, investors will flock to the asset, increasing the price, and decreasing the expected return and the reward-to-risk ratio.

10. AIMR suggested answer:

a. Systematic risk refers to fluctuations in asset prices caused by macroeconomic factors that are common to all risky assets; hence systematic risk is often referred to as market risk. Examples of systematic risk include the business cycle, inflation, monetary policy, and technological changes.

Firm-specific risk refers to fluctuations in asset prices caused by factors that are independent of the

market such as industry characteristics or firm characteristics. Examples of firm-specific risk include litigation, patents, management, and financial leverage.

b. Trudy should explain to the client that picking only the top five best ideas would most likely result

in the client holding a much more risky portfolio. The total risk of the portfolio, or portfolio variance, is the combination of systematic risk and firm-specific risk. i.) The systematic component depends on the sensitivity of the individual assets to market movements as measured by beta. Assuming the portfolio is well-diversified, the number of assets will not affect the systematic risk component of portfolio variance. The portfolio beta depends on the individual security betas and the portfolio weights of those securities. ii.) On the other hand, the components of the firm-specific risk (sometimes called nonsystematic risk) are not perfectly positively correlated with each other and as more asset are added to the portfolio those additional assets tend to reduce portfolio risk. Hence, increasing the number of securities in a portfolio reduces firmspecific

risk. For example, a patent expiring for one company would not affect the other securities in the portfolio. An increase in oil prices might hurt airline stock but aid an energy stock. As the number of randomly selected securities increases, the total risk (variance) of the portfolio approaches its systematic variance.

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple

steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is

found without rounding during any step in the problem.

Core Questions

1. $E(R_i) = .136 = .04 + .075\beta_i$; $\beta_i = 1.28$

2. $E(R_i) = .12 = .05 + (E(R_{mkt}) - .05)(.85)$; $E(R_{mkt}) = .1324$

B-66 SOLUTIONS

3. $E(R_i) = .11 = R_f + (.13 - R_f)(.70)$; $R_f = .0633$

4. $E(R_i) = .16 = .055 + 1.3(MRP)$; $MRP = .0808$

5. $\beta_P = .15(1.2) + .20(.6) + .25(1.5) + .40(.9) = 1.035$

6. Portfolio value = $200(\$60) + 300(\$85) + 100(\$25) = \$40,000.00$

$x_A = 200(\$60)/\$40,000 = .3000$ $x_B = 300(\$85)/\$40,000 = .6375$

$x_C = 100(\$25)/\$40,000 = .0625$

$\beta_P = .3000(1.2) + .6375(.9) + .0625(1.6) = 1.03$

7. $\beta_P = 1.0 = 1/3(0) + 1/3(1.2) + 1/3(\beta_X)$; $\beta_X = 1.80$

8. $E(R_i) = .058 + (.12 - .058)(1.15) = .1293$

9. $E(R_i) = .045 + (.11 - .045)(1.2) = .1230$

Dividend yield = $\$1.20/\$48 = .0229$

Capital gains yield = $.1230 - .0229 = .1001$

Price next year = $\$48(1 + .1001) = \52.80

10. a. $E(R_P) = (.15 + .06)/2 = .1050$

b. $\beta_P = 0.5 = x_S(1.1) + (1 - x_S)(0)$; $x_S = 0.5/1.1 = .4545$; $x_{rf} = 1 - .4545 = .5455$

c. $E(R_P) = .12 = .15x_S + .06(1 - x_S)$; $x_S = .6667$; $\beta_P = .6667(1.1) + .3333(0) = 0.73$

d. $\beta_P = 1.8 = x_S(1.1) + (1 - x_S)(0)$; $x_S = 1.8/1.1 = 1.64$; $x_{rf} = 1 - 1.64 = -0.64$

The portfolio is invested 164% in the stock and -64% in the risk-free asset. This represents borrowing at the risk-free rate to buy more of the stock.

Intermediate Questions

11. $\beta_P = x_W(1.2) + (1 - x_W)(0) = 1.2x_W$

$E(R_W) = .15 = .07 + \text{MRP}(1.20)$; $\text{MRP} = .08/1.2 = .0667$

$E(R_P) = .07 + .0667 \beta_P$; slope of line = $\text{MRP} = .0667$; $E(R_P) = .07 + .0667 \beta_P = .07 + .08x_W$

x_W

$E[r_P]$

β_P

x_W

$E[r_P]$

β_P

0%

.0700

0.00

100%

.1500

1.20

25

.0900

0.30

125

.1700

1.50

50

.1100

0.60

150

.1900

1.80

75

.1300

0.90

12. $E[R_{ii}] = .06 + .07\beta_i$

$.18 > E[R_Y] = .06 + .07(1.50) = .1650$; Y plots above the SML and is undervalued.

reward-to-risk ratio $Y = (.18 - .06) / 1.50 = .0800$

$.11 < E[R_Z] = .06 + .07(0.80) = .1160$; Z plots below the SML and is overvalued.

reward-to-risk ratio $Z = (.11 - .06) / .80 = .0625$

13. $[(.18 - R_f)/1.50 = (.11 - R_f)/0.80]$; $R_f = .0300$

CHAPTER 12 B-67

14. $(E(R_A) - R_f)/\sigma_A = (E(R_B) - R_f)/\sigma_B$

$\sigma_A/\sigma_B = (E(R_A) - R_f)/(E(R_B) - R_f)$

15. Here we have two equations with two unknowns:

$E(R_{\text{Oxy Co.}}) = .16 = R_f + 1.10(R_m - R_f)$; $E(R_{\text{More-On Co.}}) = .12 = R_f + .75(R_m - R_f)$

$.16 = R_f + 1.10R_m - 1.10R_f = 1.10R_m - .10R_f$; $.12 = R_f + .75(R_m - R_f) = R_f + .75R_m - .75R_f$

$R_f = (1.10R_m - .16)/.10$; $R_m = (.12 - .25R_f)/.75 = .16 - .3333R_f$

$R_f = [1.10(.16 - .3333R_f) - .16]/.10$

$4.6667R_f = .16$

$R_f = .0343$

$R_m = (.12 - .25R_f)/.75 = .16 - .3333R_f = .16 - .3333(.0343) = .1486$

16. From the chapter, $\beta_i = \text{Corr}(R_i, R_M) \times (\sigma_i / \sigma_M)$. Also, $\text{Corr}(R_i, R_M) = \text{Cov}(R_i, R_M) / (\sigma_i \times \sigma_M)$.

Substituting this second result into the expression for β_i produces the desired result.

17. The relevant calculations can be summarized as follows:

Returns

Return deviations
 Squared deviations
 Product
 of
 deviations
 Year
 Security
 Market
 Security
 Market
 Security
 Market

1995 12% 6% 3% 2% 0.00090 0.00040 0.00060
 1996 -9 -12 -18% -16% 0.03240 0.02560 0.02880
 1997 -6 0 -15% -4% 0.02250 0.00160 0.00600
 1998 30 -4 21% -8% 0.04410 0.00640 -0.01680
 1999 18 30 9% 26% 0.00810 0.06760 0.02340
 Totals 45% 20% 0.10800 0.10160 0.04200

Average returns: Variances: Standard deviations:
 Security: $45/5 = 9.00\%$ $0.10800/4 = 0.02700$ $\sqrt{0.02700} = 16.43\%$
 Market: $20/5 = 4.00\%$ $0.10160/4 = 0.02540$ $\sqrt{0.02540} = 15.94\%$
 Covariance = $\text{Cov}(R_i, R_M) = 0.04200/4 = 0.01050$
 Correlation = $\text{Corr}(R_i, R_M) = 0.01050 / (.1643 \times .1594) = .40$
 Beta = $\beta = .40(16.43/15.94) = 0.41$

B-68 SOLUTIONS

18. $E[R_p] = .15 = w_X(.19) + w_Y(.122) + (1 - w_X - w_Y)(.06)$
 $\beta_p = .9 = w_X(1.5) + w_Y(1.1) + (1 - w_X - w_Y)(0)$
 solving these two equations in two unknowns gives $w_X = 0.86400$, $w_Y = -0.36000$
 $w_{RF} = 0.49600$

amount of stock Y to sell short = $0.36000(\$100,000) = \$36,000$

19. $E[R_I] = .25(.01) + .50(.21) + .25(.16) = .1475$; $.1475 = .05 + .08\beta_I$, $\beta_I = 1.22$

$\sigma = .25(.04 - .1475)^2 + .5(.21 - .1475)^2 + .25(.16 - .1475)^2 = .00672$; $\sigma_I = [.00672]^{1/2} = .0820$

$E[R_{II}] = .25(-.20) + .5(.11) + .25(.34) = .0900$; $.0900 = .05 + .08\beta_{II}$, $\beta_{II} = 0.50$

$\sigma_{II} = .25(-.20 - .0900)^2 + .5(.11 - .0900)^2 + .25(.34 - .0900)^2 = .03685$; $\sigma_{II} = [.03685]^{1/2} = .1920$

Although stock II has more total risk than I, it has much less systematic risk, since its beta is much smaller than I's. Thus, I has more systematic risk, and II has more unsystematic and more total risk.

Since unsystematic risk can be diversified away, I is actually the "riskier" stock despite the lack of volatility in its returns. Stock I will have a higher risk premium and a greater expected return.

20. $E(R) = .06 + 1.20[.13 - .06] = 14.40\%$

Unexpected
 Returns
 Systematic
 Portion
 Unsystematic
 Portion

Year $R - E(R)$ $R_M - E(R_M)$ $\beta \times [R_M - E(R_M)]$ $R - E(R) - \beta \times [R_M - E(R_M)]$

2001 11.60% 6.00% 7.20% 4.40%
 2002 -2.40% -5.00% -6.00% 3.60%
 2003 -35.40% -26.00% -31.20% -4.20%
 2004 -17.40% -9.00% -10.80% -6.60%
 2005 23.60% 9.00% 10.80% 12.80%

21. AIMR suggested answer:

Furhman Labs: $E(R) = 5.0\% + 1.5(11.5\% - 5.0\%) = 14.75\%$ Overvalued

Garten Testing: $E(R) = 5.0\% + 0.8(11.5\% - 5.0\%) = 10.20\%$ Undervalued

*Supporting calculations

Furhman: Required - Forecast = $13.25\% - 14.75\% = -1.50\%$ Overvalued

Garten: Required - Forecast = $11.25\% - 10.20\% = 1.05\%$ Undervalued

If the forecast return is less (greater) than the required rate of return, the security is overvalued (undervalued).

22. AIMR suggested answer:

Subscript OP refers to the original portfolio, ABC to the new stock, and NP to the new portfolio.

$E(R_{NP}) = .9(.67) + .1(1.25) = 0.728\%$

$COV = 0.40(2.37)(2.95) = 2.7966$

²

$NP \sigma = .9_2(.0237_2) + .1_2(.0295_2) + 2(.9)(.1)(.0237)(.0295)(.40) = .000514$

$\sigma_{NP} = 2.27\%$

CHAPTER 12 B-69

23. AIMR suggested answer:

Subscript OP refers to the original portfolio, GS to government securities, and NP to the new portfolio.

$E(R_{NP}) = .9(.67) + .1(0.42) = 0.645\%$

$COV = 0(2.37)(0) = 0$

²

$NP \sigma = .9_2(.0237_2) + .1_2(0_2) + 2(.9)(.1)(.0237)(.0295)(0) = .000455$

$\sigma_{NP} = 2.13\%$

24. AIMR suggested answer:

Adding the risk-free government securities would cause the beta of the new portfolio to be lower.

The new portfolio beta will be a weighted average of the individual security betas in the portfolio; the presence of the risk-free securities would lower the weighted average.

25. AIMR suggested answer:

The comment is not correct. Although the standard deviations and expected returns of the two securities under consideration are the same, the covariances between each security and the original

portfolio are unknown, making it impossible to draw the conclusion stated. For instance, if the covariances are different, selecting one security over another may result in a lower standard deviation for the portfolio as a whole. In such a case, the security would be the preferred investment

if all other factors are equal.

Spreadsheet Problem