I. Use AD-IA diagram to represent the new Keynesian model to answer the following questions. Remember that the central bank adjust the short-term nominal interest rate according to a rule, 

\[ i_t = r^n_t + \phi_n \pi_t + \phi_y \frac{Y_t - Y^n_t}{Y^n_t}. \]

1. What happens if there is a permanent increase in government expenditures in the short-run and how do inflation and output adjust to the medium run?

2. Let’s continue to represent oil price shocks as large increases in the mark-up of nominal prices over marginal costs so that a rise in oil prices leads to a rise in the natural rate of unemployment.
   a. How do potential output and the natural rate of interest change?
   b. How do short-run output and inflation change?

3. How does an increase in expected future inflation affect output, interest rates and inflation in the short run?

II. Consider the algebraic model:

\[ IS \quad \frac{Y_t - Y^n_t}{Y^n_t} = -a(r_t - r^n_t) \]

\[ AS \quad \pi_t = \beta \pi_{t+1}^e + \kappa \frac{Y_t - Y^n_t}{Y^n_t} \]

\[ MP \quad i_t = r^n_t + \phi_n \pi_t \]

and remember \( r_t = i_t - \pi_{t+1}^e \). (Notice, the IS relationship is simplified.)

1. Show that zero is the medium-run inflation rate.

2. Reconsider your answer if \( \phi_n = 1 \). Substitute this into the MP equation, and then substitute the MP into the IS to get the AD relationship. Next, use \( r_t = i_t - \pi_{t+1}^e \) to get
\[ \frac{Y_t - Y_t^n}{Y_t^n} = -a(\pi_t - \pi_t^e). \]

Now, substitute the AD into the AS to get

\[ \pi_t = \beta \pi_{t+1} - \kappa \alpha (\pi_t - \pi_{t+1}). \]

Rearranging, you have the equation

\[ (1 + \kappa \alpha) \pi_t = (\beta + \kappa \alpha) \pi_{t+1}. \]

In the medium run, expectations are fulfilled – this means that \( \pi_{t+1}^e = \pi_{t+1} \). The parameter \( \beta \) is less than one. Rewriting the equation once more, medium-run inflation dynamics are

\[ \pi_{t+1} = \left( \frac{1 + \kappa \alpha}{\beta + \kappa \alpha} \right) \pi_t. \]

Show that inflation keeps rising if inflation is ever positive under this monetary policy rule. (If \( \phi_x > 1 \), inflation converges to zero in the medium run.)

3. Most central banks adopt an inflation target that is positive (for example, 2 percent). To put the inflation target into the new Keynesian model, we just change the monetary policy rule to

\[ i_t = r_t^n + \phi \pi_t (\pi_t - \pi^T) + \phi_y \frac{Y_t - Y_t^n}{Y_t^n}. \]

The IS and AS relationships do not change.