The important motivations for the model explained in this paper are placed in italics in the introduction. These are that

1. Monetary transmission (that is, the effects of monetary policy on the economy) depends on private sector expectations of the future path of the short-term interest rate.

2. The natural levels of real output and the real interest rate are important guides to monetary policy and may fluctuate a lot (natural levels are the medium-run values).

First, a short guide to their notation:

output gap \( \tilde{y}_t = \frac{Y_t - Y_n}{Y_n} \)

long-term real interest rate \( r^f_t \)

short-term real interest rate \( r_t \)

natural rate of interest \( r^n_t \)

real interest rate gap (long-term) \( \tilde{r}^f_t = r^f_t - r^n_t \)

real interest rate gap (short-term) \( \tilde{r}^r_t = r_t - r^n_t \)

nominal interest rate \( i_t \)

Now, let’s change the interest rates to the notation we usually use:

long-term real interest rate \( r^f_t \)

short-term real interest rate \( r_t \)

natural rate of interest \( r^n_t \)

real interest rate gap (long-term) \( r^f_t - r^n_t \)

real interest rate gap (short-term) \( r_t - r^n_t \)

nominal interest rate \( i_t \)

That is, let’s drop the extra r’s, call the nominal rate i and write out the interest rate gaps to make it easier. We’ll leave the output gap as \( \tilde{y}_t \) (although other authors will use \( x_t \) for it).
Next, you need to know (recall) what Tobin’s q is. Tobin’s q is the ratio of the value of a firm’s capital to what it would cost to replace it. For example, a manufacturing plant may be worth $11 million in its current use but the cost to build an identical one could be $10 million. Tobin’s q for this example would be 1.1. If the plant is only worth $9 million in it’s current use, then Tobin’s q would be 0.9. When Tobin’s q is greater than one, investment is high (firms want to build more plants and buy more equipment); when it is less than one, investment is low.

The gap in Tobin’s q is the percentage difference between q and 1: \( \tilde{q}_t = \frac{q_t - 1}{1} = q_t - 1 \)

We’ll leave this as \( \tilde{q}_t \).

The AD equation:

The equation for aggregate demand tells us that consumption demand decreases if the real interest rate rises and investment rises when it rises.

Consumption depends on human wealth and financial wealth. Human wealth is the present discounted value of current and expected future income. A rise in the long-term real interest rate lowers human wealth. It also reduces future real income. Overall, financial wealth decreases with a rise in the interest rate because value of stocks decreases with a rise in the real interest rate and fall in expected output. The long-term real interest rate is used because consumption is forward-looking and depends on the next few to several years, rather than just the next month.

Investment depends on q, and q depends on the long-term real interest rate. If the long-term real interest rises, the value of the capital already in place falls so that q falls.

Let’s write the AD as

\[ \tilde{y}_t = -a(r_t^e - r_t^n) + b\tilde{q}_t \]

The first term is the consumption gap and the second the investment gap.

The next equation (on page 31) is just definitional:

\[ \tilde{r}_t = r_t - rr_t^n \Rightarrow r_t - r_t^n = (i_t - \pi_{t+1}^e) - r_t^n \]

The term, \( E,\pi_{t+1} \) is expected inflation for next year – it’s the same as our \( \pi_{t+1}^e \).

The long-term real interest rate depends on current and expected future short-term real interest rate through the yield curve. If short-terms rates go up and are expected to remain higher, then long-term rates rise. If they go up and are expected to come back down, then long-term rates do not rise much.
The AS equation:

This one does not need rewriting \( \pi_t = \beta \pi_{t+1} + \kappa \tilde{y}_t + u_t \) but it is new.

The new part is that inflation this month depends on expected future inflation for next month rather than the inflation rate expected last month for this month. Inflation today depends on how prices and wages are set today by firm’s and worker’s expectations about future inflation. The output gap part is not new. The \( u_t \) explicitly measures cost shocks.

Monetary Policy equation:

On page 36, you see the interest rate target for the central bank. In our notation, it is

\[ i_t^T = r^n_t + \phi_x \pi_t + \phi_y \tilde{y}_t. \]

We will discuss in class why it is important that \( \phi_x > 1 \) and \( \phi_y \geq 0 \).

The equation on page 37 in our notation is

\[ i_t = (1 - \rho) i_t^T + \rho i_{t-1} \]

and tells us that central banks usually adjust the short-term nominal interest rate towards the target rate gradually.

A couple things to notice:

1. The interest rate and output gaps depend on the natural rate of interest and potential output which can change over time. This means that monetary policy has to change in response to these.

2. Sticky inflation is incorporated in writing the interest rate target in terms of the nominal short-term interest rate, and expectations about future short-term rates come in through the AS relationship.