ABSTRACT. This study employs a new modified rescaled range (R/S) test to examine the long memory behavior of stock returns in four foreign markets. Unlike the conventional R/S analysis, the modified R/S analysis is robust to short-term dependence and conditional heteroskedasticity, that are shown to exist in stock return data. Empirical results show that the conventional and new modified R/S analyses can lead to entirely different conclusions about the presence of long cycles in foreign stock returns. While the conventional R/S analysis seems to indicate the presence of long cycles, no significant evidence of long cycles can be found using the modified R/S analysis once short-term dependence and conditional heteroskedasticity in the data are accounted for. Implications of the findings are discussed.

Financial economists often seek better understanding of the nature of stock price dynamics and improvement in modeling stock price movements. Studies have been devoted to examine the distributional properties of stock returns and identify stochastic processes that are consistent with these properties. These studies generally report significant departures from normality, and different probability distributions have been explored to model stock returns, including the stable Paretoan, scaled-t, and compound normal dis-

Yin-Wong Cheung is Assistant Professor of Economics, University of California, Santa Cruz, CA 95064; Kon S. Lai is Associate Professor of Economics and Michael Lai is Assistant Professor of Finance, California State University, Los Angeles, CA 90032.

Journal of International Financial Markets, Institutions & Money, Vol. 3(1) 1993 © 1993 by The Haworth Press, Inc. All rights reserved.
tributions. Some of the studies also show that changes in stock
prices can well be described by mixed diffusion-jump models [e.g.,
Tucker (1992)]. All these models of distributions assume that data
observations represent independent realizations of some random
variables. Such assumption is not valid, however, in the presence of
temporal dependence in the return data.

Analyses of nonlinear dependence in stock prices have enjoyed
much attention recently. The interest in nonlinear dynamics arises
from the observation that the often wide and nonperiodic cyclical
fluctuations of stock prices cannot be adequately explained by linear
models. The usual findings of leptokurtosis in stock returns may be
further indirect evidence of nonlinear dynamics. Scheinkman and
LeBaron (1989) note that ARCH (autoregressive conditional heter-
okedastic) processes can exhibit dependence similar to that of
chaotic systems. Empirical evidence of chaotic dynamics or ARCH-
type dependence in stock returns has been reported by Hsieh (1991)
and Scheinkman and LeBaron (1989).

Another possible form of temporal dependence in asset returns
relates mainly to the long memory (or long cycle) property of the
time series. Series showing this type of dependence with long
memory is characterized by nonperiodic long cycles. While such
long-term dependence is hard to detect using standard statistical
techniques, rescaled range (R/S) analysis [Hurst (1951), Mandel-
brot (1972), Mandelbrot and Wallis (1966), and Wallis and Matalas
(1970)], which can uncover irregular cyclical dependence, can be
used. To the extent that a long memory process generates nonper-
iodic cycles, long memory can be viewed as a special form of
nonlinear dependence. Mandelbrot (1977) describes long memory
processes as having 'fractal dimensions.' Lo (1991) notes that some
chaotic process such as the 'tent map' can display the long memory
property.

A number of studies have explored the long memory property of
financial prices, including stock prices [Aydogas and Booth (1988),
Greene and Fielitz (1977), and Lo (1991)], gold prices [Booth et al.
(1982a)], exchange rates [Booth et al. (1982b) and Cheung
(1992a)], commodity futures [Heims et al. (1984)], and Eurodollar
and T-bill futures [Lee and Mathur (1992)]. Empirical evidence on
the presence of long memory in the U.S. stock returns has first been
presented by Greene and Flieltz (1977) based on the R/S analysis discussed by Hurst (1951), Mandelbrot (1972), Mandelbrot and Wallis (1969), and Wallis and Matalas (1970). Aydogan and Booth (1988) reexamine the evidence and suggest that the finding of long memory in the U.S. stock returns may be spurious because of the existence of preasymptotic behavior in statistical estimates. Aydogan and Booth (1988) also discuss other potential problems of the R/S analysis such as sensitivity to short-term autocorrelation and nonhomogeneity in the data. More recently, Lo (1991) employs a new R/S procedure with improved robustness and fails to find long memory in the U.S. stock market once the effects of short-term dependence and conditional heteroskedasticity are accounted for.

This study supplements previous evidence by examining international evidence. Specifically, the study extends Lo's (1991) modified R/S analysis to investigate the long memory behavior of stock returns in several major foreign markets. We examine if Lo's (1991) finding is common to other national stock markets or unique to the U.S. stock market. Four countries are considered; they include Germany, Italy, Japan, and the United Kingdom.

Interest in the long memory behavior in stock returns arises from various concerns. Recent studies by Diebold and Rudebusch (1989) and Shea (1991) find that economic fundamentals such as national output and interest rates display long memory. Since the extent to which movements in stock prices reflect changes in economic fundamentals is still a debated issue, it is interesting to examine whether or not long memory is present in stock returns as well. Moreover, studies by, e.g., Fama and French (1988) and Poterba and Summers (1988) report evidence of anomalous behavior in stock returns: they display positive correlations over short horizons but negative correlations over long horizons. Lo (1991) observes that the finding of negative correlations for long-horizon stock returns can be a symptom of long memory dynamics.

The relevance of long memory behavior can have further implications for time series modeling. When asset prices exhibit long memory dynamics, proper modeling of the dynamics using, e.g., fractional processes can potentially improve forecasts upon usual time series forecasting models (Cheung (1992a)). This is particularly relevant to long-term forecasts.
Furthermore, if long memory is indeed present in stock price series, statistical inferences concerning asset pricing models based on standard testing procedures may no longer be valid. In addition, theoretical and empirical models that allow for long memory price dynamics should be explored. Mandelbrot (1971) notes that in the presence of long memory, the arrival of new market information cannot be freely arbitrated away and martingale models of asset prices cannot be obtained from arbitrage. Mandelbrot (1971) also shows that variability in the imperfectly arbitrated price may not be stationary and the return distribution is nonnormal. Kaen and Rosenman (1986) observe that findings of long cycles in asset prices can be consistent with the competence-difficulty (C-D) gap hypothesis of rule-governed behavior on the part of investors. According to the hypothesis, the C-D gap measures a spread between an economic agent's competence to make optimal decisions and the complexity of decision problems under uncertainty. When the C-D gap is wide, the agent is likely to follow some rule-governed behavior, which can produce persistent price movements in the same direction. Kaen and Rosenman argue that due to the irregular arrival of new important information to the asset market, persistent price movements will at times reverse direction suddenly, thus leading to nonperiodic cycles in asset prices.

I. ON THE USE OF A MODIFIED R/S PROCEDURE

Early studies on the long memory behavior of stock returns by Aydogan and Booth (1988) and Greene and Fielitz (1977) employ the conventional R/S analysis, first proposed by Hurst (1951) and later refined by Mandelbrot (1972), Mandelbrot and Wallis (1969), and Wallis and Matalas (1970). Desirable properties of the R/S testing procedure have been discussed by, e.g., Mandelbrot (1972, 1975) and Wallis and Matalas (1970).

A problem with the conventional R/S analysis, as pointed out by Lo (1991), is that the distribution of its test statistic is not well-defined and the analysis is not robust to possible heterogeneity in the underlying data generating process. Consequently, reliable statistical inferences are hard to make. The problem of heterogeneously distributed processes is relevant, since stock prices are known to
display conditional heteroskedasticity [e.g., Hsieh (1991) and Scheinkman and LeBaron (1989)]. Tests for long memory should therefore properly account for the conditional heteroskedastic effects. The preciseness of the classical R/S analysis has also been called in question by Aydogan and Booth (1988) for the problem of preasymptotic behavior. Moreover, as shown by Lo (1991), the presence of short-term dependence can seriously bias the classical R/S test toward finding long memory too often. Short-term dependence in stock returns can arise from, e.g., stop-loss orders and marginal calls as well as from noise trading. It can also result from extrapolative expectations and portfolio insurance.

To deal with some of the aforementioned problems, Lo (1991) proposes the use of a new modified R/S technique. In contrast with the classical R/S procedure, the modified one is attractive in that it has well-defined distributional properties and is robust to short-term dependence. In addition, the modified procedure is robust to a wide class of heterogeneously distributed processes, including those with conditional heteroskedasticity. The modified R/S test examines the null hypothesis of a short memory process against long memory alternatives. The test imposes little distributional structure on the data process. Specifically, short-term dependence, nonnormal innovations, and conditional heteroskedasticity are all allowed for under the null hypothesis. Monte Carlo results reported by Cheung (1992b) support that the modified R/S test is robust to variance shifts and ARCH effects. Using the improved R/S technique, Lo (1991) illustrates that once the short-term dependence and conditional heteroskedasticity are properly accounted for, little evidence of long memory can be found in the U.S. stock returns, in contrast to Greene and Fielitz (1977) but similar to Aydogan and Booth (1988). To see whether Lo’s (1991) finding has more general relevance, this study extends Lo’s analysis to investigate the long memory behavior of stock returns in several foreign stock markets. The modified R/S test for long memory is discussed more in the next section.

II. STATISTICAL ANALYSIS

The modified R/S test examines the null hypothesis of a short memory and possibly heterogeneously distributed process with a
constant mean [Lo (1991)]. Let \( x_t \) be the time series under examination. Under the null hypothesis

\[
x_t = \mu + \phi_t
\]

where \( \mu \) is a constant and \( \{\phi_t\} \) a zero-mean disturbance series satisfying three general conditions:

\begin{align*}
\text{(C1)} & \quad \sup \mathbb{E} \left[ |\phi_t|^\beta \right] < \infty \text{ for sample } \beta > 0; \\
\text{(C2)} & \quad 0 < \sigma^2 = \lim_{T \to \infty} \mathbb{E} \left[ \left( \sum_{t=1}^{T} \phi_t \right)^2 / T \right] < \infty; \\
\text{(C3)} & \quad \{\phi_t\} \text{ is strong-mixing with mixing coefficient } \alpha_k \\
& \quad \text{such that } \sum_{k=1}^{\infty} \alpha_k^{1.2/\beta} < \infty.
\end{align*}

The expectation operator is denoted by \( \mathbb{E}[\cdot] \). These conditions restrict the maximal degree of dependence and heterogeneity allowable in the data so that some form of the law of large numbers and the functional central limit theorem can be applied. The conditions impose little distributional structure on the stochastic process. In particular, short-term autocorrelations and time-dependent heteroskedasticity such as conditional heteroskedasticity are allowed for [see Lo (1991) for a more detailed discussion of the statistical assumptions].

Let \( T \) be the sample mean of a given data series \( \{x_t, t = 1, 2, \ldots, T\} \). The modified R/S statistic, denoted by \( Q_T \), is given by the range of cumulative sums of deviations of the time series from its mean, rescaled by a consistent estimate of its standard deviation:

\[
Q_T = R/s_T(q)
\]

(2)

where \( R \), the range of cumulative sums of deviations from the sample mean, is given by

\[
R = \max_{1 \leq i \leq T} \sum_{t=i}^{T+1} (x_t - \bar{x}) - \min_{1 \leq i \leq T} \sum_{t=i}^{T+1} (x_t - \bar{x})
\]

(3)

and \( s_T^2(q) \) is a heteroskedasticity and autocorrelation consistent variance estimator given by
\[ s_y(q) = \left[ \sum_{i=1}^{T} (x_i - \bar{x})^2 / T + 2 \sum_{i=1}^{T} \sum_{j=1}^{i-1} \gamma_{ij}(q) \right]^{1/2} \]  

(4)

with the weighting function \( \gamma_{ij}(q) = \frac{1 - q^{|i-j|}}{q} \) and \( q \) being a truncation lag determined by [Andrews (1991)].

The numerator in equation (2) measures the memory in the series using cumulative sums of deviations from the mean. A long memory series will stay above or below its mean for a long period of time such that the range of the cumulative sums can become rather large. A major difference between the modified R/S statistic and the classical one lies in the normalization of the range measure. The denominator in equation (2) normalizes the range measure not only by the sample variance, which is considered in the classical R/S analysis, but also by a weighted sum of sample autocovariances for \( q \geq 0 \). This modification provides the robustness of the modified R/S analysis to both short-term dependence and heteroskedasticity. If \( x_t \) displays short-term dependence, the estimator of the variance of the cumulative sums should include both the sample variance and the sample autocovariances of the individual terms. The variance estimator, as given in equation (4), can be shown to be consistent in the presence of short-term dependence and time-dependent heteroskedasticity [Andrews (1991)].

The modified R/S analysis differs from the classical one in another important respect. The modified R/S test is based on R/S values computed using the entire series directly, while the classical R/S test is regression-based, which examines estimates of the Hurst (1951) coefficient obtained from regressing R/S values of different subseries on their corresponding length. In contrast to the classical one, the modified R/S test statistic thus constructed has a well-defined distribution, useful for statistical inference. Mandelbrot (1975) examines a R/S statistic similar to the modified R/S statistic applied in
this paper, but without the adjustments for short-term dependence and conditional heteroskedasticity. While Mandelbrot shows the consistency of the classical R/S statistic, its distributional property is not derived. To get around the problem, the classical R/S analysis examines least squares estimates of the Hurst coefficient, which indirectly relates the R/S statistic with the series length. In modified R/S analysis, however, the theoretical distribution of the modified R/S statistic can be derived analytically and applied directly for inferences.

Under the null hypothesis of no long memory, the limiting distribution of the $Q_T$ statistics standardized by the sample size can be established. Specifically, the statistic $Q_T/\sqrt{T}$ converges to the range of a Brownian bridge $V_i$ with its distribution given by $F(v)$ as follows:

$$F(v) = 1 + 2\sum_{j=1}^{\infty}(1 - 4j^2v^2)\exp\{-2jv^2\}. \tag{6}$$

Critical values for the modified R/S test at different significance levels can be computed using equation (6) and are tabulated in Lo (1991). For a large value of the estimate of $Q_T/\sqrt{T}$, we reject the null hypothesis of no long memory in the series. Monte Carlo results presented by Lo (1991) indicate that the modified R/S test has reasonably good power against various long memory processes.

### III. DATA AND EMPIRICAL RESULTS

The data examined in this study are national stock price indices for four countries: Germany, Italy, Japan, and the United Kingdom. The sample series consists of monthly observations of 34 years (from 1957 through 1990), drawn from the IMF's *International Financial Statistics* data tape. Each price series is transformed into a monthly return series by taking first differences in the logarithms of the prices. Since we are interested in the low-frequency dynamics here, the use of a long data set is desirable in that the long memory behavior, if it exists, can presumably take a long time span to manifest itself in the data.

Some preliminary data analysis of the stock return series is car-
TABLE 1. Some Descriptive Statistics of National Stock Returns

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>U.K.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>0.1906</td>
<td>0.2235</td>
<td>0.2384</td>
<td>0.3637</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.2674</td>
<td>-0.2223</td>
<td>-0.3254</td>
<td>-0.2579</td>
</tr>
<tr>
<td>Median</td>
<td>0.0030</td>
<td>0.0027</td>
<td>0.0129</td>
<td>0.0113</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0057</td>
<td>0.0040</td>
<td>0.0085</td>
<td>0.0077</td>
</tr>
<tr>
<td>t-Test for Mean = 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[p-value]</td>
<td>[0.0123]**</td>
<td>[0.1813]</td>
<td>[0.0001]***</td>
<td>[0.0022]***</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.0865</td>
<td>0.0026</td>
<td>-1.1654</td>
<td>0.3041</td>
</tr>
<tr>
<td>[p-value]</td>
<td>[0.4764]</td>
<td>[0.8142]</td>
<td>[0.0000]***</td>
<td>[0.0137]***</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>4.4966</td>
<td>3.1029</td>
<td>9.9615</td>
<td>8.3854</td>
</tr>
<tr>
<td>[p-value]</td>
<td>[0.0000]***</td>
<td>[0.0000]***</td>
<td>[0.0000]***</td>
<td>[0.0000]***</td>
</tr>
<tr>
<td>The Kolmogorov Test</td>
<td>0.9854</td>
<td>0.9564</td>
<td>0.9483</td>
<td>0.9594</td>
</tr>
<tr>
<td>[p-value]</td>
<td>[0.0001]***</td>
<td>[0.0001]***</td>
<td>[0.0001]***</td>
<td>[0.0001]***</td>
</tr>
</tbody>
</table>

The skewness is computed as the third sample moment standardized by the cube of the standard deviation. The kurtosis is the fourth sample moment divided by the square of the variance minus three. For a normal distribution both coefficients should be equal to zero. The Kolmogorov test statistic is for the hypothesis that the data examined come from a normal distribution. The numbers in [ ] give the p-values of the respective test statistics. Statistical significance is indicated by * for 10%, by ** for 5% and by *** for 1%.
percent significance level. The excess kurtosis coefficients are all positive, suggesting that the return distributions have a much flatter tail than the normal distribution. The results from the Kolmogorov test for normality also support the presence of significant departures from normality in all the stock return series under examination.

Further analysis is conducted by fitting to the stock return data an autoregressive (AR(p)) model given by

\[ x_t = c_0 + \sum_{j=1}^{p} c_j x_{t-j} + \epsilon_t. \]  

(7)

In our analysis the lag parameter \( p \) was first determined using a model selection procedure based on the Schwarz information criterion. The corresponding residuals were then tested for the presence of autocorrelations using the usual Ljung-Box test. The estimated lag length would be used when the residuals could pass the autocorrelation test. If they could not, the lag length would be increased until autocorrelations in residuals were removed. Moreover, the squared residual series is tested for possible ARCH effects using the standard TR² test. The test can be implemented as follows: Estimate an AR(\( r \)) process in terms of the squared residuals, and the TR² statistic is computed as the product of the number of effective observations and the coefficient of multiple determination from the AR(\( r \)) regression. In addition, White's (1980) test for general heteroskedasticity is performed. This test is based on the method-of-moments approach. Unlike the TR² test for the ARCH effects, the White (1980) test does not assume any specific model of the structure of heteroskedasticity.

Table 2 contains the estimation and test results for autoregressions. The Ljung-Box statistics suggest that the residuals from all four regressions show no significant serial correlation, therefore supporting the adequacy of the respective autoregressive lag specification. The reported t-statistics for the autoregressions are computed from White's (1980) heteroskedasticity-consistent covariance matrix estimator. This allows proper inferences about the autoregressions to be made even in the presence of heteroskedasticity. As shown by the t-statistics, there are significant first- to third-order autocorrelations in all but the Japanese stock return series. The ARCH test statistics further indicate the presence of substantial
<table>
<thead>
<tr>
<th>Regressors and Statistics</th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>U.K.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.4135</td>
<td>0.2668</td>
<td>0.6676</td>
<td>0.5301</td>
</tr>
<tr>
<td></td>
<td>(1.9195)*</td>
<td>(0.9596)*</td>
<td>(2.2019)**</td>
<td>(2.0113)**</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>0.2286</td>
<td>0.3381</td>
<td>0.1899</td>
<td>0.3637</td>
</tr>
<tr>
<td></td>
<td>(4.1929)**</td>
<td>(5.8820)**</td>
<td>(1.3333)</td>
<td>(4.1386)**</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>-0.1608</td>
<td>0.0384</td>
<td>-0.1380</td>
<td>-1.8027*</td>
</tr>
<tr>
<td></td>
<td>(-2.7461)**</td>
<td>(0.5782)</td>
<td>(-1.8027)*</td>
<td></td>
</tr>
<tr>
<td>( \gamma_3 )</td>
<td>0.1214</td>
<td>0.2084**</td>
<td>0.0885</td>
<td>(1.7039)*</td>
</tr>
<tr>
<td></td>
<td>(2.0584)**</td>
<td>(2.0584)**</td>
<td>(2.0584)**</td>
<td>(2.0584)**</td>
</tr>
<tr>
<td>LB(6)</td>
<td>1.2385</td>
<td>5.1730</td>
<td>2.3100</td>
<td>8.9118</td>
</tr>
<tr>
<td>[p-value]</td>
<td>[0.9749]</td>
<td>[0.5218]</td>
<td>[0.8891]</td>
<td>[0.1786]</td>
</tr>
<tr>
<td>LB(12)</td>
<td>12.6232</td>
<td>18.4096</td>
<td>14.2872</td>
<td>17.2243</td>
</tr>
<tr>
<td>[p-value]</td>
<td>[0.0380]</td>
<td>[0.0103]</td>
<td>[0.2067]</td>
<td>[0.1414]</td>
</tr>
<tr>
<td>LB(16)</td>
<td>20.3303</td>
<td>20.8492</td>
<td>16.5341</td>
<td>24.2493</td>
</tr>
<tr>
<td>[p-value]</td>
<td>[0.3145]</td>
<td>[0.2853]</td>
<td>[0.5553]</td>
<td>[0.1455]</td>
</tr>
<tr>
<td>ARCH(6)</td>
<td>20.7713</td>
<td>18.0006</td>
<td>126.4439</td>
<td>36.0132</td>
</tr>
<tr>
<td>[p-value]</td>
<td>[0.0020]**</td>
<td>[0.0032]**</td>
<td>[0.0000]**</td>
<td>[0.0000]**</td>
</tr>
<tr>
<td>ARCH(12)</td>
<td>29.7867</td>
<td>25.6119</td>
<td>127.5359</td>
<td>36.7057</td>
</tr>
<tr>
<td>[p-value]</td>
<td>[0.0000]**</td>
<td>[0.0122]**</td>
<td>[0.0000]**</td>
<td>[0.0000]**</td>
</tr>
<tr>
<td>ARCH(18)</td>
<td>34.2562</td>
<td>37.1443</td>
<td>126.6452</td>
<td>45.3758</td>
</tr>
<tr>
<td>[p-value]</td>
<td>[0.0036]**</td>
<td>[0.0109]</td>
<td>[0.0000]**</td>
<td>[0.0007]**</td>
</tr>
<tr>
<td>White's Test</td>
<td>19.3023</td>
<td>58.2586</td>
<td>68.5810</td>
<td>95.0514</td>
</tr>
<tr>
<td>[p-value]</td>
<td>[0.0002]**</td>
<td>[0.0000]**</td>
<td>[0.0000]**</td>
<td>[0.0000]**</td>
</tr>
</tbody>
</table>

An ARCH(p) model is estimated for individual stock return series (\( \gamma \)). The lag parameter \( p \) of the model is selected using the Schwarz information criterion, with a maximum lag equal to 10 allowed. The figures in parentheses are \( t \)-statistics, computed using standard errors obtained from White's (1980) heteroskedasticity-consistent covariance matrix estimator. Statistical significance is indicated by * for 10%, by ** for 5% and by *** for 1%. The asymptotic distribution of the Ljung-Box statistic, LB(6), is \( \chi^2(6) \) under the null hypothesis of no serial correlation in the residual. The ARCH(6) statistic, obtained as \( \chi^2 \) from regressing the squared residual on a constant and its lagged values at 6 lags, is distributed asymptotically as \( \chi^2(6) \) under the null hypothesis of no ARCH effects. The White (1980) test provides statistics for a TR(2) test for general heteroskedasticity. The numbers in [ ] give the corresponding p-values of the LB statistics, the ARCH test statistics and the White test statistics.
ARCH effects in all the four stock return series. The results from the White (1980) test also confirm the presence of significant heteroskedasticity in all the return series under consideration: the null hypothesis of no heteroskedasticity can be rejected at the 1 percent significance level.

The results in Table 2 suggest in general the presence of either short-term dependence or heteroskedasticity or both in the stock return data. In view of the results, it is desirable that R/S tests for long memory should properly account for these stochastic properties in the data; otherwise, reliable statistical inferences cannot be drawn. In this regard, these results provide support for the use of the modified R/S test for its robustness to both short-term dependence and heteroskedasticity.

The results of the modified R/S test are reported in Table 3. For comparison, the value of the conventional R/S statistic (with \( q = 0 \)) is also reported for each stock return series. As shown in the results reported, if the analysis was based on the classical R/S test only, the null hypothesis of no long memory in stock returns could be rejected for Germany, Italy, and the United Kingdom at the 5 percent significance level, though not for Japan. As a result, researchers would be led to conclude that there was statistically significant evidence of long memory in the national stock markets in these countries. Such a conclusion would be misleading, however, because short-term dependence and heteroskedasticity were both shown to exist in the stock return data and the conventional R/S test

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>U.K.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical R/S test</td>
<td>2.1188**</td>
<td>1.9726**</td>
<td>1.2413</td>
<td>1.8397**</td>
</tr>
<tr>
<td>Modified R/S test</td>
<td>1.6196</td>
<td>1.5629</td>
<td>1.0569</td>
<td>1.4669</td>
</tr>
<tr>
<td>The q lag selected</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

Critical values at the 10% and 5% are respectively given by 1.620 and 1.747. Statistical significance for the one-sided test for long memory are indicated by * for 10% and ** for 5%. The lag parameter \( q \) used for the modified R/S test is determined by Andrews’s (1991) data-dependent rule (equation (5)). For the classical R/S test, \( q = 0 \) by construction.
could be biased toward rejecting the null hypothesis of no long memory too often. This problem can, nonetheless, be minimized by applying the modified R/S test. We observe that in constructing the modified R/S statistic, Andrews's (1991) lag selection rule generally suggests $q > 0$ be employed to adjust for short-term dependence and conditional heteroskedasticity in the data series. The empirical results obtained from the modified R/S test are in sharp contrast with those from the conventional R/S test. According to the modified R/S test results, the null hypothesis of no long memory cannot be rejected by the return data in any of the four stock markets, therefore suggesting little evidence of long cycles in the stock returns in these foreign markets.

IV. CONCLUSIONS

Previous empirical evidence on long cycles in stock returns is based primarily on the U.S. stock market. This study contributes to the literature by providing some international evidence from major foreign stock markets. A new R/S test developed by Lo (1991) is employed to examine the time series behavior of stock returns in the German, Italian, Japanese and U.K. markets. Unlike the conventional R/S analysis, the modified R/S analysis is robust to short-term dependence and conditional heteroskedasticity, that are generally shown to exist in the foreign stock return series. Our empirical results show that the conventional and new modified R/S analyses can lead to entirely different conclusions about the presence of long cycles in foreign stock returns. While the conventional R/S analysis seems to indicate the presence of long cycles in stock returns, no significant evidence of long cycles can be found using the modified R/S analysis once short-term dependence and conditional heteroskedasticity in the data are adjusted for.

Some additional remarks are as follows:

1. Our statistical results illustrate that the modified R/S analysis is in general preferred to the conventional one for providing more reliable evidence on long memory dynamics, especially when significant short-term dependence and conditional heteroskedasticity exist in the data.

2. The empirical failure to find long cycles in foreign stock
returns does not entirely deny their presence [see also the negative findings of long cycles in the U.S. stock market reported by Aydognan and Booth (1988) and Lo (1991)]. If long cycles are in fact present in stock returns, however, new supportive evidence is apparently needed to confirm their empirical relevance.

3. Studies by, e.g., Fama and French (1988) and Poterba and Summers (1988) provide empirical evidence indicating that stock returns display positive serial correlation over short horizons and negative correlation over longer horizons in the U.S. and some other industrialized countries. The evidence points to the presence of predictable components in long-horizon stock returns. Lo (1991) finds, however, that the potential long-run predictability of the U.S. stock returns in form of long cycles is not significant and the potential predictability can be the result more of short-term dependence than of long-term dependence. Given that Lo (1991) examines the U.S. stock returns only, the international evidence reported in this study supplements and reinforces the finding by Lo (1991).

4. While the modified R/S test is not sensitive to ARCH-type nonlinear dependence, little is known about the possible effects of chaotic dynamics on the test. The robustness of the modified R/S test to chaotic dynamics is an interesting topic for future research.

REFERENCES


