

Stock Market Volatility and Fractional Integration

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A fractional integration framework and a relationship between the variability of innovations in real stock prices and real dividends implied by the present value model are used to examine the issue of stock market volatility raised by Shiller (1981) and LeRoy and Porter (1981). It is found that both stock price and dividend data are neither trend stationary nor difference stationary; they are fractionally integrated. The data also show that low interest rates and investors' myopic behaviour only have a limited role in explaining excessive market volatility. On the other hand, the evidence for excess market volatility seems substantial even after controlling for sampling uncertainty.

Keywords: ARFIMA model; long memory; present value model; stock price dynamics; variance bounds

SUMMARY

In their seminal papers, Shiller (1981) and LeRoy and Porter (1981) report that variations in stock prices appear too large to be explained by changes in the fundamental value constructed from the dividend stream. The finding of excess market volatility contradicts the notion that the stock market is efficient and the price reflects the true value of the underlying stock. In fact, results reported by Shiller and LeRoy-Porter are usually interpreted as evidence against the rational or efficient market hypothesis and as indirect evidence that stock prices are also driven by 'fads' and 'fashions.'

Since these authors published their papers, the excess market volatility result has been re-evaluated using different methodologies. Marsh and Merton (1986) provide, perhaps, the most forceful argument against the excess market volatility result. Marsh and Merton show that the original Shiller's result is driven by the model Shiller used to describe the behaviour of dividend data. Specifically, there is no evidence of excess market volatility if a different model is used to describe

dividend data. Therefore, whether the market is too volatile or not depends on the characterization of dividend dynamics. If Shiller's original characterization is correct, the stock market is too volatile and not efficient. If Marsh and Merton's claim is right, then there is no evidence against market efficiency.

In this study we use a stochastic process, called a fractionally integrated autoregressive and moving average (ARFIMA) process, to model dividend dynamics. One advantage of using the ARFIMA process is that it can describe a wide range of data dynamics. In particular, the processes adopted by both Shiller and Marsh-Merton to characterize dividend data are special cases of this general stochastic process. Therefore, the use of ARFIMA processes allows us to discriminate effectively between the two previous views on dividend dynamics and provides a more accurate description of dividend data.

In contrast to the existing results, we find both the real stock price and the real dividend data are fractionally integrated, a property that is different from the characterizations adopted by either Shiller or Marsh and Merton. We use a relation between

the variability of innovations in real stock prices and real dividends, which is derived under the fractional integration framework, to evaluate stock price volatility. Overall, there is substantial evidence for excess market volatility when the real interest rate is higher than 3%. Our findings support the view that the market is excessively volatile unless the market uses a low real interest rate to discount future dividend payments. This implies that price movements may not correspond to changes in the fundamental value of the underlying stock. This study, however, does not address the question of whether this mispricing behaviour represents some exploitable profit opportunities.

INTRODUCTION

In their seminal papers, Shiller (1981) and LeRoy and Porter (1981) report that variations in stock prices appear too large to be explained by changes in the fundamental value constructed from the dividend stream. Their results are based on variance bounds tests derived from the present value model. The finding of excess market volatility is interpreted as evidence against the rational or efficient market hypothesis and as indirect evidence that stock prices are also driven by 'fads' and 'fashions' (Shiller, 1984).¹ Recent studies such as Mankiw *et al.* (1991) and LeRoy and Steigerwald (1992) also report evidence of excess market volatility. LeRoy (1989) and Shiller (1989) provide an excellent review on the variance bounds test literature.

Marsh and Merton (1986) provide, perhaps, the most forceful argument against Shiller's original variance bounds test result.² These authors show that Shiller's (1981) result is driven by the assumption of trend stationary dividends. They argue that the dividend process is better described as difference stationary (that is, the dividend data contain a unit root and are integrated of order one) when firm managers smooth dividend payments over time. When dividend data are difference stationary, the observed stock market volatility is no longer excessive. Kleidon (1986) also attributes the observed excess volatility to the presence of a unit root but not to market inefficiency. That is, the validity of the market inefficiency interpretation

depends on the assumed temporal dynamics of dividends; the presence of long-term persistence in dividends, as implied by difference stationarity, does not favour the excess volatility interpretation.

However, the issue of whether dividends are trend stationary or integrated is still unsettled. DeJong and Whiteman (1991) find that the unit root hypothesis for dividend data is rejected in three studies and not rejected in eight. However, the failure to reject the unit root hypothesis is sometimes attributed to the low power of standard unit root tests. Using Bayesian techniques both DeJong and Whiteman (1991) and Koop (1991) find little evidence for unit roots in the dividend data.

In this study we examine the stochastic process generating dividends and its implications for the relationship between stock price and dividend variations from a different perspective. Temporal dynamics are modelled by long memory, fractionally integrated autoregressive and moving average (ARFIMA) processes. The ARFIMA process is a generalized ARMA process and can describe a wide range of data persistence. For instance, a fractional process includes trend stationary and difference stationary ARMA processes as special cases. That is, the use of fractional time series models can avoid the potential bias caused by the stringent classification of trend stationarity or difference stationarity adopted in the previous studies. This aspect of fractional models is important for studying market volatility because of the crucial role played by dividend dynamics.

For instance, in the short run, dividend smoothing can introduce persistence and give rise to temporal properties not associated with sustainable earnings. Also, smoothed dividend payments may add extra noise to stock prices as they distort the true underlying present value relationship. However, in a longer horizon, dividend smoothing mechanisms will be affected by the dynamics of sustainable earnings. Thus, although firm managers can smooth dividends and induce persistence over time, it would be interesting to know if such dividend-setting behaviour implies a unit root persistence, a weaker than unit root persistence, or a stronger than unit root persistence in the data. The ARFIMA model can provide a flexible way to model data persistence induced by the dividend setting behaviour without imposing a strong prior.

The remainder of the paper is organized as

