A MULTIVARIATE ARCH MODEL OF FOREIGN EXCHANGE RATE DETERMINATION

by

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ABSTRACT

This paper develops a multivariate vector-autoregressive model for five major bilateral exchange rates: the German mark, the French franc, the Swiss franc, the Italian lira, and the British pound. All rates are against the U.S. dollar. We find that logarithmic differences of the exchange rates follow essentially a vector random walk. This does not mean, however, that the rates are completely independent of one another. The variance of the rates is found to be varying autoregressively over time (following a so-called ARCH model). Because the number of parameters in a multivariate ARCH model is, in general, extremely large, the dimension of the problem has been reduced by assuming that the innovations in the vector autoregression follow a two-factor model. ARCH is assumed to affect only the common factor, not the currency-specific factors.

The estimated model provides a good explanation of exchange rate movements and the correlations among them. We test for structural change in the first week of March, 1979, when the European Monetary System went into effect. We find that there is indeed structural change and that the factor loadings on the common factor changed significantly. There are no significant changes in the manner in which the variance of a common factor is varying autoregressively. European currencies are tied more tightly together by EMS, as intended.

The importance of these results for forecasting exchange rates is not that the magnitudes and directions of changes in the rates can be predicted, but rather that, because the variance is predictable, the confidence with which one can forecast variations about the mean is changing over time. In addition, the varying volatility of the rates may be expected to affect the spread between forward and future spot rates, a topic which will be investigated in a subsequent paper.

* This paper represents an extension and further development of the research presented in "The Dynamics of Exchange Rate Volatility," by Diebold and Nerlove, forthcoming in the Journal of Applied Econometrics. We are indebted to R.F. Engle and two anonymous referees for helpful comments on that work which led to these extensions.
\[ H_t = H_0 + (I \otimes \varepsilon_{t-1}) C_1 (I \otimes \varepsilon_{t-1}) + \ldots \]

\[ + (I \otimes \varepsilon_{t-p}) C_p (I \otimes \varepsilon_{t-p}), \quad (4) \]

where \( C_k \) is an \((N^2 \times N^2)\) matrix with \((N \times N)\) blocks \( C_{k,ij} \). It is convenient to vectorize the lower triangle of \( H_t \) and write:

\[ h_t = a_0 + A_1 \eta_{t-1} + \ldots + A_p \eta_{t-p} \quad (5) \]

where:

\[ h_t = \text{vec}(\text{LT}(H_t)) \]

\[ \dim(h_t) = \dim(a_0) = \dim(\eta_{t-1}) = \frac{N^2 + N}{2}, \quad i = 1, \ldots, p \]

\[ \dim(A_i) = (\frac{N^2 + N}{2} \times \frac{N^2 + N}{2}), \quad i = 1, \ldots, p. \]

The operators "vec," "dim," and "LT" are the vectorization, dimension, and lower triangle operators, respectively, and the vector \( \eta_{t-1} \) contains all squared innovations and innovation cross products at lag \( i \).

Consistent, asymptotically efficient and normal parameter estimates are obtained by maximizing the log likelihood, which is a direct multivariate analog of the univariate case. The point log likelihoods are given by:

\[ \log L_t = -(N/2) \log 2\pi + (1/2) \log |H_t^{-1}| - 1/2 \varepsilon_t^T H_t^{-1} \varepsilon_t, \quad (6) \]

and the likelihood for the sample is the sum of the point log likelihoods. The problem is that the number of parameters in this general form may be very large,

\[ K = \frac{N^2 + N}{2} + p\left(\frac{N^2 + N}{2}\right)^2. \]

For five rates and ARCH of order 6, for example, \( K = 1365 \). It is clear that the dimensionality of the parameter space must be reduced if we are to get anywhere.

If graphs of several exchange rate movements are examined simultaneously, we observe that periods of tranquility and periods of volatility roughly correspond for the several rates. This suggests a common factor with ARCH may explain both the high covariance and the common periods of tranquility/
volatility. Consider the five-variate system:

\[
y_t = \lambda^* F_t + \epsilon_t
\]  
\( (5\times1) \quad (5\times1) \quad (1\times1) \quad (5\times1) \)

where:

\[
E(F_t) = E(e_{jt}) = 0, \text{ for all } j \text{ and } t
\]

\[
E(F_t F_{t'}) = 0, \ t \text{ not equal to } t'
\]

\[
E(F_t e_{jt'}) = 0, \text{ for all } j, t, t'
\]

\[
E(e_{jt} e_{kt'}) = \gamma_{j} \text{ if } j = k, t = t'
\]

\[
0 \text{ otherwise.}
\]

We adopt the normalization unconditional var \( (F_t) = 1, \text{ all } t, \text{ by setting}
\]

\[
P = \sum_{j} \alpha_j. \quad \text{(All expectations are otherwise understood to be}
\]

\[
\text{conditional.) If, in addition, assuming } p = 6,
\]

\[
F_t | F_{t-1}, \ldots, F_{t-6} \sim N (0, \sigma_t^2),
\]  
\( (8) \)

where

\[
\sigma_t^2 = \sigma_0^2 + \alpha_1 F_{t-1}^2 + \ldots + \alpha_6 F_{t-6}^2,
\]  
\( (9) \)

with \( \sigma_0 > 0 \) and \( \alpha_j > 0 \) for \( j = 1, \ldots, 6, \)

we have

\[
H_t = \text{cov}(y_t | F_{t-1}) = \sigma_t^2 \lambda^* + \Gamma,
\]  
\( (10) \)

where \( \Gamma = E e_t e_{t}^* \), and the \( j \)-th element of \( H_t \) is given by

\[
H_{jkt} = (\lambda_{j}^* \lambda_{k}^* \sigma_0) + \lambda_{j}^* \lambda_{k}^* \left\{ \alpha_1 F_{t-1}^2 + \ldots + \alpha_6 F_{t-6}^2 \right\} + \gamma_{j},
\]  
\( (11) \)

where \( \gamma_{j} = 0 \) for \( j \neq k \text{ and } \gamma_{kk} > 0 \).

The intuitive motivation for such a model is strong. The common factor \( F \)
represents general influences which tend to affect all exchange rates. The
impact of the common factor on exchange rate \( j \) is reflected in the value \( \lambda_j \).
The "unique factors," represented by the \( e_{jt} \)'s, reflect uncorrelated country-
specific shocks. The conditional variance-covariance structure \( (H_t) \) arises
from their joint dependence on the common factor \( F \) and its elements are functions of the parameters \( \lambda \) and \( \Gamma \).

3. **Maximum-Likelihood Estimates. A Test for Structural Change Due to EMS.**

A likelihood function for the data in terms of the factor loadings and variances of the currency specific factors, and ARCH parameters may be obtained by casting the model in state-space form. The Kalman filter enables us to obtain the conditional innovations for the vector process and their variance-covariance matrix, given some initial values and thus permits us to construct the likelihood function of the parameters in the usual way. The approach is related to the DYMIMIC model of Watson and Engle (1983).

The state-space form of the model is

\[
F_t = v_t \\
y_t = \lambda F_t + e_t
\]

(transition equation)  (observation equation)  

where \( F_t = v_t \) is a scalar such that

\[
v_t \sim n(0, \sigma_v^2), \sigma_v^2 = \sigma_0 + \sum_{j=1}^{p} \alpha_j v_{t-j}^2, \]

and

\[
e_t \sim N(0, \Gamma), \Gamma = \text{diag}(\gamma_j).
\]

\( \Gamma \) does not depend on \( t \). The log likelihood function is (with \( N = 5 \) and \( p = 6 \))

\[
\log L(\cdot) = -\frac{TN}{2} \log 2\pi - \frac{1}{2} \log \det H_t^{-1} - \frac{1}{2} y_t' H_t^{-1} y_t.
\]

\( H_t \) is obtained recursively as follows, starting from initial values

\[
\lambda_0, \Gamma_0, \alpha_0, \alpha_j, F_{1-j} \quad j = 1, \ldots, p.
\]
\[ \hat{H}_1 = \sigma_1^{-2} \lambda_0 \gamma_0 + \Gamma_0 \]  
\[ \sigma_1 = \sigma_0 + \sum_{j=1}^{p} \alpha_j H_{1-j} \]  
\[ F_1 = \sigma_1^{-2} \lambda_0^{-1} H_1 y_1. \]  

The recursion continues with \( \hat{F}_1 \) in place of \( F_0^0 \). In this way we obtain a series \( \hat{H}_1, \ldots, \hat{H}_T \) to insert in (16). The function \( \log L() \) can thus be evaluated for any set of parameter values \( \lambda_0, \Gamma_0, \sigma_0, \alpha_j, j = 1, \ldots, p \), and initial values \( F_{1-j}^0, j = 1, \ldots, p \). If \( T \) is large relative to \( p \), the choice of \( F_{1-j}^0, j = 1, \ldots, p \), will not affect the outcome very much. The initial values of all the parameters are obtained in a two-step procedure which first assumes ARCH to be absent, then computes estimates of the series \( F_t \) and, finally, estimates an ARCH process for these values.  

Results are presented in Tables 2 - 4 for the full sample, January 1, 1974, through July 1, 1987 (710 observations), and for two sub-samples, separated by March 13, 1979, the date when EMS was officially put into effect (270 and 440 observations, respectively). Table 1 presents tests for structural change between the two periods assuming unconstrained 3rd, 6th, or 12th order ARCH processes in the common factor. Four alternative hypotheses are considered:  

\( H_1 \): The factor loadings, \( \lambda \), individual factor variances, \( \Gamma \), and ARCH parameters, \( \alpha \), are different in the two periods.  

\( H_2 \): Only the ARCH parameters, \( \alpha \), differ in the two periods.  

\( H_3 \): Only the factor loadings, \( \lambda \), and individual factor variances, \( \Gamma \), are different in the two periods.  

\( H_4 \): All parameters are constant throughout both periods (the null hypothesis).  

Irrespective of the order of ARCH, the model in which all parameters remain constant throughout is decisively rejected in favor of a model in which
at least the common factor loadings and currency-specific factor variances differ. But variation in ARCH parameters, as compared with variation in factor loadings and specific variances, is significant only when a 3rd order ARCH process is assumed. It is not surprising that when the number of ARCH parameters is so limited, the estimates tend to pick up structural change in other parameters. On the other hand, the results for 6th order and 12th order ARCH are not very different. In results not presented here, we have found many insignificant $\alpha$'s in the 12th order case and $\sum_{j=1}^{p} \alpha_j$ perilously close to one. (Remember we have constrained $\alpha_0 = 1 - \sum_{j=1}^{p} \alpha_j$ to be positive.) For parsimony, therefore, the remaining results presented are for the 6th order case.

Table 2 presents estimates of $\sqrt{\gamma_0}$ and $\sqrt{\gamma_j}$, $j = 1, \ldots, 6$, for the two subperiods and the whole sample. All ARCH weights are significant for the whole period when the $\lambda$'s and $\gamma$'s are either fixed or allowed to vary, although $\alpha_0$ only barely so. Several of these coefficients become insignificant, however, when the period is broken up at the date when EMS was introduced.

Tables 3 - 4 present estimates of the variances of the currency specific factors and the factor loadings for the five currencies, respectively. Recall that the variance of the common factor has been normalized to one (and the observations scaled by a factor of 1000, as well) so that the sum of squared loadings reported in the last line of Table 3 reflects this. The sum of squared common factor loadings is a measure of the contribution of that factor to the sum of the variances of the observed rate changes over time. A better measure is the percent of total sum of variances due to the common factor reported in the last line of Table 4. For the whole sample, with all parameters assumed to be constant, we see this is 74%, but distinctly smaller in the first subperiod, only 51%. The difference, as one can see from the last two columns, is accounted for almost entirely by allowing the $\lambda$'s and $\gamma$'s to
vary while holding the a's fixed. Turning to Table 3, we see that all common factor loadings rise in the second period when the ARCH loadings across periods are held fixed, although some, such as for the Italian lira and British pound, rise more than the others. Returning to Table 4, note that the variances of all the individual currency-specific factors fall in the second period with the exception of the British pound.

It is clear from these results that EMS principally had the effect of tying the movements of European currencies more closely together, even those of the British pound and Swiss franc which were not formally part of the system. At the same time, volatility has not been reduced nor have the properties of its persistence been altered significantly.

4. **Implications for Forecasting**

Clearly, if a time series follows a random walk, with or without drift (non-zero mean), the best forecast which can be made is the last observed value plus the mean. In the case of our model, the means of the common factor and of the currency-specific factors have been found to be insignificantly different from zero and assumed to be zero in the present calculation. Thus, the best forecast of the logarithmic first difference for each currency and for any basket (linear combination) of currencies, based on past observations is zero. But while the logarithmic first differences are uncorrelated over time they are *not* independent as our finding of ARCH in the common factor shows. The dependence of the logarithmic differences over time suggests the possibility that some *nonlinear* method might be used to reduce the forecast variance and/or predict, to some extent, the direction of change. Unfortunately, in the case of these rates, the latter is not the case; only the *variance* of the common factor depends on past observations. But this dependence does imply that we can predict a currency's future variances and covariances (or the variance of a
basket of currencies) to some degree.

Figure 2 graphs estimates of the values of the common factor obtained for the five currencies, allowing λ's and γ's to take on different values pre- and post-EMS. Figure 3 graphs the conditional variance of the common factor, as estimated by our model. The best forecast of any linear combination of currencies, with weights (w_1, w_2, ..., w_5)' at time t for time t + 1 is zero. But the best forecast of the variance of such a combination is

\[ w \hat{\sigma}^2_{t+1} + w' \hat{\sigma} w. \]  

(20)

Thus an approximate 95% confidence interval for the one-step ahead forecast of the basket (FFr, DM, LIR, SFr, BP) with weights \( w' = (0.2221, 0.4904, 0.1031, 0, 0.1844) \) can be constructed by taking the interval

\[ \pm 1.96 \left( w \hat{\sigma}^2_{t+1} w \right)^{1/2}. \]  

(21)

The weights \( w \) are arbitrarily determined by the weight which each of the currencies bears in the ECU (European Currency Unit). A graph of the confidence interval for this case is given in Figure 4. While zero is the best forecast throughout the period for changes in the basket, the confidence with which the value could be forecast decreased markedly after EMS both because of increased volatility of the rates themselves and because of the increased importance of the common factor in the determination of the five rates. Our ability to forecast this changing variance is enhanced by the latter.

The finding that movements in the major rates can be well approximated by a multivariate random walk with ARCH is certainly consistent with market efficiency: prices adjusting rapidly and fully reflecting all available information. But what precisely is the significance of ARCH? If ARCH simply reflects misspecification of the model, in this case a random-walk model, then our result may cast doubt on the efficient market hypothesis.
In an efficient exchange market, traders respond nearly instantaneously to incoming information; the supply of, and demand for, each currency is always in balance at each moment. But, of course, there must be differences of opinion about the significance of the information for any trades to take place. "New" information coming into the market affects supply and demand; "old" information is already incorporated in these schedules and thus in the previous equilibria. Most interpretations of market efficiency involve the idea that what is "new" is not predictable from what is "old," but predictability is usually interpreted in a linear sense; hence the conclusion that price changes should be serially uncorrelated. Linearity is quite restrictive, however. It is well known that a time series may be predictable in terms of other variables or its own past in a nonlinear way, yet appear to be uncorrelated serially or with other variables if the relationship is nonlinear. This could be why we find evidence of ARCH, which is a form of nonlinear serial dependence. An alternative explanation more consistent with the theory of efficient markets runs as follows: Incoming information is of different kinds. For some kinds there is more disagreement and uncertainty about the significance or relevance of the information. When relatively clear (i.e., easily and unambiguously interpretable) signals are abundant, exchange rate volatility is likely to be low. When there is disagreement about the meaning of incoming information or when clearly relevant and significant information is scarce, we would expect greater market volatility. Our hypothesis, then, is that the new information coming into the market is serially correlated, both in quantity and quality. When the "world" is unsettled, there is a great deal of news of dubious relevance or significance. When the "world" is tranquil, signals are clear. The "state of the world" is a serially correlated thing; hence, we find ARCH.

Finally, it is apparent from our results that the major institutional change which occurred during the period, EMS, greatly affected the relationship among
various rates without, at the same time, affecting the underlying factors governing their volatility, or, at least, the dependence of this volatility over time.
FOOTNOTES

1. Detailed characterizations of the conditional and unconditional moment structures of ARCH processes are provided by Engle (1982) and Milhoj (1986).

2. This is shown by Diebold (1986a), who obtains the result by applying a central limit theorem for dependent, identically distributed, random variables due to White (1984).

3. Diebold and Pauly (1988) deal with a similar problem for a two-currency model: the French mark and the Italian lira, both against the German mark. Their conclusions are rather different from those presented here.

REFERENCES


Table 1: TESTS OF STRUCTURAL CHANGE

<table>
<thead>
<tr>
<th>CASE A: 3rd order ARCH process in ${F_t}$</th>
<th>Tail Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$ vs $H_4$</td>
<td>L.R. $(\chi^2_{13}) = 486.38$ &lt;0.0001</td>
</tr>
<tr>
<td>$H_1$ vs $H_3$</td>
<td>L.R. $(\chi^2_{3}) = 7.82$ 0.0499</td>
</tr>
<tr>
<td>$H_1$ vs $H_2$</td>
<td>L.R. $(\chi^2_{10}) = 449.98$ &lt;0.0001</td>
</tr>
<tr>
<td>$H_2$ vs $H_4$</td>
<td>L.R. $(\chi^2_{3}) = 36.39$  &lt;0.0001</td>
</tr>
<tr>
<td>$H_3$ vs $H_4$</td>
<td>L.R. $(\chi^2_{10}) = 478.56$ &lt;0.0001</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CASE B: 6th order ARCH process in ${F_t}$</th>
<th>Tail Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$ vs $H_4$</td>
<td>L.R. $(\chi^2_{16}) = 481.83$ &lt;0.0001</td>
</tr>
<tr>
<td>$H_1$ vs $H_3$</td>
<td>L.R. $(\chi^2_{6}) = 8.60$  0.1974</td>
</tr>
<tr>
<td>$H_1$ vs $H_2$</td>
<td>L.R. $(\chi^2_{10}) = 454.08$ &lt;0.0001</td>
</tr>
<tr>
<td>$H_2$ vs $H_4$</td>
<td>L.R. $(\chi^2_{6}) = 27.75$  0.0001</td>
</tr>
<tr>
<td>$H_3$ vs $H_4$</td>
<td>L.R. $(\chi^2_{10}) = 473.23$ &lt;0.0001</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>CASE C: 12th order ARCH process in ${F_t}$</th>
<th>Tail Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$ vs $H_4$</td>
<td>L.R. $(\chi^2_{22}) = 483.95$ &lt;0.0001</td>
</tr>
<tr>
<td>$H_1$ vs $H_3$</td>
<td>L.R. $(\chi^2_{12}) = 15.03$  0.2398</td>
</tr>
<tr>
<td>$H_1$ vs $H_2$</td>
<td>L.R. $(\chi^2_{10}) = 448.69$ &lt;0.0001</td>
</tr>
<tr>
<td>$H_2$ vs $H_4$</td>
<td>L.R. $(\chi^2_{12}) = 35.27$  0.0004</td>
</tr>
<tr>
<td>$H_3$ vs $H_4$</td>
<td>L.R. $(\chi^2_{12}) = 468.93$ &lt;0.0001</td>
</tr>
</tbody>
</table>

Notes:  
$H_1$: All the parameters are allowed to change over the two sub-periods.  
$H_2$: Only the ARCH-parameters $a$'s are allowed to change over the two sub-periods.  
$H_3$: Only the "factor-parameters" $\lambda$'s and $\gamma$'s are allowed to change over the two sub-periods.  
$H_4$: All the parameters are assumed to be constant over the whole sample.

Full sample: 1 January 1974 to 1 July 1987.  
First sub-period: 1 January 1974 to 6 March 1979.  
Second sub-period: 13 March 1979 to 1 July 1987.

L.R. $(\chi^2_k)$: Likelihood ratio statistic which has an asymptotic $\chi^2$-distribution with $k$ degrees of freedom under the null hypothesis.
Table 2: PARAMETER COMPARISON, $\alpha$
(For ARCH-parameters, $\alpha$, from the Model with a 6-th Order ARCH Process in $\{r_t\}$)

<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>First Sub-period</th>
<th>Second Sub-period</th>
<th>First* Sub-period</th>
<th>Second* Sub-period</th>
<th>Full Sample**</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{\alpha_1}$</td>
<td>0.4224 (8.09)</td>
<td>0.5822 (7.70)</td>
<td>0.3079 (3.16)</td>
<td>0.6024 (7.38)</td>
<td>0.3142 (3.32)</td>
<td>0.4322 (7.45)</td>
</tr>
<tr>
<td>$\sqrt{\alpha_2}$</td>
<td>0.4969 (10.9)</td>
<td>0.5553 (6.29)</td>
<td>0.4642 (5.83)</td>
<td>0.5519 (6.39)</td>
<td>0.4651 (6.41)</td>
<td>0.5239 (11.21)</td>
</tr>
<tr>
<td>$\sqrt{\alpha_3}$</td>
<td>0.4457 (9.75)</td>
<td>0.1525 (0.51)</td>
<td>0.4521 (6.59)</td>
<td>0.0277 (0.02)</td>
<td>0.4572 (6.05)</td>
<td>0.4306 (9.56)</td>
</tr>
<tr>
<td>$\sqrt{\alpha_4}$</td>
<td>0.2406 (2.99)</td>
<td>0.4092 (3.84)</td>
<td>0.1843 (1.25)</td>
<td>0.4327 (4.30)</td>
<td>0.1524 (0.87)</td>
<td>0.2760 (3.33)</td>
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<tr>
<td>$\sqrt{\alpha_5}$</td>
<td>0.3859 (6.80)</td>
<td>0.3063 (2.76)</td>
<td>0.3256 (3.44)</td>
<td>0.2752 (2.32)</td>
<td>0.3578 (3.79)</td>
<td>0.3490 (4.67)</td>
</tr>
<tr>
<td>$\sqrt{\alpha_6}$</td>
<td>0.2954 (4.19)</td>
<td>0.0758 (0.24)</td>
<td>0.1898 (1.04)</td>
<td>0.0859 (0.34)</td>
<td>0.2539 (1.76)</td>
<td>0.1979 (1.72)</td>
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<tr>
<td>$\sqrt{\alpha_0}$</td>
<td>0.0820</td>
<td>0.0624</td>
<td>0.3094</td>
<td>0.0614</td>
<td>0.2602</td>
<td>0.1161</td>
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<tr>
<td>$\sum_{i=1}^{6} \alpha_i$</td>
<td>0.9180</td>
<td>0.9376</td>
<td>0.6906</td>
<td>0.9386</td>
<td>0.7398</td>
<td>0.8839</td>
</tr>
</tbody>
</table>

* Only $\alpha$'s vary across sub-sample
** $\lambda$'s and $\gamma$'s vary across sub-periods

Note: Full sample - 1 January 1974 to 1 July 1987
First sub-period - 1 January 1974 to 6 March 1979
Second sub-period - 13 March 1979 to 1 July 1987
t-statistics are in parentheses
<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>First Sub-period</th>
<th>Second Sub-period</th>
<th>Full Sample*</th>
<th>First** Sub-period</th>
<th>Second** Sub-period</th>
</tr>
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<tbody>
<tr>
<td>$\lambda_{FFr}$</td>
<td>12.65 (7.12)</td>
<td>7.56 (3.28)</td>
<td>14.80 (8.55)</td>
<td>12.61 (7.00)</td>
<td>7.09 (8.57)</td>
<td>15.13 (19.14)</td>
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<tr>
<td>$\lambda_{DM}$</td>
<td>13.53 (6.81)</td>
<td>9.75 (3.32)</td>
<td>15.50 (8.33)</td>
<td>13.48 (6.74)</td>
<td>9.08 (8.64)</td>
<td>15.84 (20.13)</td>
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<tr>
<td>$\lambda_{LIR}$</td>
<td>10.87 (6.92)</td>
<td>4.88 (3.14)</td>
<td>13.33 (7.47)</td>
<td>10.83 (6.84)</td>
<td>4.60 (6.04)</td>
<td>13.63 (19.37)</td>
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<tr>
<td>$\lambda_{SFr}$</td>
<td>14.57 (7.33)</td>
<td>12.34 (3.25)</td>
<td>16.04 (8.26)</td>
<td>14.51 (6.87)</td>
<td>11.68 (8.45)</td>
<td>16.40 (19.85)</td>
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<tr>
<td>$\lambda_{BP}$</td>
<td>9.52 (6.60)</td>
<td>5.08 (3.11)</td>
<td>11.31 (7.66)</td>
<td>9.50 (6.75)</td>
<td>4.77 (6.57)</td>
<td>11.56 (12.93)</td>
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<tr>
<td>$\Sigma \lambda_i^2$</td>
<td>764.19</td>
<td>354.10</td>
<td>1022.45</td>
<td>758.88</td>
<td>313.05</td>
<td>1068.19</td>
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</tbody>
</table>

* Only $\alpha$'s varies across sub-sample
** $\lambda$'s and $\gamma$'s vary across sub-periods

Note: Full sample - 1 January 1974 to 1 July 1987.
First sub-period - 1 January 1974 to 6 March 1979.
Second sub-period - 13 March 1979 to 1 July 1987.
t-statistics are in parentheses
Table 4: PARAMETER COMPARISON, γ

(For Individual Factor Variances, \( \Gamma \), from the Model with a 6-th Order ARCH Process in \( \{ R_t \} \))

<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>First Sub-period</th>
<th>Second Sub-period</th>
<th>Full Sample*</th>
<th>First** Sub-period</th>
<th>Second** Sub-period</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{\gamma_{FFR}} )</td>
<td>5.27</td>
<td>6.40</td>
<td>4.75</td>
<td>5.25</td>
<td>6.40</td>
<td>4.75</td>
</tr>
<tr>
<td></td>
<td>(27.34)</td>
<td>(26.83)</td>
<td>(26.72)</td>
<td>(18.63)</td>
<td>(23.38)</td>
<td></td>
</tr>
<tr>
<td>( \sqrt{\gamma_{DM}} )</td>
<td>4.07</td>
<td>5.38</td>
<td>2.27</td>
<td>4.08</td>
<td>5.52</td>
<td>2.28</td>
</tr>
<tr>
<td></td>
<td>(19.35)</td>
<td>(9.33)</td>
<td>(19.22)</td>
<td>(13.12)</td>
<td>(9.05)</td>
<td></td>
</tr>
<tr>
<td>( \sqrt{\gamma_{LIR}} )</td>
<td>7.66</td>
<td>10.41</td>
<td>5.10</td>
<td>7.67</td>
<td>10.40</td>
<td>5.09</td>
</tr>
<tr>
<td></td>
<td>(36.08)</td>
<td>(23.07)</td>
<td>(28.17)</td>
<td>(34.63)</td>
<td>(22.37)</td>
<td>(25.63)</td>
</tr>
<tr>
<td>( \sqrt{\gamma_{SFR}} )</td>
<td>7.91</td>
<td>9.57</td>
<td>6.03</td>
<td>7.93</td>
<td>9.42</td>
<td>6.04</td>
</tr>
<tr>
<td></td>
<td>(32.35)</td>
<td>(16.85)</td>
<td>(26.31)</td>
<td>(31.58)</td>
<td>(15.69)</td>
<td>(27.26)</td>
</tr>
<tr>
<td>( \sqrt{\gamma_{BP}} )</td>
<td>10.34</td>
<td>8.46</td>
<td>11.40</td>
<td>10.33</td>
<td>8.46</td>
<td>11.40</td>
</tr>
<tr>
<td></td>
<td>(37.59)</td>
<td>(22.29)</td>
<td>(29.56)</td>
<td>(37.07)</td>
<td>(21.85)</td>
<td>(29.75)</td>
</tr>
<tr>
<td>( \Sigma \gamma_i )</td>
<td>272.58</td>
<td>341.41</td>
<td>219.91</td>
<td>272.60</td>
<td>339.90</td>
<td>220.11</td>
</tr>
<tr>
<td>( \Sigma \lambda_i )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Sigma \gamma_i )</td>
<td>73.71%</td>
<td>50.91%</td>
<td>82.30%</td>
<td>73.57%</td>
<td>47.94%</td>
<td>82.91%</td>
</tr>
</tbody>
</table>

* Only \( \alpha \)'s vary across sub-sample  
** \( \lambda \)'s and \( \gamma \)'s vary across sub-periods

Note: Full sample - 1 January 1974 to 1 July 1987.  
First sub-period - 1 January 1974 to 6 March, 1979.  
Second sub-period - 13 March 1979 to 1 July 1987.  
t-statistics are in parentheses
FIG. 2: The estimated common factor
FIG. 3: Conditional Variance of the Common Factor
The basket is defined as

\[ (227FF + 49040M + 103170 + 184790) \]

**Figure 4.95% Confidence Interval of the Basket of Currencies**
The basket is defined as: 222FF + .480DM + .103Lira + .184BP

FIG.6: Forecasting the basket of currencies