Productivity and the Role of the Global Acquisition Market

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Abstract

This paper presents a model of domestic and foreign acquisitions with heterogeneous firms. The model shows that acquisitions positively affect aggregate productivity by transferring capital from the least efficient firms to higher efficiency firms. However, contrary to the existing literature, these acquiring firms are in a mid-range of productivity. The results of the model show that the most productive firms do not find domestic acquisitions profitable in either a closed or open economy, and do not find foreign acquisitions profitable in relatively integrated open economies. This is a result of a variable-elasticity (linear) demand system, and an interaction between acquisitions that reduce production costs and acquisitions that enhance foreign market access. In using the model to evaluate the effects of trade liberalization on aggregate productivity and welfare, I identify a novel role of foreign acquisitions in tempering, and in some cases reversing, the beneficial effects of trade liberalization.

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1 Introduction

Without question, cross-border mergers and acquisitions (M&As) are one of the fastest growing aspects of globalization. In absolute terms, according to the OECD (2001), the value of cross-border M&As increased five-fold over the period 1990-1999. Relatively, the growth of cross-border M&As has also been substantial, where the share of North American firms that acquired cross-border rather than domestically increased 133% between 1985 and 2004.[1] Further, as noted by Navaretti and Venables (2006), cross-border M&As make-up a majority of foreign direct investment (FDI) between developed countries, and are increasing as a share of FDI to developing countries and transition economies.

However, a fact often lost in this discussion is that despite the rapid growth of cross-border M&As, domestic acquisitions are more common.[2] Further, despite the well-known "proximity" benefits of foreign investment, many incentives for cross-border M&As derive from factors beyond foreign market access. For example, market power, risk sharing, brand expansion, and cost reduction are all incentives that may motivate mergers of both the domestic and foreign variety. Hence, M&As to obtain foreign market access must not be examined in a vacuum, where all of the aforementioned incentives may influence the market clearing price for assets on a merger market.

In this paper, I examine the interaction between acquisitions for foreign market access and acquisitions for cost reduction. In doing so, I derive two novel theoretical contributions related to acquisition behavior. First, I show that using a common variable-elasticity (linear) demand system, firms that benefit from acquisitions for market access are not the same firms that benefit from acquisitions for cost reduction. Hence, the relative size of trade costs determines the qualitative sorting of firms into acquisition choices. Further, I use the model to evaluate the effects of trade liberalization on aggregate productivity and welfare, where I identify a novel role of foreign acquisitions in tempering, and in some cases reversing, the beneficial effects of trade liberalization.

The key to model is how acquisitions may reduce cost, and when they do, how these cost reductions result in higher profits. Generally, if production by a merged firm is less than the combined production of the old firms (a common prediction in almost all acquisition models), the acquisition allows the acquiring firm to reallocate production amongst the best assets of both firms.[3] Indeed, I adopt a functional form of variable costs in which an acquiring firm integrates the capital of the acquired firm into its own constant-returns production function. Hence, the acquisition of additional capital increases the efficiency of other factors that are variable at the time of production.

Critically, the demand function governs how cost reductions map into higher sales, and overall, higher profits. In many models of firm heterogeneity and investment, and often as a matter of analytical convenience, the constant-elasticity (CES) assumption is used to characterize demand. Under CES, high productivity firms operate on the relatively flat portion of the demand curve. Thus, even a small reduction in costs has the potential to yield huge gains in profits. This is in stark contrast with common non-CES demand systems, where the absolute elasticity of demand may be falling with quantity. In these cases, high productivity firms earn very little from a cost reduction as they already produce near the point where revenues are maximized. Therein lies the

[1] Author's calculation using the *Thomson SDC Platinum* database.

[2] For example, using the *Thomson SDC Platinum* database, the yearly mean and median foreign share of acquisitions over target SIC2-country pairs for worldwide mergers over the period 1980-2006 is uniformly less than 0.5.

[3] This might include reducing the amount of overtime shifts, firing the least productive workers, retiring the oldest machinery, or reducing output at inefficient plants and increasing output at more efficient plants.

first novel result in the paper. Using a linear demand system, I show that high productivity firms never engage in domestic acquisitions. Instead, mid productivity firms, who are relatively less constrained by demand, purchase the assets of low productivity firms, who themselves stand to earn little in the market all together.

A novel corollary of this result is that high productivity firms must search elsewhere to expand profits via an acquisition. Indeed, the incentives to acquire for additional foreign market access are largest for high productivity firms. Hence, a foreign market, if sufficiently protected by trade costs, will be a suitable alternative for high productivity firms. In this case, high productivity firms acquire abroad motivated by foreign market access, mid productivity firms acquire domestically motivated by cost reduction, and low productivity firms sell and exit. In contrast, when trade costs are relatively small, market access incentives are too small relative to the acquisition price to justify a foreign acquisition by high productivity firms. In this case, high productivity firms do nothing, and mid productivity firms acquire. Overall, trade costs not only have a natural effect on the domestic-foreign composition of acquisition demand, but also have an effect on the qualitative features of how firms - especially high productivity firms - select into investment choices. Indeed, the results of the model provide an simple example in which firms that benefit the most from increased market access (high productivity firms) cannot acquire abroad since they have been bid out of the acquisition market by firms motivated by other incentives.

The second novel contribution of the paper is the way in which trade costs influence aggregate efficiency and welfare, and in particular, how the presence of foreign acquisitions may overturn the canonical selection effects present in Melitz (2003). In previous models of trade and firm heterogeneity, the predominant form of selection and productivity improvement is increased competition, and the evolutionary effect of getting rid of producers which can no longer earn positive profits. Indeed, when trade costs fall in these models, competition toughens with increased import competition, and low productivity firms exit. In my model, this channel of selection is entirely closed, as the functional form of the model guarantees that all firms can profitably produce if they so choose (no fixed overhead costs). However, with an acquisition market, relatively unproductive firms may have an incentive to sell their assets to firms which are better able to utilize those assets. Given this transfer of resources from inefficient firms to more efficient firms, changes to the acquisition market have an effect on aggregate productivity.

Via this channel of reallocation, the effects of trade liberalization are provocative. I particular, I show that the presence of foreign acquisitions at a minimum tempers, and in some cases reverses, the beneficial effects of trade liberalization. The reason is simple. Trade liberalization, which mitigates the proximity advantage of foreign acquisitions, leads to a reduction in foreign demand for assets currently used by inefficient domestic firms, and hence, a reduction in the price that inefficient domestic firms receive for their assets in equilibrium. When only domestic acquisitions occur, this issue is not present, and trade liberalization necessarily improves aggregate productivity and welfare. However, when trade costs are large enough to support foreign acquisitions by some measure of firms, trade liberalization may reduce aggregate productivity and welfare by the mechanism described above. Indeed, when parameters are such that only foreign acquisitions occur, I show that trade liberalization always reduces aggregate productivity and welfare. Overall, the results of the model show that the degree to which trade liberalization is productivity and welfare enhancing depends crucially on the composition of the demand side of the acquisition market.

Related literature

Broadly, this paper adds to the growing literature examining the role of firm heterogeneity in trade and investment decisions (Jovanovic and Rousseau, 2002; Nocke and Yeaple, 2007; Breinlich, 2008). While the acquisition framework in the paper is particular stylized (loosely based on neoclassical investment models such as Hayashi, 1982), the results are much more general. The governing issue is that investment - in this case acquisitions - may affect variable efficiency, and when it does, the structure of demand matters. Without cost improvements, demand issues would not be relevant and the model would be based solely on issues discussed in Brainard (1997), and deliver predictions similar to Helpman, Melitz, and Yeaple (2004). Overall, the model suggests that industry specific factors related to demand and costs have a strong influence on equilibrium firm behavior. This is not unlike Nocke and Yeaple (2007), where the degree to which capabilities are transferrable across borders - an industry level attribute - has a critical effect on firm-level incentives for greenfield investment vis-a-vis acquisitions. Head and Reis (2003) is also similar, where low productivity firms may optimally invest abroad, hiring low-wage workers to offset their poor labor productivity. Finally, in a companion empirical paper, Spearot (2010), I show that when products are differentiated, mid-productivity firms are most likely to acquire another firm. When products are homogeneous, high productivity firms are most likely to acquire.

In terms of the policy results, the role of foreign acquisitions in tempering and potentially reversing the positive effects of trade liberalization, to my knowledge, is entirely new. In particular, the results are in stark contrast with the canonical results from Melitz (2003) and Melitz and Ottaviano (2008) which do not include a foreign-domestic reallocation of resources. One notable exception is Nocke and Yeaple (2007), who derive a result in which trade liberalization can reduce aggregate productivity. However, their result is via a completely different mechanism which is not directly related to foreign acquisition demand.[4]

Tangentially, the paper is related to the classic literature on mergers and acquisitions, in which the predominant explanation for M&A behavior is market power.[5] While I do not add to the literature discussing market power, my paper does identify the set of firms that can use cost-reductions as a viable merger defense. That is, unless acquisitions are for scope (new varieties) or market access, high productivity firms cannot use variable cost efficiencies as a merger defense.

Related to scope, a number of authors have modeled mergers and acquisitions as a form of expansion to additional varieties (for example, Nocke and Yeaple, 2006). My paper makes the strong assumption that acquisitions occur within the industry in which the firm currently operates. However, as long as there is a sufficient degree of substitution between the target industry and the industry of the acquiring firm, the incentives summarized above, and detailed below, will be relevant for acquisition decisions.[6]

[4] In Nocke and Yeaple (2007), the result is driven by the margin between exporting and selling. Lower trade costs increase the profitability of exporting, which reduces the measure of inefficient firms that sell. This implies a productivity loss.

[5] See the work of Salant, Switzer and Reynolds (1983) and Deneckre and Davidson (1985) for a classic discussion of the incentives for mergers driven by market power. For market power integrated with cost efficiencies, see Perry and Porter (1985) and Farrell and Shapiro (1990)

[6] A proof of this statement is available upon request.

Outline

The rest of the paper is organized as follows. In section two, I develop the closed economy acquisition model, highlighting the basic acquisition framework and why this framework leads to a new sorting of firms by productivity. In section three, I extend the closed economy model to an open economy model with two identical countries, where in particular, I focus on the role of trade costs in determining the qualitative characteristics of acquisition incentives. In section four, I evaluate the effects of trade costs on the equilibrium sorting of firms into acquisition decisions, aggregate productivity, and welfare. In section five, I briefly conclude.

2 Model

The basic closed economy model presented in this paper consists of three stages. In stage one, entry decisions are made. Firm-level productivity is uncertain and each potential entrant is ex-ante identical. Firms enter until their expected post-entry profits are equal to the fixed cost of entry. Upon entry, firms receive a fixed "lump" of capital that may be used in the product market.

In stage two, acquisition decisions are made. Post-entry, productivity is realized and firms are allowed to trade industry-specific capital on a perfectly competitive acquisition market. However, since investment behavior tends to be "lumpy" (Doms and Dunne, 1998), I assume that capital from the entry stage is indivisible in the acquisition stage; firms may not buy or sell fractions of capital. Additionally, due to unmodeled organizational factors, it is assumed that a firm only has enough resources to acquire one firm in the acquisition stage. Thus, firms are restricted to three options: sell all capital and exit, buy the capital of an exiting firm, or do nothing.

Finally, in stage three, each active firm supplies its individual variety to the product market. Active firms are monopolists in their own variety, taking other industry variables as given. At this point, any capital accrued during the entry and acquisition stages is fixed, and firms only procure variable factors.

The model is solved by backward induction, and will be introduced in this order.

2.1 Product Market Equilibrium

Consumers

Consumers have quasi-linear preferences over a differentiated industry and a numeraire good, x_0. Similar to Melitz and Ottaviano (2008), preferences of this sort can be written as:

$$U = x_0 + \theta \int_{i\in\Omega} q_i di - \frac{1}{2}\eta \left(\int_{i\in\Omega} q_i di \right)^2 - \frac{1}{2}\gamma \int_{i\in\Omega} (q_i)^2 \, di \tag{1}$$

In (1), Ω represents the measure of varieties, q_i is the consumption of variety i, and the parameters θ (> 0) and η (> 0) determine the substitution pattern between the differentiated industry and the numeraire. Finally, γ (> 0) represents the degree to which varieties are substitutable. If γ were zero, all firms would price at the same level, since products would be homogeneous in the eyes of the consumer.

In an economy with L consumers who each supply one unit of labor at a numeraire wage, the

inverse demand function for variety i can be derived as:

$$p_i = \underbrace{\theta - \eta M \overline{q}}_{A} - \frac{\gamma}{L} q_i = A - b q_i \tag{2}$$

In (2), p_i is the price of variety i, M is the measure of all varieties sold in the product market, and $\overline{q}$ is the average quantity sold of each variety. Naturally, competition will be "tougher" when M and/or $\overline{q}$ are high. Thus, the overall level of market "toughness" is captured in A, the residual demand level facing each firm. As all firms are small outside their own variety, firms take A as given. In equilibrium, A will be pinned down by a yet-to-be-presented free entry condition.

Firms

Capital influences firm decisions through the cost function. Similar to Perry and Porter (1985), the cost function of each firm takes the following form:

$$C\left(q_i | \alpha_i, K_i\right) = \frac{1}{2} \cdot \frac{q_i^2}{\alpha_i K_i} \tag{3}$$

In (3), α_i is firm-level productivity. Productivity is continuously distributed according to $G(\alpha)$, defined over $\alpha \in (0, \infty)$. The variable K_i represents capital accumulated during the initial stage and acquisition stage for firm i.[7] Firm-level productivity is transferrable across all holdings of capital within the firm.

A firm with productivity draw α_i and capital level K_i faces the following profit maximization problem in stage 3:

$$\begin{aligned} \pi\left(\alpha_i, K_i\right) &= \max_{q_i} \left\{ \left(A - b \cdot q_i\right) \cdot q_i - \frac{1}{2} \frac{q_i^2}{\alpha_i K_i} \right\} \\ st &: \quad q_i \geq 0 \end{aligned} \tag{4}$$

Solving (4) and dropping $i's$ for notational convenience, profits in the product market are written as:

$$\pi\left(\alpha, K\right) = \frac{A^2 \alpha K}{2\left(2b\alpha K + 1\right)} \tag{5}$$

In the closed economy, there are two types of firms active in the product market. Firms that "do nothing" (N) in the acquisition stage retain their initial capital level from entry, k, while firms that buy capital (B) in the acquisition stage double their initial capital level, holding $2k$. Firms that sell capital (S) in the acquisition stage are not active in the product market. Subject to these capital

[7] This cost-structure can be recovered from a Cobb-Douglas production function, given that the level of capital is fixed in the product market stage. In stage three, firms only procure variable factors at a price v per unit. The cost function is written as $C\left(X_i | v\right) = v \cdot X_i$. With equal intensity of capital and variable factors, the Cobb-Douglas production function can be written as $q_i = \left(2\alpha_i X_i\right)^{\frac{1}{2}} K_i^{\frac{1}{2}}$. Solving for X_i, and substituting into the above cost function, we get $C\left(Q_i | \alpha_i, v, K_i\right) = \frac{v}{2} \cdot \frac{q_i^2}{\alpha_i K_i}$. Normalizing v to equal 1 gives the desired result, $C\left(Q_i | \alpha_i, K_i\right) = \frac{1}{2} \cdot \frac{q_i^2}{\alpha_i K_i}$

positions, the profits for N and B, respectively, are expressed as:

$$\pi^N(\alpha) = \frac{A^2\alpha k}{(4b\alpha k+2)} \tag{6}$$

$$\pi^B(\alpha) = \frac{A^2\alpha k}{(4b\alpha k+1)} \tag{7}$$

where,

$$\pi^B(\alpha) > \pi^N(\alpha) \text{ for } \alpha \in (0,\infty)$$

Generally, since monopolists operate on the elastic portion of the demand curve, firms have incentive to increase production after a cost-lowering acquisition (an acquisition halves variable costs at every quantity). Shortly, I will evaluate the difference in $\pi^B(\alpha)$ and $\pi^N(\alpha)$ as a function of productivity.

Finally, given the properties of the cost function in (3), profits exhibit diminishing returns to capital. Thus, $\pi^N(\alpha)$ and $\pi^B(\alpha)$ have the following intuitive ranking:

$$\frac{1}{2}\pi^B(\alpha) < \pi^N(\alpha) < \pi^B(\alpha) \tag{8}$$

This property will be used when characterizing optimal firm-level acquisition decisions as a function of productivity.

2.2 Acquisition Stage Equilibrium

Optimal Acquisition Choice

Since firms are "small", I assume an acquisition market in which firm-level decisions have no effect on the market-clearing price per lump of capital, R_a, or the residual demand level, A. First, taking A and R_a as given, I derive optimal firm acquisition decisions as a function of productivity. Then, for a given A, I show that a unique value of R_a clears the acquisition market. Finally, I prove that there exists a unique value of A, subject to firm-level acquisition decisions and the market clearing price per firm, $R_a(A)$.

In the acquisition stage, firms must choose between three options: Sell their firm (S), do nothing (N), or buy capital (B). Respectively, the profits of each option in the acquisition stage are written as:

$$\Pi^S(R_a) = R_a \tag{9}$$

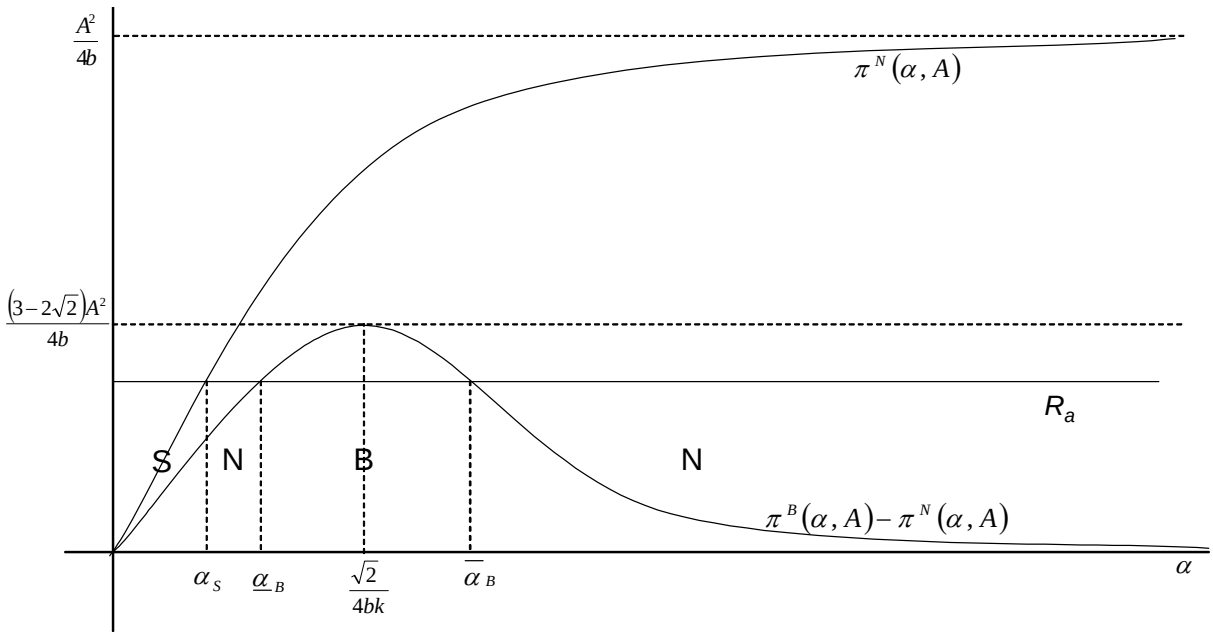
$$\Pi^N(\alpha, A) = \pi^N(\alpha, A) \tag{10}$$

$$\Pi^B(\alpha, A, R_a) = \pi^B(\alpha, A) - R_a \tag{11}$$

Here, the dependence of $\pi^N(\alpha, A)$ and $\pi^B(\alpha, A)$ on A in (6) and (7) is made explicit to emphasize that A is fixed for the moment. In (9), firms sell their capital, collect R_a, and exit the market. In (10), firms do nothing in the acquisition market and earn profits given their initial capital endowment, k. In (11), firms buy capital, earning $\pi^B(\alpha, A)$ in the product market after paying R_a for an additional lump of capital.

A firm of productivity α chooses the acquisition option which maximizes profits in the acquisition market. Defining $V(\alpha, A, R_a)$ as optimal acquisition market profits as a function of α, the acquisition

Figure 1: Optimal Acquisition Choice



decision of each firm is characterized by the following:

$$V(\alpha, A, R_a) = \max\left\{R_a, \pi^N(\alpha, A), \pi^B(\alpha, A) - R_a\right\} \tag{12}$$

In characterizing $V(\alpha, A, R_a)$, the critical component is the difference in profits between a domestic acquisition and doing nothing. Hence, I refer to $\pi^B(\alpha, A) - \pi^N(\alpha, A)$ as the benefit of an acquisition. As a function of model parameters, $\pi^B(\alpha, A) - \pi^N(\alpha, A)$ is written as:

$$\pi^B(\alpha, A) - \pi^N(\alpha, A) = \frac{A^2 \alpha k}{2(2b\alpha k + 1)(4b\alpha k + 1)} \tag{13}$$

It is straightforward to show that $\pi^B(\alpha, A) - \pi^N(\alpha, A)$ approaches zero for low and high α, and reaches its maximum on the interior at $\frac{\sqrt{2}}{4bk}$. The intuition for this property is as follows. On one end of productivity, the least efficient firms are limited by an intrinsically steep marginal cost schedule. Whether or not they acquire, they are still quite unproductive, and the absolute gains from an acquisition are tiny. In contrast, the most efficient firms are constrained not by costs, but by the structure of market demand. Specifically, the highest productivity firms operate on a less-elastic portion of the demand curve, which limits the incentive to expand production after a cost-lowering acquisition. Firms in a mid-range of productivity are constrained by neither, and earn relatively high returns from an acquisition. Thus, with linear demand, firms within a mid-range of productivity benefit the most from a cost-lowering acquisition.

The optimal acquisition decision derived from $V(\alpha, A, R_a)$ is illustrated in Figure 1. Here, for

$\alpha < \alpha_S$, the profits from selling are greater than profits from doing nothing. Also, the benefit of buying, $\pi^B(\alpha, A) - \pi^N(\alpha, A)$, is less than the acquisition price. Thus, selling is the dominant option for the least efficient firms. There will exist a positive measure of these selling firms for $R_a > 0$.

For "small" R_a ($R_a \leq \frac{(3-2\sqrt{2})A^2}{4b}$), firms with productivity between $\underline{\alpha}_B$ and $\overline{\alpha}_B$ find an acquisition profitable. For these firms, the benefit of an acquisition, $\pi^B(\alpha, A) - \pi^N(\alpha, A)$, is greater than the acquisition price. Additionally, for small R_a, there exist two disjoint regions of productivity such that doing nothing is optimal. These regions are labeled by N in Figure 1. For "large" R_a ($R_a > \frac{(3-2\sqrt{2})A^2}{4b}$), no firms find an acquisition profitable. The acquisition price is too large, where $\pi^B(\alpha, A) - \pi^N(\alpha, A) < R_a$ for all α. Naturally, since there exist selling firms and no buying firms, large R_a cannot be an acquisition market clearing price. Thus, I henceforth restrict attention to "small" R_a.

The features in Figure 1 follow closely the intuition discussed for firms of low, middle, and high productivity. Precisely, mid productivity firms have the highest incentive to acquire another firm. They are relatively less constrained by intrinsically high costs, which is the problem for low productivity firms. Further, they have additional room on the revenue side to expand production, which is not the case for the highest productivity firms.[8]

Equilibrium

Once again turning attention to Figure 1, α_S, $\underline{\alpha}_B$, and $\overline{\alpha}_B$ represent kinks in $V(\alpha, A, R_a)$. More precisely, these represent firms that are indifferent between acquisition options. Hence, α_S is implicitly defined as:

$$\pi^N(\alpha_S, A) = R_a \tag{14}$$

where,

$$For\ \alpha < \alpha_S\ ,\ S \succ N \tag{15}$$

The preference condition $S \succ N$ is a straightforward result when observing that stage three profits are increasing in productivity.

Similarly, $\underline{\alpha}_B$, and $\overline{\alpha}_B$ can be defined by:

$$\pi^B(\underline{\alpha}_B, A) - \pi^N(\underline{\alpha}_B, A) = R_a \tag{16}$$

$$\pi^B(\overline{\alpha}_B, A) - \pi^N(\overline{\alpha}_B, A) = R_a \tag{17}$$

where,

$$For\ \alpha \in (\underline{\alpha}_B, \overline{\alpha}_B)\ ,\ B \succ N \tag{18}$$

The condition $B \succ N$ is immediate from the shape of $\pi^B(\alpha, A) - \pi^N(\alpha, A)$.

Using the indifference conditions in (14), (16), and (17), and the preference conditions in (15) and (18), the following lemma proves that the features illustrated in Figure 1 are representative of optimal acquisition choice.

[8] This last point is critically dependent on the shape of the demand curve, and not the restriction that acquisition behavior is lumpy. See Spearot (2010) for a discussion of the role demand plays in acquisition decisions, with empirical support.

Lemma 1 *In the closed economy, given A and R_a, optimal acquisition choice is the following:*

$$\begin{array}{ll} For & \alpha \in [0, \alpha_S(A, R_a)), \textit{ firms sell} \\ & \alpha \in [\alpha_S(A, R_a), \underline{\alpha}_B(A, R_a)], \textit{ firms do nothing} \\ & \alpha \in (\underline{\alpha}_B(A, R_a), \overline{\alpha}_B(A, R_a)), \textit{ firms buy} \\ & \alpha \in [\overline{\alpha}_B(A, R_a), \infty) \textit{firms do nothing} \end{array}$$

Proof. See Appendix ■

With Lemma 1, given M_E entrants, the demand $(K_D(A, R_a))$ and supply $(K_S(A, R_a))$ of acquired capital are written as:

$$K_D(A, R_a) = M_E k \left(G(\overline{\alpha}_B(A, R_a)) - G(\underline{\alpha}_B(A, R_a))\right) \tag{19}$$

$$K_S(A, R_a) = M_E k G(\alpha_S(A, R_a)) \tag{20}$$

The acquisition price, R_a, affects $K_D(A, R_a)$ and $K_S(A, R_a)$ through the acquisition cutoffs $\alpha_S(A, R_a)$, $\underline{\alpha}_B(A, R_a)$ and $\overline{\alpha}_B(A, R_a)$. Of course, the acquisition market clears if,

$$K_D(A, R_a) = K_S(A, R_a). \tag{21}$$

For a given value of A, there is a unique $R_a(A)$ that clears the acquisition market. This is proven in the following Lemma:

Lemma 2 *Holding A fixed, there exists a unique $R_a(A)$ that clears the acquisition market.*

Proof. See Appendix ■

The intuition behind Lemma 2 is a simple case of supply and demand. The measure of buying firms is decreasing in the acquisition price, and the measure of selling firms is increasing in the acquisition price. Given that no firms are willing to sell at $R_a = 0$ and no firms are willing to buy at $R_a \geq \frac{(3-2\sqrt{2})A^2}{4b}$, the demand and supply functions cross only once at the equilibrium acquisition price, $R_a(A)$.

Finally, with the acquisition market clearing condition in-hand, I now show that there exists a unique equilibrium value of A. In particular, the equilibrium value of A is pinned down by the following free-entry condition:

$$\int_0^\infty V(\alpha, A, R_a(A)) dG(\alpha) = F_E \tag{22}$$

The following lemma proves that there exists a unique equilibrium value of A.

Lemma 3 *There exists a unique equilibrium value of A (> 0).*

Proof. See Case 1 in Appendix D. ■

The intuition for Lemma 3 is fairly simple. The effects of A on (22) are only through the profit functions within $V(\alpha, A, R_a(A))$, and not the productivity cutoffs or the acquisition price. Regarding the latter two, the productivity cutoffs cancel out in a similar fashion to the envelope theorem (they are optimally chosen given A and $R_a(A)$), and the acquisition price $R_a(A)$ is simply a transfer and hence has no direct effect on average profits. Regarding the effect of A on profit

functions, higher A increases all profit functions (from zero), and thus as long as F_E isn't too large, there exists a unique value of A that satisfies the free entry condition.

Finally, given the definition of A in (2), I can prove that there exists a unique measure of entering firms, M_E:

Lemma 4 *There exists a unique equilibrium value of entering firms, M_E (> 0).*

Proof. See Appendix. ■

With M_E firms entering, since a $G(\alpha_S)$ share of those firms exit, the measure of active firms M is pinned down by $M = M_E\,(1 - G(\alpha_S))$.

With Lemmas 1-4, the following Proposition summarizes the equilibrium in the closed economy.

Proposition 1 *The closed economy acquisition equilibrium consists of a unique A, R_a, M_E, α_S, $\underline{\alpha}_B$, and $\overline{\alpha}_B$ such that:*

$$\begin{aligned} For \quad & \alpha \in [0, \alpha_S), \textit{ firms sell} \\ & \alpha \in [\alpha_S, \underline{\alpha}_B]\,, \textit{ firms do nothing} \\ & \alpha \in (\underline{\alpha}_B, \overline{\alpha}_B)\,, \textit{ firms buy} \\ & \alpha \in [\overline{\alpha}_B, \infty), \textit{ firms do nothing} \end{aligned}$$

Proof. Follows directly from Lemmas 1-4. ■

The highlight of Proposition 1 is that the highest productivity firms acquire nothing. These firms operate on a less-elastic portion of the demand curve, which limits the incentive to expand production after a cost-lowering acquisition. In contrast, as discussed in the introduction, the highest productivity firms *would* acquire if I assumed CES demand or a setting in which firms were price takers. Thus, when acquisitions reduce production costs, the structure of competition and demand are important components of the equilibrium acquisition decisions of heterogeneous firms.

3 Open Economy

In the previous section, acquisition incentives are derived for firms that procure capital and sell products in the same market. In an open economy, firms may also buy capital as well as sell products in distant markets. In this section, I detail how the incentives to acquire another firm are affected by the location of capital and product markets. In particular, I focus on how the costs of trade affect acquisition decisions, and hence, how the costs of trade affect aggregate productivity and welfare through reallocation in the acquisition market.

To capture the standard features of the trade and investment literature as simply as possible, I assume that there are two countries which are identical in every dimension, each with segmented markets for products (varieties) and assets. Thus, if profitable, firms may sell varieties in either country, and/or buy capital for use in either country. Focusing on a firm in the home market for the remainder of the paper, let K_i^H and K_i^F represent capital holdings at home and in foreign, respectively. Further, if the firm wishes to export to another market, a trade cost, t, is incurred per unit of exports.

Again, the model will consist of three stages: entry, acquisitions, and product market. Solving the model backward from the product market stage, the inverse demand function for each variety in each country is $p_i = A - b \cdot q_i$, where given the assumption of symmetry, the $A's$ will be identical

in each country. As in the closed economy, the $A's$ will be pinned down in industry equilibrium by a free entry condition.

For any value of trade cost, each firm solves the following profit maximization problem:

$$\pi\left(\alpha_i, K_i^H, K_i^F\right) = \max_{q_i, q_i^x, q_i^F} \left\{ \begin{array}{c} \overbrace{\underbrace{(A - b \cdot q_i) \cdot q_i}_{\text{home}} + \underbrace{\left(A - b \cdot \left(q_i^x + q_i^F\right)\right) \cdot \left(q_i^x + q_i^F\right)}_{\text{foreign}}}^{\text{revenues}} \\ \underbrace{-\frac{1}{2}\frac{1}{\alpha_i} \cdot \underbrace{\frac{(q_i^x + q_i)^2}{K_i^H}}_{\text{home}} - \frac{1}{2}\frac{1}{\alpha_i} \cdot \underbrace{\frac{\left(q_i^F\right)^2}{K_i^F}}_{\text{foreign}}}_{\text{production costs}} - \underbrace{t \cdot q_i^x}_{\text{trade costs}} \end{array} \right\} \tag{23}$$

$$\textit{such that} \quad : \quad q_i \geq 0 \; q_i^F \geq 0 \; \textit{and} \; q_i^x \geq 0$$

In (23), q_i is home production of variety i for sale in the home market, q_i^x is home production of variety i for sale in the foreign market, and q_i^F is foreign production of variety i for sale in the foreign market. These production levels are dictated by costs in each country, which are a function of the capital holdings of firm i in each country, K_i^H at home and K_i^F abroad. These capital holdings will be determined in the acquisition stage.[9] Lastly, one critical feature in (23) is that export production at home increases the costs of non-export production at home, and vice versa. This will be a particularly important feature when discussing foreign acquisitions.

Prior to acquisition decisions, all firms are endowed with k units of capital at home, and zero units of capital abroad. For the sake of tractability, I will assume that firms choose one of the following options in the acquisition market: (S) Sell all domestic capital and exit, (N) do nothing, (B) buy an additional "lump" of domestic capital, or (B^*) buy an additional "lump" of foreign capital.

Generally, if firms remain "domestic", in the sense that they purchase no foreign capital in the acquisition market ($K_i^F = 0$), profits are written as:

$$\pi\left(\alpha, K^H, 0\right) = \left\{ \begin{array}{ll} \frac{A^2 \alpha K^H}{4b\alpha K^H + 2} & \alpha \leq \frac{t}{2bK^H(A-t)} \\ \\ \frac{A^2 \alpha K^H}{2b\alpha K^H + 2} - \frac{t(4bA\alpha K^H - 2b\alpha K^H t - t)}{4b(2b\alpha K^H + 2)} & \alpha > \frac{t}{2bK^H(A-t)} \end{array} \right\} \tag{24}$$

In (24), where $i's$ have been dropped for notational convenience, when $\alpha \leq \frac{t}{2bK^H(A-t)}$, firms do not export. If a firm's productivity is too low, the maximum trade-cost adjusted marginal revenue of serving the foreign market is lower than the equilibrium marginal cost of only serving the domestic market. If $\alpha > \frac{t}{2bkK^H(A-t)}$, the opposite is the case, where firms have low enough production costs such that additional domestic production intended for exports is optimal. Note that this cutoff decreases as the level of domestic capital holdings, K^H, increases. Conditional on productivity, exporting is more likely when firms hold more capital at home.

[9] Note that I have written (23) such that no firms from home produce varieties abroad and export them back home. This will never occur in equilibrium, as given acquisitions choices, firms will never have lower production costs abroad than at home, and thus will never profitably export from foreign to home.

Product market profits of options (N) and (B) are defined as:

$$\begin{aligned} \pi^N(\alpha) &\equiv \pi(\alpha, k, 0) \\ \pi^B(\alpha) &\equiv \pi(\alpha, 2k, 0) \end{aligned}$$

As in the closed economy, $\pi^B(\alpha) > \pi^N(\alpha)$, since variable costs are lower after an acquisition. Of course, the difference in $\pi^B(\alpha)$ and $\pi^N(\alpha)$ will be weighed against the acquisition price, and the added option of foreign acquisitions.

Finally, firms that purchase a foreign firm (B^*) hold k units of capital at home, and k units of capital abroad. Subject to these capital holdings, profits in the product market are written as:

$$\pi^{B^*}(\alpha) \equiv \pi(\alpha, k, k) = \frac{A^2 \alpha k}{(2b\alpha k + 1)} \tag{25}$$

Since firms have an equal amount of capital in each country (identical marginal cost curves), profit maximization will never include exports. Thus, trade costs do not enter equilibrium profits when buying foreign capital.

3.1 Acquisition Stage

Firms must choose between four options in the acquisition stage. Respectively, the profits of selling, doing nothing, buying domestic capital, and buying foreign capital, can be written as:

$$\begin{aligned} \Pi^S(R_a) &= R_a \\ \Pi^N(\alpha) &= \pi^N(\alpha) \\ \Pi^B(\alpha, R_a) &= \pi^B(\alpha) - R_a \\ \Pi^{B^*}(\alpha, R_a) &= \pi^{B^*}(\alpha) - R_a - \delta \end{aligned}$$

Firms pay a fixed cost, R_a, for each lump of capital. This acquisition price will be endogenously determined, in equilibrium, using a market clearing condition for assets in each country. Given the assumption that countries are symmetric in every dimension, similar to the $A's$ I will impose that the acquisition price is the same in each country. For the moment, since firms are assumed to be small outside their own variety, R_a will be taken as given. Finally, note that in the last equation, firms acquiring abroad pay an additional fixed cost of serving the foreign market, δ. This is to embody the additional organizational, legal, or marketing costs associated with serving a foreign market.

A firm of productivity α chooses the acquisition option which maximizes profits in the acquisition market. Defining $V(\alpha)$ as optimal acquisition market profits (suppressing R_a and A for brevity in labeling $V(\alpha)$), the acquisition choice problem is written as:

$$V(\alpha) = \max\left\{R_a, \pi^N(\alpha), \pi^B(\alpha) - R_a, \pi^{B^*}(\alpha) - R_a - \delta\right\} \tag{26}$$

Before characterizing the relevant kinks in $V(\alpha)$, I will discuss the components of $V(\alpha)$ as they pertain to domestic and foreign acquisitions relative to the option of doing nothing.

Domestic Acquisitions

The benefit of a domestic acquisition relative to doing nothing is written as $\pi^B(\alpha) - \pi^N(\alpha)$. Defining $\Delta\Pi(\alpha) = \pi^B(\alpha) - \pi^N(\alpha)$, as a function of model parameters, I can write $\Delta\Pi(\alpha)$ as follows:

$$\begin{aligned} \Delta\Pi(\alpha) &= \pi^B(\alpha) - \pi^N(\alpha) \\ &= \left\{ \begin{array}{lll} \frac{A^2\alpha k}{2(2b\alpha k+1)(4b\alpha k+1)} & if \quad 0 & \leq \alpha < \frac{t}{4bk(A-t)} \\ \frac{A^2\alpha k}{(2b\alpha k+1)} - \frac{t(8bA\alpha k-4b\alpha kt-t)}{8b(2b\alpha k+1)} - \frac{A^2\alpha k}{(4b\alpha k+2)} & \frac{t}{4bk(A-t)} & \leq \alpha < \frac{t}{2bk(A-t)} \\ \frac{A^2\alpha k}{2(b\alpha k+1)(2b\alpha k+1)} - \frac{tk\alpha(4A-t)}{8(b\alpha k+1)(2b\alpha k+1)} & \frac{t}{2bk(A-t)} & \leq \alpha \end{array} \right\} \end{aligned} \tag{27}$$

In (27), the incentives to acquire a domestic firm are split-up into three regions of productivity. For low values of α, firms cannot export before or after an acquisition, and the incentive to acquire a domestic firm is identical to the closed economy model. For firms in a middle range of productivity, the acquisition of additional capital provides the cost-improvement required to make exporting profitable. For high productivity firms, exporting is profitable before and after an acquisition. The incentives available to these firms are a combination of the incentives under free trade ($\frac{A^2\alpha k}{2(b\alpha k+1)(2b\alpha k+1)}$), and the negative effect of positive trade costs ($\frac{tk\alpha(4A-t)}{8(b\alpha k+1)(2b\alpha k+1)}$).

Despite the multiple regions of productivity, the following lemma proves that the incentives to acquire a domestic lump of capital are qualitatively identical to those in the closed economy.

Lemma 5 *There exists a productivity level $\widehat{\alpha}$ such that*

$$\begin{array}{lll} \frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} > 0 & if & \alpha < \widehat{\alpha} \\ \frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} < 0 & if & \widehat{\alpha} < \alpha \end{array}$$

Furthermore,

$$\Delta\Pi(0) = 0 \quad \lim_{\alpha\to\infty} \Delta\Pi(\alpha) = 0 \quad \tfrac{1}{2}\pi^B(\alpha) < \pi^N(\alpha) < \pi^B(\alpha)$$

Proof. See Appendix ■

As in the closed economy, the incentive to acquire domestic capital is highest for mid productivity firms. Low productivity firms are constrained by their steep cost curve pre and post-acquisition. High productivity firms are constrained by bounds on variety specific revenues. Mid productivity firms are relatively less-constrained by each, and have the largest incentive to acquire domestic capital.

Foreign Acquisitions

With domestic acquisitions, despite having two potential markets in which to sell varieties, the incentives to buy additional capital for the home market are qualitatively identical to the closed economy. In contrast, with foreign acquisitions, firms may buy capital in a market in which they currently own no capital. Hence, this section will examine how the added element of proximity influences the acquisition decisions of heterogeneous firms.

The profits from acquiring foreign capital relative to doing nothing, $\Delta\Pi^*(\alpha)$, are written as:

$$\begin{aligned} \Delta\Pi^*(\alpha) &= \pi^{B^*}(\alpha) - \pi^N(\alpha) - \delta \\ &= \left\{ \begin{array}{ll} \frac{A^2\alpha k}{2(2b\alpha k+1)} - \delta & \alpha \leq \frac{t}{2bk(A-t)} \\ \\ \frac{A^2\alpha k}{2(b\alpha k+1)(2b\alpha k+1)} - \delta + \frac{t(4bA\alpha k - 2b\alpha kt - t)}{8b(b\alpha k+1)} & \alpha > \frac{t}{2bk(A-t)} \end{array} \right\} \end{aligned} \tag{28}$$

In (28), $\Delta\Pi^*(\alpha)$ is split into two regions of productivity. The first region is for firms of low enough productivity such that exporting is not profitable given k units of capital at home. Here, a foreign acquisition simply provides additional market access. In contrast, higher productivity firms export before a foreign acquisition, and thus acquiring a foreign firm not only affords *additional market access*, but also *diverts export production* to a newly purchased foreign affiliate. The former is embodied in the last term, which is a positive benefit of direct market access for potential exporters ($\frac{t(4bA\alpha k-2b\alpha kt-t)}{8b(b\alpha k+1)}$). The latter is embodied in the first term ($\frac{A^2\alpha k}{2(b\alpha k+1)(2b\alpha k+1)}$), which as will be explained shortly, is identical to the benefits of a domestic acquisition under free trade.

It is the production diversion, and the corresponding effect on production costs, which is crucial to the relationship between trade costs and foreign acquisition incentives. This can be seen by deriving the relevant properties of $\Delta\Pi^*(\alpha)$.

Lemma 6 $\Delta\Pi^*(\alpha)$ *has the following properties:*

$$\begin{aligned} \Delta\Pi^*(0) &= -\delta \\ \lim_{\alpha\to\infty} \Delta\Pi^*(\alpha) &= \frac{(2A-t)\,t}{4b} - \delta \\ \frac{1}{2}\pi^{B*}(\alpha) &\leq \pi^N(\alpha) < \pi^{B*}(\alpha) \end{aligned}$$

Furthermore, if $t < \frac{2}{2+\sqrt{2}}A$, *there exists an* $\widehat{\alpha}^*$ *such that:*

$$\begin{aligned} \frac{\partial\Delta\Pi^*(\alpha)}{\partial\alpha} > 0 \quad & \textit{if} \quad \alpha < \widehat{\alpha}^* \\ \frac{\partial\Delta\Pi^*(\alpha)}{\partial\alpha} < 0 \quad & \qquad \alpha > \widehat{\alpha}^* \end{aligned}$$

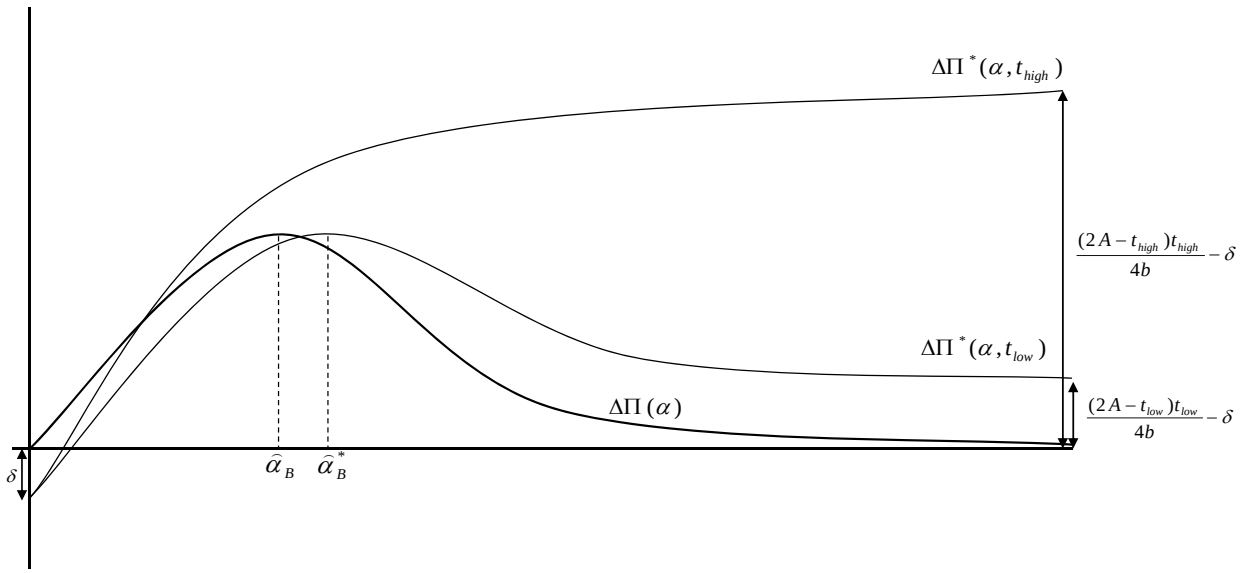
Conversely, if $t \geq \frac{2}{2+\sqrt{2}}A$, *then* $\frac{\partial\Delta\Pi^*(\alpha)}{\partial\alpha} > 0$ *for all* α.

Proof. See Appendix ■

Lemma 6 is an entirely novel result, and one which is central to the open economy model. It states that if trade costs are low ($t < \frac{2}{2+\sqrt{2}}A$), mid productivity firms will have the highest incentive to acquire a foreign firm. In contrast, for high trade costs ($t \geq \frac{2}{2+\sqrt{2}}A$), high productivity firms have the highest incentive to acquire a foreign firm.

The intuition for this result is best explained by how trade costs affect incentives for market access and variable factor efficiency. If trade costs are non-zero, purchasing a foreign firm always provides additional market access. Similar to Helpman, Melitz, and Yeaple (2004), the incentive to gain additional market access is increasing in productivity. High productivity firms are the

Figure 2: Acquisition Incentives - Open Economy



largest exporters, and given that exporting costs are independent of productivity, these firms have the highest incentive to avoid these costs. Thus, *absent any other incentives*, high productivity firms always have the largest incentive to acquire abroad.

However, in this particular model, there are additional incentives on the cost-side. Not only does purchasing a foreign firm provide additional market access, but it will divert export production that would otherwise be produced at home. Critically, given a fixed amount of capital at home, lower production at home decreases the average variable cost of producing at home.[10] Hence, by acquiring foreign capital, a firm can reduce domestic production costs in a fashion similar to a domestic acquisition. And, as has been discussed above, mid productivity firms have the highest incentive to acquire to reduce production costs when faced with a bounded market for their variety.

Thus, the central issue is whether market access (proximity) or production cost (concentration) considerations dominate. As characterized in Lemma 6, the relative size of trade costs provides a resolution to this issue. This is also illustrated in Figure 2. When trade costs are relatively high (t_{high} in Figure 2), exports are relatively low, and the incentives to gain additional market access dominate. Thus, high productivity firms have the highest incentive to acquire abroad. In contrast, when trade costs are low (t_{low} in Figure 2), foreign sales via exports are already significant and the incentives to gain additional market access are modest. Hence, the incentives to reduce production costs are now relatively large, and mid productivity firms have the highest incentive to acquire abroad

Essentially, one can think of foreign acquisitions as a skewed version of domestic acquisitions. If trade costs are zero, they function identically to domestic acquisitions, (except for the fixed cost

[10]Indeed, this is the critical difference with other cost assumptions, such as total costs written as $C(q) = \frac{1}{\alpha k}q$. Here, diverting export production would yield no cost-side effects.

δ). However, as trade costs increase, the incentives to acquire a foreign firm become skewed toward firms of higher productivity. Eventually, if trade costs are high enough, the incentive to acquire a foreign firm is an increasing function in productivity, and looks nothing like the incentive to acquire a domestic firm.

Optimal Acquisition Choice

With acquisition incentives in-hand, I will characterize the indifference points in $V(\alpha)$ as they pertain to the choice between selling, doing nothing, acquiring domestically, and acquiring abroad. First, I will evaluate preferences over S, N, and B. Then, I will do the same for S, N, and B^*. Finally, I evaluate preferences over B and B^*.

Firms are indifferent between doing nothing and selling at α_S. This cutoff is defined by the following:

$$\pi^N(\alpha_S) = R_a \tag{29}$$

where,

$$For\ \alpha < \alpha_S\ ,\ S \succ N \tag{30}$$

Firms prefer selling if the acquisition price is greater than the return from staying in the market.

At $\underline{\alpha}_B$ and $\overline{\alpha}_B$, firms are indifferent between doing nothing and buying domestic capital. However, since there is another acquisition option to influence the acquisition price, it is not guaranteed that $\underline{\alpha}_B$ and $\overline{\alpha}_B$ are defined. Precisely, if $\max_{\alpha} \Delta\Pi(\alpha) < R_a$, $\underline{\alpha}_B$ and $\overline{\alpha}_B$ are not defined. In contrast, if $\max_{\alpha} \Delta\Pi(\alpha) > R_a$,

$$\Delta\Pi(\underline{\alpha}_B) = R_a \tag{31}$$

$$\Delta\Pi(\overline{\alpha}_B) = R_a \tag{32}$$

where,

$$For\ \alpha \in (\underline{\alpha}_B, \overline{\alpha}_B), B \succ N \tag{33}$$

The following lemma summarizes the relative position of the productivity cutoffs α_S, $\underline{\alpha}_B$ and $\overline{\alpha}_B$, and the implications this has for the choice between selling, domestic acquisitions, and doing nothing.

Lemma 7 *If* $\max_{\alpha} \Delta\Pi(\alpha) > R_a$, *then*

$$\begin{array}{ll} For & \alpha \in [0, \alpha_S),\ S \succ N \succ B \\ & \alpha \in [\alpha_S, \underline{\alpha}_B],\ N \succ S \succ B \\ & \alpha \in (\underline{\alpha}_B, \overline{\alpha}_B),\ B \succ N \succ S \\ & \alpha \in [\overline{\alpha}_B, \infty),\ N \succ S \succ B \end{array}$$

Proof. Using the properties in Lemma 5, identical to the closed economy. ■

As foreshadowed in the results in Lemma 5, the incentives to acquire domestically, and the resulting preference conditions between domestic acquisitions, doing nothing and selling, do not change relative to the closed economy.

Moving forward, I now characterize the indifference points in $V(\alpha)$ between foreign acquisitions and doing nothing. From Lemma 6, we see that the shape of $\Delta\Pi^*(\alpha)$ is critically dependent on the value of trade costs. For low t, the upper and lower limits of $\Delta\Pi^*(\alpha)$ are tiny (and possibly negative), with a maximum at a mid value of α. In contrast, for high t, $\Delta\Pi^*(\alpha)$ is strictly increasing

in α. Hence, when t is relatively low, there exists the possibility of two productivity cutoffs at which firms are indifferent between doing nothing and acquiring abroad. In contrast, when t is relatively high, there will be at most one indifference point.

To put more structure on the choice between foreign acquisitions and doing nothing, first suppose that $\max_{\alpha} \Delta\Pi^*(\alpha) > R_a$ and $\lim_{\alpha\to\infty} \Delta\Pi^*(\alpha) < R_a$. Here, the maximum of $\Delta\Pi^*(\alpha)$ is above the acquisition price, and the upper and lower limits of $\Delta\Pi^*(\alpha)$ are below the acquisition price. For this case, define $\underline{\alpha}_{B^*}$ and $\overline{\alpha}_{B^*}$ as the productivity levels at which firms are indifferent between foreign acquisitions and doing nothing. Precisely, these two cutoffs are defined by:

$$\Delta\Pi^*(\underline{\alpha}_{B^*}) = R_a \tag{34}$$

$$\Delta\Pi^*(\overline{\alpha}_{B^*}) = R_a \tag{35}$$

where given Lemma 6 and the assumption that $\max_{\alpha} \Delta\Pi^*(\alpha) > R_a$ and $\lim_{\alpha\to\infty} \Delta\Pi^*(\alpha) < R_a$,

$$For\ \alpha \in (\underline{\alpha}_{B^*}, \overline{\alpha}_{B^*}),\ B^* \succ N \tag{36}$$

Next, in contrast with the previous case, assume that $\max_{\alpha} \Delta\Pi^*(\alpha) > R_a$ and $\lim_{\alpha\to\infty} \Delta\Pi^*(\alpha) > R_a$. Here, the parameters of the model are such that the maximum and upper limits of $\Delta\Pi^*(\alpha)$ are both above the acquisition price. In this case, there exists a unique productivity level, $\underline{\alpha}_{B^*}$, at which firms are indifferent between foreign acquisitions and doing nothing. Precisely,

$$\Delta\Pi^*(\underline{\alpha}_{B^*}) = R_a$$

where,

$$For\ \alpha \in (\underline{\alpha}_{B^*}, \infty),\ B^* \succ N \tag{37}$$

Again, this case is more likely when t is high. Finally, if $\max_{\alpha} \Delta\Pi^*(\alpha) < R_a$, foreign acquisitions are never profitable relative to doing nothing, and neither $\underline{\alpha}_{B^*}$ or $\overline{\alpha}_{B^*}$ are defined.

The preference conditions between selling, doing nothing, and foreign acquisitions are proven in the following lemma.

Lemma 8 *If* $\max_{\alpha} \Delta\Pi^*(\alpha) > R_a$ *and* $\lim_{\alpha\to\infty} \Delta\Pi^*(\alpha) < R_a$, *then*

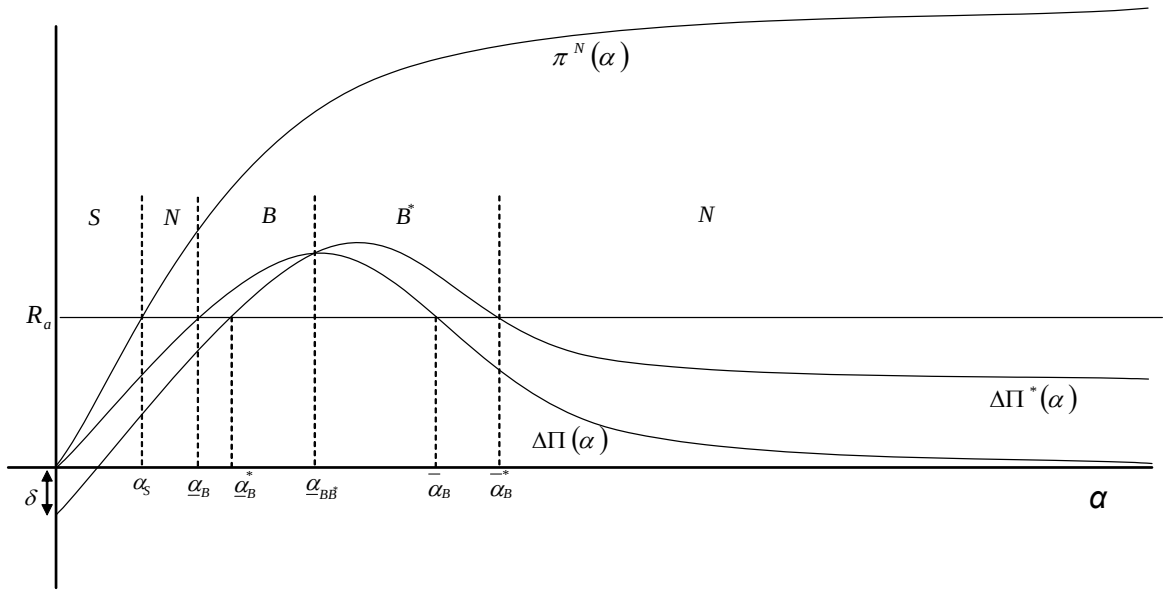
$$\begin{aligned} For\quad & \alpha \in [0, \alpha_S),\ S \succ N \succ B^* \\ & \alpha \in [\alpha_S, \underline{\alpha}_{B^*}],\ N \succ S \succ B^* \\ & \alpha \in (\underline{\alpha}_{B^*}, \overline{\alpha}_{B^*}),\ B^* \succ N \succ S \\ & \alpha \in [\overline{\alpha}_{B^*}, \infty),\ N \succ S \succ B^* \end{aligned}$$

If $\max_{\alpha} \Delta\Pi^*(\alpha) > R_a$ *and* $\lim_{\alpha\to\infty} \Delta\Pi^*(\alpha) > R_a$, *then*

$$\begin{aligned} For\quad & \alpha \in [0, \alpha_S),\ S \succ N \succ B^* \\ & \alpha \in [\alpha_S, \underline{\alpha}_{B^*}],\ N \succ S \succ B^* \\ & \alpha \in (\underline{\alpha}_{B^*}, \infty),\ B^* \succ N \succ S \end{aligned}$$

Proof. See appendix. ■

Figure 3: Optimal Acquisition Choice - Open Economy



In Lemma 8, the preference conditions between selling, doing nothing, and acquiring a foreign firm are a function of the relative shape of $\Delta\Pi^*(\alpha)$ around R_a, which is ultimately a function of the relative value of trade costs. That is, when t is small and the incentives to avoid trade costs are low, mid productivity firms are the only firms that prefer a foreign acquisition to doing nothing. In contrast, when t is high and the incentives to avoid trade costs are high, the only firms that prefer a foreign acquisition to doing nothing are high productivity firms.

Lemmas 7 and 8 describe the indifference conditions of each acquisition option, respectively, relative to doing nothing and selling. However, I have yet to characterize the decision between domestic acquisitions and foreign acquisitions. This indifference condition is summarized in the following lemma:

Lemma 9 *If* $\frac{(2A-t)t}{4b} < \delta$, $B \succ B^*$ *for all* α. *If* $\frac{(2A-t)t}{4b} > \delta$, $B^* \succ B$ *if* $\alpha > \alpha_{BB^*}$, *where* α_{BB^*} *is defined by:*

$$\Delta\Pi^*(\alpha_{BB^*}) = \Delta\Pi(\alpha_{BB^*})$$

Proof. See Appendix. ■

In Lemma 9, when comparing domestic acquisitions and foreign acquisitions, the highest productivity firms are more likely to prefer a foreign acquisition to a domestic acquisition. Intuitively, as they are larger potential exporters, higher productivity firms have a larger incentive to avoid trade costs by owning equal amounts of capital at home and abroad. However, there do exist parameter values ($\frac{(2A-t)t}{4b} < \delta$) such that domestic acquisitions are always prefered relative to foreign acquisitions. In this case, the maximum incentive to avoid trade costs ($\frac{(2A-t)t}{4b}$) is less than the added fixed cost of foreign investment (δ).

To pin-down the equilibria of the model, I must compare the values of $\underline{\alpha}_B$, $\overline{\alpha}_B$, $\underline{\alpha}_{B^*}$, $\overline{\alpha}_{B^*}$, and α_{BB^*}, and the corresponding preference relationships in Lemmas 7-9. Given a fixed A and R_A,

there are many possible rankings of $\underline{\alpha}_B$, $\overline{\alpha}_B$, $\underline{\alpha}_{B^*}$, $\overline{\alpha}_{B^*}$, and α_{BB^*}, and thus, many possible equilibria which are qualitatively different (all possible equilibria are summarized in Appendix D). While I will later detail a number of different equilibria using simulations, the novel features of the acquisition model are displayed in Figure 3, where I have illustrated one possible solution to the acquisition choice problem for a specific pair of t and δ in which both domestic and foreign acquisitions occur, and $\underline{\alpha}_B$, $\overline{\alpha}_B$, $\underline{\alpha}_{B^*}$, $\overline{\alpha}_{B^*}$, and α_{BB^*} are all defined. In Figure 3, the least productive firms choose to sell and exit. Firms in a mid-range of productivity find some type of acquisition profitable, where the most-efficient firms within this group prefer foreign acquisitions. Of the remaining active firms, the least productive and most productive choose to do nothing in the acquisition market. For the highest productivity firms, even though they have the highest incentive to avoid trade costs, these incentives are not sufficient to compensate for the acquisition price, which itself has been bid-up by mid productivity firms primarily for purposes of cost reduction.

A novel feature of Figure 3 is that the set of firms that acquire domestically and the set of firms that acquire abroad are non-nested. In other words, acquiring domestically is not the next best option for every firm that acquires abroad, and vice versa. For some firms, the next best option to a domestic or foreign acquisition is no acquisition. The crucial implication of this feature for the forthcoming policy analysis is that there exist margins through which both domestic and foreign acquisition demand can influence *total* acquisition demand. For example, in response to higher trade costs, if $\overline{\alpha}_{B^*}$ is defined, it will rise into the region of productivity which supports no acquisitions. In contrast, if $\overline{\alpha}_{B^*}$ is not defined, the only expansion of foreign acquisitions will occur through either $\underline{\alpha}_{B^*}$ or α_{BB^*}, where in the latter case the expansion of foreign acquisitions will only affect the composition of acquisition demand, not total acquisition demand.

Figure 3 also describes a novel pattern of trade and investment. That is, exporters are not necessarily less productive than those firms investing abroad. This suggests that in relatively integrated markets, such that the costs of international commerce are low, exporters should be the most productive.[11] On a basic level, this prediction may seem counterintuitive since the most productive firms tend to have the largest incentive to avoid trade costs. However, if trade costs are relatively low, this incentive is relatively small compared to the benefits mid-productivity firms can earn from acquisitions that are primarily for cost reduction. Hence, when trade costs are low, high-productivity firms are bid out of the acquisition market.

3.2 Equilibrium

To close the model, I now characterize the industry equilibrium components of the acquisition market, R_a and A. Despite the rich and diverse possibilities of acquisition behavior, I can show that there exists a unique solution to both of these aggregate measures, and thus, a unique solution to the acquisition market itself. For the sake of simplicity, let Θ_S represent the measure of firms that sell and exit the market, and let Θ_B and Θ_{B^*} represent the measure of firms that buy domestic and foreign capital, respectively. Note that Θ_{B^*} may only be comprised of one productivity cutoff,

[11] This could be an interesting prediction to test in future work using detailed census data. Indeed, this is similar to the discussion in Nocke and Yeaple (2007) in which the sorting of firms into modes of foreign investment depends crucially on the form of heterogeneity. The analogous result in this paper is that the basic characteristics of cost-structure and demand at the industry level motivate a similar degree of care in approaching empirical work. Indeed, one could use industry-specific regressions and non-parametric methods to evaluate the probability of exporting as function of productivity. As detailed by the above model, industries in which markets are relatively integrated may exhibit a non-monotone relationship between the probability of exporting and productivity.

and that either Θ_B and Θ_{B^*} may be empty. Finally, let Θ_N represent the measure of firms that choose not to participate in the acquisition market.

Subject to firm-level acquisition decisions, the acquisition market must clear. Formally, this condition is written as:

$$\overbrace{\int_{\alpha\in\Theta_B} dG(\alpha) + \int_{\alpha\in\Theta_{B^*}} dG(\alpha)}^{Demand} = \overbrace{\int_{\alpha\in\Theta_S} dG(\alpha)}^{Supply} \tag{38}$$

It is straightforward to show that there exists a unique acquisition market clearing price, R_a. Domestic and foreign acquisition demand are both decreasing in the acquisition price, both equaling zero when the acquisition price is high enough. In contrast, supply is increasing in the acquisition price from zero. Thus, there exists an intersection of demand and supply at $R_a(A)$ such that there is as positive level of acquisition activity. Note that since it is guaranteed that $R_a(A) > 0$, it must be the case that either Θ_B or Θ_{B^*} remain non-empty. That is, some acquisitions always occur in equilibrium.

The aggregate demand level A is once again determined by a free entry condition. The free entry condition in each country, after imposing the acquisition market clearing condition, is written as:

$$\int_{\alpha\in\Theta_N} \pi^N(\alpha)dG(\alpha) + \int_{\alpha\in\Theta_B} \pi^B(\alpha)dG(\alpha) + \int_{\alpha\in\Theta_{B^*}} (\pi^{B^*}(\alpha) - \delta)dG(\alpha) = F_E \tag{39}$$

Using (39), uniqueness of A can be proven in a manner similar to the closed economy, and is done so in Appendix D for all possible cases of acquisition behavior (also summarized in Appendix D). To summarize the technique, the proof similar to the envelope theorem (though does not invoke it due to the discrete nature of the firm's optimization problem). Precisely, A has no effect on the free entry condition through the productivity cutoffs contained within Θ_N, Θ_B, and Θ_{B^*} since the productivity cutoffs are optimally chosen given A. Thus, the only relevant effects of A through the free entry condition are through the profit functions, where all profit functions are increasing in A. Thus, so long as F_E is not too high, there exists a unique value of A which satisfies the free entry condition in (39).

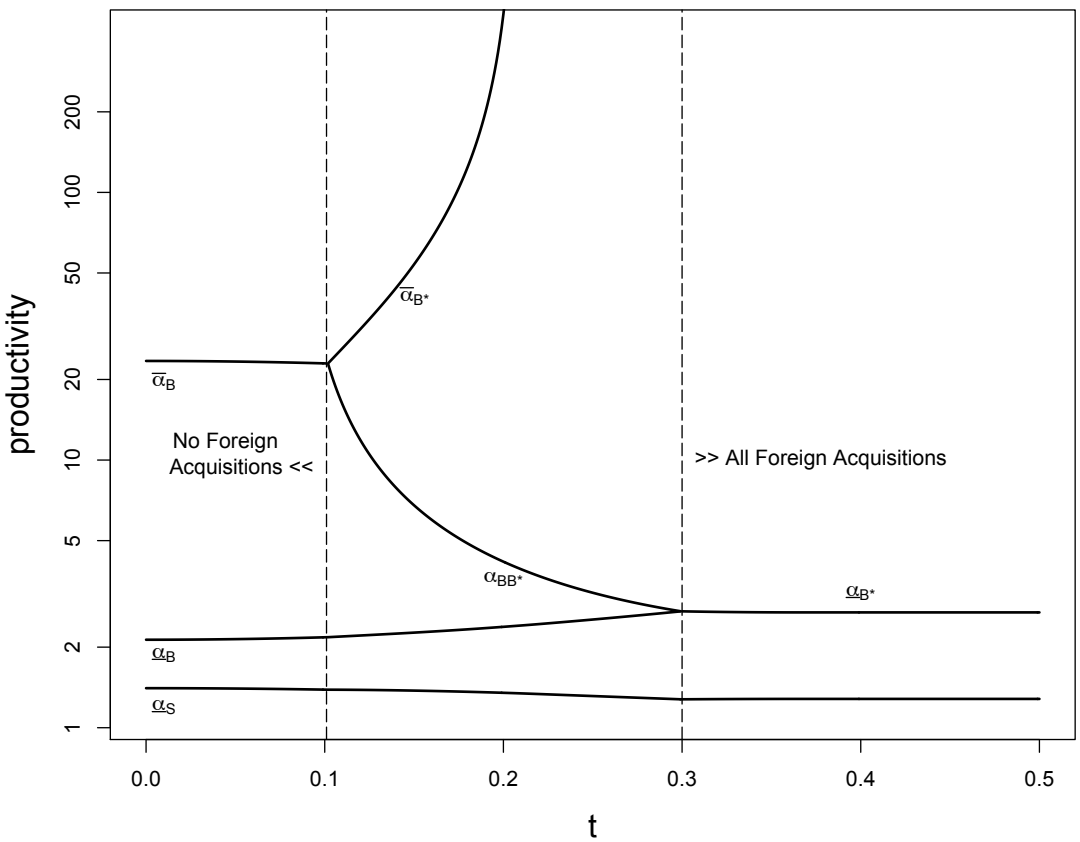
4 Trade Liberalization

In this section, I evaluate the effect of trade costs on acquisition behavior, where I begin by showing that trade costs have intuitive effect on the composition of domestic and foreign acquisitions. I continue by illustrating a novel effect of trade costs on aggregate productivity and welfare. In particular, I show that the degree to which liberalization of trade barriers improves aggregate productivity and welfare depends crucially on whether foreign acquisitions are an important component of the acquisition market, and whether foreign acquisitions affect total acquisition demand rather than simply the composition of acquisition demand.

4.1 Acquisitions and Trade liberalization

To examine the effect of trade costs on firm-level decisions and aggregate measures, I have chosen to conduct a number of simulation exercises. The details of the simulation are contained in the

Figure 4: Equilibrium Acquisition Choice and Trade Costs



appendix.[12] Of note, I choose relatively simple values of baseline parameters to evaluate the effect of trade costs, and in particular, a differential effect of trade costs as a function of the relative degree and type of foreign acquisition penetration. To begin, I will detail the acquisition equilibrium itself over a wide range of trade costs.

Figure 4 details the effect of trade costs on the acquisition decisions of firms, in equilibrium. To begin, focus on the region labeled "No Foreign Acquisitions". Here, trade costs are relatively low compared with the fixed cost of foreign investment, δ. Thus, in equilibrium, only domestic acquisitions occur within this region. Further, the scale of domestic acquisition activity falls with trade costs. The intuition is that trade costs reduce the effective size of the world market. As a smaller market reduces in the incentive to acquire another firm, acquisition demand falls, pushing down the equilibrium acquisition price, resulting in fewer inefficient firms selling. Indeed, this is

[12]While I choose to use simulations to make a clear and concise point, the model can be solved rigorously (with a substantial amount of algebra). For analytical results of model with small t and small δ, please see the working paper version of the paper available at the author's website.

exactly the response discussed in Breinlich (2008).[13]

Within the middle region of Figure 4, we have a case in which trade costs are now large enough to support acquisitions abroad. Here, as trade costs increase, the measure of firms which choose domestic acquisitions $(\underline{\alpha}_B, \alpha_{BB^*})$ continues to shrink while the measure of firms which choose foreign acquisitions $(\alpha_{BB^*}, \overline{\alpha}_{B^*})$ widens (from nothing). Thus, within the middle region of Figure 4, the share of foreign acquisitions increases with higher trade costs. Further, within this same middle region of Figure 4, relatively high values of t yield a case in which $\overline{\alpha}_{B^*}$ is no longer defined. Thus, for relatively low trade costs within this region, foreign acquisitions have a direct effect on total acquisition demand. However, for relatively high trade costs within this region, $\overline{\alpha}_{B^*}$ is undefined, and only domestic acquisitions affect total acquisition demand (through the productivity cutoff $\underline{\alpha}_B$). This is a novel feature of the acquisition framework outlined in the previous section, and will be a crucial issue within the forthcoming discussion of aggregate productivity.

Finally, in the right-most region of Figure 4, domestic acquisitions no longer occur, since whenever they are profitable relative to the acquisition price, foreign acquisitions are more profitable. Given the parameters of the simulation, the response of the acquisition market to trade costs is fairly tepid within this region. In moment, I will illustrate a few other cases in which the acquisition market is more responsive when only foreign acquisitions occur.

4.2 Trade liberalization, Aggregate Productivity, and Welfare

In previous models of trade and firm heterogeneity, the predominant form of selection and productivity improvement is increased competition, and the evolutionary effect of getting rid of producers which can no longer profitably produce. Indeed, when trade costs fall in these models, competition toughens with increased import competition, and low productivity firms exit. In my model, this channel of selection is entirely closed, as all firms can profitably produce if they so choose. However, relatively unproductive firms may have an incentive to sell their assets to firms which are better able to utilize those assets. Critically, falling trade costs will affect foreign and domestic acquisition demand in opposite directions. In equilibrium, I will show that these competing effects can result in a rich and decidedly non-monotonic relationship between trade costs and aggregate productivity.

To evaluate the effects of exogenous parameters, I first define the following measure of aggregate productivity, $\widetilde{\alpha}$:

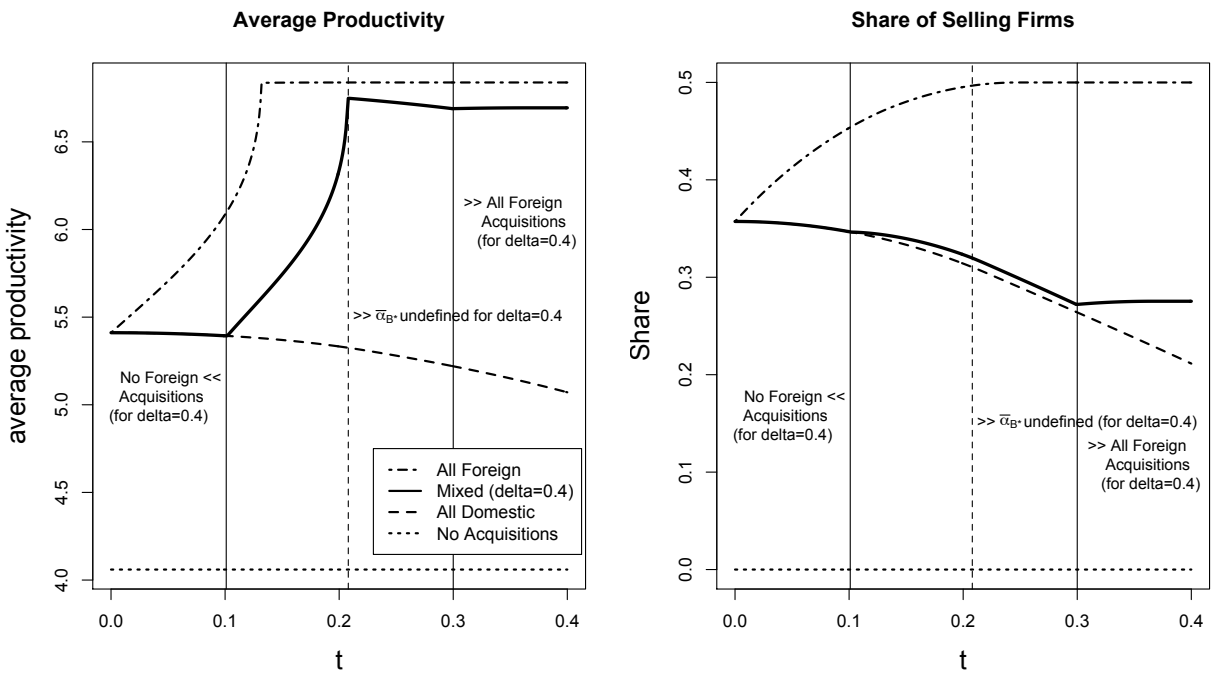
$$\widetilde{\alpha} = s_{NA}\widetilde{\alpha}_{NA} + s_A\widetilde{\alpha}_A$$

Here, $\widetilde{\alpha}_{NA}$ represents the average productivity applied to units of capital which do not change ownership in the acquisition market, and $\widetilde{\alpha}_A$ reflects the average productivity applied to capital which changes ownership (acquired) in the acquisition market. Further, s_{NA} and s_A represent the respective shares of each group. Thus, $\widetilde{\alpha}$ will represent average productivity of firms operating in the product market, weighted by capital holdings.[14] In terms of model parameters, $\widetilde{\alpha}$ is written

[13] He also tests this prediction on a firm-level sample of US and Canadian firms, where the response of the acquisition market to trade costs is confirmed in the data. That is, trade liberalization expands acquisition activity. However, one important feature within the US-Canada case study is that domestic acquisitions comprise roughly 95% of all acquisitions. Thus, Breinlich's empirical work is well motivated by a model with only domestic acquisitions.

[14] It can be shown that this measure is equivalent to average sales per worker. See Spearot (2010) for a derivation.

Figure 5: Trade Costs, Aggregate Productivity, and Acquisition Activity



as,[15]

$$\widetilde{\alpha} = \int_{\alpha_S}^{\infty} \alpha g(\alpha) d\alpha + \int_{\alpha \in \Theta_B} \alpha g(\alpha) d\alpha + \int_{\alpha \in \Theta_{B^*}} \alpha g(\alpha) d\alpha$$

Figure 5 illustrates the effects of trade costs on capital-weighted average productivity, and the simple fraction of firms that decide to sell and exit. The latter may also be interpreted as a measure of productivity, where given that selling firms are the least productive, a higher fraction of selling firms implies that the measure of remaining firms in each country (in terms of intrinsic productivity) is, on average, more productive.

In both panels of Figure 5, I have presented the results from four different simulations. The dark black line represents the same simulation as is used in Figure 4. The dotted-dashed line represents the case in which foreign investment costs are negligible, and thus, only foreign acquisitions occur.

[15] The naive function is written as

$$\begin{aligned}
\widetilde{\alpha} \quad = \quad & \frac{1 - \Pr(\alpha \in \Theta_S)}{1 - \Pr(\alpha \in \Theta_S) + \Pr(\alpha \in \Theta_B) + \Pr(\alpha \in \Theta_{B^*})} \cdot \frac{1}{1 - \Pr(\alpha \in \Theta_S)} \int_{\alpha_S}^{\infty} \alpha g(\alpha) d\alpha \\
& + \frac{\Pr(\alpha \in \Theta_B)}{1 - \Pr(\alpha \in \Theta_S) + \Pr(\alpha \in \Theta_B) + \Pr(\alpha \in \Theta_{B^*})} \cdot \frac{1}{\Pr(\alpha \in \Theta_B)} \int_{\alpha \in \Theta_B} \alpha g(\alpha) d\alpha \\
& + \frac{\Pr(\alpha \in \Theta_{B^*})}{1 - \Pr(\alpha \in \Theta_S) + \Pr(\alpha \in \Theta_B) + \Pr(\alpha \in \Theta_{B^*})} \cdot \frac{1}{\Pr(\alpha \in \Theta_{B^*})} \int_{\alpha \in \Theta_{B^*}} \alpha g(\alpha) d\alpha
\end{aligned}$$

Via the acquisition market clearing condition, $1 - \Pr(\alpha \in \Theta_S) + \Pr(\alpha \in \Theta_B) + \Pr(\alpha \in \Theta_{B^*}) = 1$, and $\widetilde{\alpha}$ is simplified as written above.

The dashed line represents the case when foreign investment costs are prohibitive and only domestic acquisitions occur. Finally, the dotted (horizontal) line represents the case in which there is no acquisition market. For this "benchmark" case, trade costs do not affect aggregate productivity since all firms can profitably produce. Thus, the traditional method by which firms select in and out (as in Melitz and Ottaviano) of the market is shut down, and hence, aggregate productivity is invariant to trade costs.

A number of striking features are evident in Figure 5. Most notably, higher trade costs can actually increase aggregate productivity, and increase the degree of reallocation that occurs. Consider first the case in which δ is negligible and only foreign acquisitions occur. This is the dotted-dashed line in the Figure 5. Here, when trade costs rise, more firms find it profitable to buy the capital of an inefficient firm in the foreign market. This yields potentially three effects, all of which tending to increase aggregate productivity. First, in the right panel of Figure 5, the increase in demand for foreign capital pushes up the acquisition price, causing a greater share of inefficient firms to sell capital and exit. This will tend to increase the average productivity of active firms in each country. Further, there is more capital available to be transferred from the inefficient to more efficient, and this yields a positive effect on the capital-adjusted aggregate productivity measure. The final effect, which is specific to this particular model, is that the measure of firms that find a foreign acquisition profitable will shift toward higher productivity firms. Thus, in the above measure of aggregate productivity, those firms which hold two lumps of capital rather than one will tend to be more productive. Overall, for the case where domestic acquisitions do not occur, with higher trade costs, not only do fewer inefficient firms choose to stay in the market, but the firms buying capital tend to be on balance more productive.

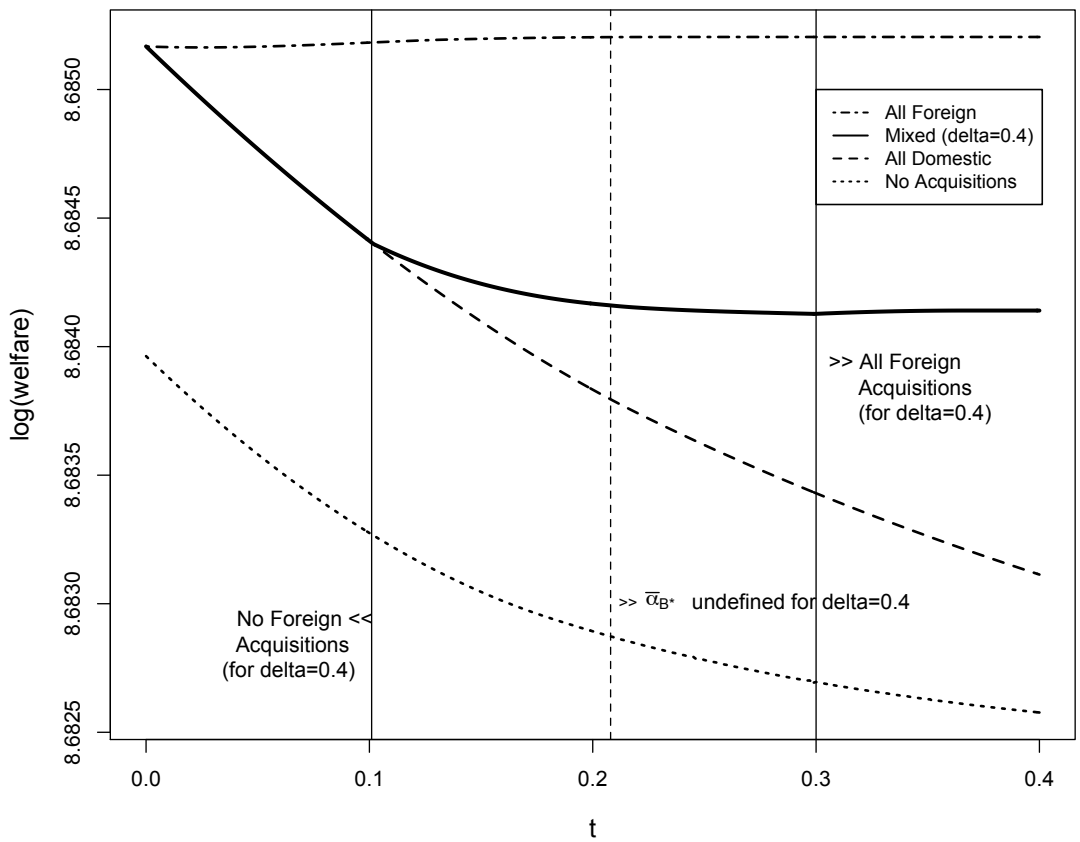
To push this point further, consider the dark black line in Figure 5, which plots aggregate productivity as a function of trade costs for the same parameters as in Figure 4. For this case, depending on the value of trade costs, the foreign share of the acquisition market may be zero, one, or between zero and one. In the left-most region of Figure 5, no foreign acquisitions occur, and higher trade costs decrease productivity. Here, since the effective world market is smaller, fewer capital transfers occur, and fewer inefficient firms sell and exit.

When increasing trade costs to the point of initial foreign acquisition penetration, aggregate productivity begins to improve with trade liberalization. In this case, changes to aggregate productivity are purely a function of the characteristics of buying firms. Indeed, in the right panel of Figure 5, it is clear that the measure of firms selling is actually falling, which would tend to reduce aggregate productivity without a transfer of capital to more productive firms. However, as long as $\overline{\alpha}_{B^*}$ is defined, the measure of acquiring firms expands to higher productivity, and aggregate productivity improves. Once trade costs increase to the point where $\overline{\alpha}_{B^*}$ isn't defined (the vertical dotted line), aggregate productivity falls with trade costs once again.

Overall, the simulations in Figure 5 detail a diverse response of aggregate productivity and acquisition behavior to trade costs. Different from the existing literature, I make an important point. Specifically, the reciprocal liberalization of trade barriers need not increase aggregate productivity. The reason is simple. Trade liberalization, which reduces the incentives that drive foreign acquisitions, may actually reduce the reallocation of capital between inefficient firms in each country and foreign firms that tend to be quite productive.

Finally, I now examine how aggregate productivity improvements map into welfare. To do so, I calculate welfare via the utility function in (1). The issue at hand is that although productivity may improve with higher trade costs, there may be a reduction in product variety by a more robust

Figure 6: Welfare and Trade Costs



reallocation in the acquisition market. Figure 6 illustrates the effects of trade costs on log welfare for the same simulations as in Figure 5. In Figure 6, the effects of trade costs on log welfare may be positive. In particular, when only foreign acquisitions occur, trade costs have a modest positive effect on welfare, which is similar to the effect on aggregate productivity. In contrast, when only domestic acquisitions occur, trade costs reduce welfare, since not only are exports more expensive, but the average active firm is also less productive. When both domestic and foreign acquisitions occur, the effects of trade costs on welfare are generally ambiguous, though it appears that in Figure 6 foreign acquisition penetration at a minimum tempers the negative effect of trade costs on welfare. Finally, regardless of type, acquisitions appear to increase welfare relative to the baseline simulation in which foreign acquisitions do not occur.[16]

[16]This is consistent across a number of different utility parameters. These figures are available upon request.

5 Conclusion

This paper presented an industry-equilibrium model of acquisition behavior in which firms trade capital after productivity has been realized. The incentives for acquisition, driven by firm heterogeneity in productivity, result from cost-lowering acquisitions and a linear demand framework. The main results of the model show that in a closed economy, mid productivity firms are the most likely to acquire another firm. In an open economy model, while foreign acquiring firms are more productive than domestic acquiring firms, both groups of firms reside in a mid-range of productivity if trade costs are relatively small.

In evaluating the changing costs of trade, I show that trade liberalization improves aggregate productivity and welfare only under certain conditions. Precisely, trade liberalization necessarily improves aggregate productivity and welfare only if foreign acquisitions do not occur. Indeed, simulation exercises show that foreign acquisitions at a minimum temper, and in some cases reverse, the negative effect of trade costs on aggregate productivity and welfare.

Overall, this paper adds a number of empirical questions onto the ever growing list of issues to evaluate with firm-level data. The basic prediction that mid productivity firms are more likely to acquire is confirmed in a companion paper, Spearot (2010), using the *Compustat* database. However, while a number of authors have established a positive effect of foreign acquisitions vis-a-vis domestic acquisitions on firm-level performance (Arnt and Anselm, 2008; Arnold and Sarzynska-Javorcik, 2009), the predictions of the open economy model, and in particular the novel relationship between trade liberalization and aggregate productivity via reallocation in the acquisition market, have yet to be tested. This is an issue I plan to take up in future empirical work.

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A Closed Economy Proofs

Proof of Lemma 1

To prove Lemma 1 for small R_a, I need to establish that $\alpha_S < \underline{\alpha}_B < \overline{\alpha}_B$. Once this ranking is established, Lemma 1 is immediate via the preference conditions in (15) and (18). To show $\alpha_S < \underline{\alpha}_B$, first note that from (14) and (16) it must be the case that:

$$\pi^B\left(\underline{\alpha}_B\right) - \pi^N\left(\underline{\alpha}_B\right) = \pi^N\left(\alpha_S\right)$$

Rearranging,

$$\frac{1}{2}\pi^B\left(\underline{\alpha}_B\right) - \pi^N\left(\underline{\alpha}_B\right) = \pi^N\left(\alpha_S\right) - \frac{1}{2}\pi^B\left(\underline{\alpha}_B\right)$$

Since $\frac{1}{2}\pi^B\left(\underline{\alpha}_B\right) < \pi^N\left(\underline{\alpha}_B\right)$, the RHS must also be negative in equilibrium. This is only possible if $\alpha_S < \underline{\alpha}_B$. By definition, $\underline{\alpha}_B < \overline{\alpha}_B$. Using the preference relationships in (15) and (18).

Proof of Lemma 2

Holding A fixed, differentiating the acquisition profit functions (14), (16), and (17) with respect to R_a, and using the properties of $\frac{\pi^B(\alpha)}{\partial\alpha} - \frac{\pi^N(\alpha)}{\partial\alpha}$, yields:

$$\begin{aligned}
\overbrace{\frac{\pi^N\left(\alpha_S\left(R_a\left(A\right),A\right)\right)}{\partial\alpha}}^{>0}\frac{\partial\alpha_S\left(R_a\left(A\right),A\right)}{\partial R_a\left(A\right)} &= 1 \\
\overbrace{\left(\frac{\pi^B\left(\underline{\alpha}_B\left(R_a\left(A\right),A\right)\right)}{\partial\alpha} - \frac{\pi^N\left(\underline{\alpha}_B\left(R_a\left(A\right),A\right)\right)}{\partial\alpha}\right)}^{>0}\frac{\partial\underline{\alpha}_B\left(R_a\left(A\right),A\right)}{\partial R_a\left(A\right)} &= 1 \\
\overbrace{\left(\frac{\partial\pi^B\left(\overline{\alpha}_B\left(R_a\left(A\right),A\right)\right)}{\partial\alpha} - \frac{\pi^N\left(\overline{\alpha}_B\left(R_a\left(A\right),A\right)\right)}{\partial\alpha}\right)}^{<0}\frac{\partial\overline{\alpha}_B\left(R_a\left(A\right),A\right)}{\partial R_a\left(A\right)} &= 1
\end{aligned}$$

These derivatives clearly yield $\frac{\partial\alpha_S(R_a(A),A)}{\partial R_a(A)} > 0$, $\frac{\partial\underline{\alpha}_B(R_a(A),A)}{\partial R_a(A)} > 0$, and $\frac{\partial\overline{\alpha}_B(R_a(A),A)}{\partial R_a(A)} < 0$. Differentiating (19) and (20) with respect to $R_a(A)$, and imposing $\frac{\partial\alpha_S(R_a(A),A)}{\partial R_a(A)} > 0$, $\frac{\partial\underline{\alpha}_B(R_a(A),A)}{\partial R_a(A)} > 0$, and $\frac{\partial\overline{\alpha}_B(R_a(A),A)}{\partial R_a(A)} < 0$,

$$\frac{\partial K_D\left(R_a\left(A\right)\right)}{\partial R_a} = M_E k g(\overline{\alpha}_B\left(R_a\left(A\right),A\right))\frac{\partial\overline{\alpha}_B\left(R_a\left(A\right),A\right)}{\partial R_a\left(A\right)} - M_E k g(\underline{\alpha}_B\left(R_a\left(A\right),A\right))\frac{\partial\underline{\alpha}_B\left(R_a\left(A\right),A\right)}{\partial R_a\left(A\right)} < 0$$

$$\frac{\partial K_S\left(R_a\left(A\right)\right)}{\partial R_a\left(A\right)} = M_E k\left(g(\alpha_S\left(R_a\left(A\right),A\right))\frac{\partial\alpha_S\left(R_a\left(A\right),A\right)}{\partial R_a\left(A\right)}\right) > 0$$

Naturally, $K_D\left(R_a\left(A\right)\right)$ is decreasing and $K_S\left(R_a\left(A\right)\right)$ is increasing in the acquisition price. Thus, if $K_D\left(R_a\right)$ is larger (smaller) than $K_S\left(R_a\right)$ at low (high) values of R_a, there exists a unique R_a that clears the acquisition market. When $R_a \to 0$, using (9), (10) and (11), it is clear that $\alpha_S\left(R_a\left(A\right),A\right) \to 0$, $\underline{\alpha}_B\left(R_a\left(A\right),A\right) \to 0$, and $\overline{\alpha}_B\left(R_a\left(A\right),A\right) \to \infty$. Hence, $K_D\left(R_a\left(A\right)\right) \to M_E k$

and $K_S\left(R_a\left(A\right)\right) \to 0$. Similarly, as $R_a \to \frac{\left(3-2\sqrt{2}\right)A^2}{4b}$, $\alpha_S\left(R_a\left(A\right), A\right) > 0$, $\underline{\alpha}_B\left(R_a\left(A\right), A\right) \to \frac{\sqrt{2}}{4bk}$, and $\overline{\alpha}_B\left(R_a\left(A\right), A\right) \to \frac{\sqrt{2}}{4bk}$. In terms of demand and supply, $K_D\left(R_a\left(A\right)\right) \to 0$, $K_S\left(R_a\left(A\right)\right) > 0$. Thus, there exists a unique value $\widehat{R}_a\left(A\right) > 0$ that clears the acquisition market.

Proof of Lemma 4

Note that in (2), $A = \theta - \eta M \overline{q}$. Since a fraction $G(\alpha_S)$ of firms sell and exit, M_E, the measure of entering firms, is defined according to $M = M_E(1 - G(\alpha_S))$. Hence, M_E is defined as:

$$M_E = \frac{\theta - A}{\eta(1 - G(\alpha_S))\overline{q}} > 0$$

As $\overline{q}$ is pinned down by variables which have been shown to be unique (α_S, $\underline{\alpha}_B$, $\overline{\alpha}_B$, and A), there also exists a unique value of $\overline{q} > 0$. Further, supposing that M is positive, $\overline{q} > 0$ guarantees that the unique value of A is less than θ. Hence, there exists a unique equilibrium measure of entering firms $M_E > 0$.

B Open Economy Proofs

Proof of Lemma 5

To begin the derivation of Lemma 5, I will focus on $\alpha \leq \frac{t}{4bk(A-t)}$. Differentiating $\frac{A^2\alpha k}{2(4\alpha kb+1)(2\alpha kb+1)}$ with respect to α yields:

$$\frac{\partial \Delta\Pi\left(\alpha\right)}{\partial \alpha} = \frac{A\alpha k\left(1 - 8b^2\alpha^2k^2\right)}{2(4\alpha kb+1)^2(2\alpha kb+1)^2}$$

The positive root of $\Delta\Pi\left(\alpha\right)$ is $\alpha = \frac{\sqrt{2}}{4bk}$. However, this maximum is irrelevant if $\frac{t}{4bk(A-t)} < \frac{\sqrt{2}}{4bk}$. This condition simplifies to $\frac{\sqrt{2}}{4bk}\left(\frac{2A-(2+\sqrt{2})t}{A-t}\right) > 0$. This is satisfied if $t < \frac{2}{2+\sqrt{2}}A$. Thus, if $t < \frac{2}{2+\sqrt{2}}A$, $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} > 0$ for $\alpha \leq \frac{t}{4bk(A-t)}$. If $t > \frac{2}{2+\sqrt{2}}A$, $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} > 0$ for $\alpha \in \left(0, \frac{\sqrt{2}}{4bk}\right)$ and $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} > 0$ for $\alpha \in \left(\frac{\sqrt{2}}{4bk}, \frac{t}{4bk(A-t)}\right)$

Now turning to $\alpha \in \left(\frac{t}{4bk(A-t)}, \frac{t}{2bk(A-t)}\right)$, over this range, $\Delta\Pi\left(\alpha\right)$ has two roots (in α). They are written as:

$$\begin{aligned}
\alpha_1 &= \frac{\sqrt{2}\left(-2A + \left(2+\sqrt{2}\right)t\right)}{4bk\left(-A + (A-t)\left(\sqrt{2}+1\right)\right)} \\
\alpha_2 &= -\frac{\sqrt{2}\left(2\left(A-t\right) + \sqrt{2}t\right)}{4bk\left(A + (A-t)\left(\sqrt{2}-1\right)\right)} < 0
\end{aligned}$$

Clearly, α_2 is not relevant. The root α_1 is relevant if the following condition holds.

$$\alpha_1 - \frac{t}{4bk(A-t)} = \frac{\sqrt{2}\,(2A-t)\overbrace{\left(\left(2+\sqrt{2}\right)t-2A\right)}^{\text{(-) if } t<\frac{2}{2+\sqrt{2}}A}}{8bk\,(A-t)\underbrace{\left(-A+(A-t)\left(\sqrt{2}+1\right)\right)}_{\text{(+) if } t<\frac{2}{2+\sqrt{2}}A \text{ (see below)}}} < 0$$

To see that $-A+(A-t)\left(\sqrt{2}+1\right) > 0$ if $t < \frac{2}{2+\sqrt{2}}A$, rearrange the first expression to read $t < (\frac{\sqrt{2}}{1+\sqrt{2}})A$. Multiplying the top and bottom by $\sqrt{2}$, we get $t < \frac{2}{2+\sqrt{2}}A$. Thus, since the numerator and denominator always have the opposite sign, it must always be the case that $\alpha_1 < \frac{t}{4bk(A-t)}$, and thus not relevant in the mid range of productivity.

Both $\alpha_1 < \frac{t}{4bk(A-t)}$ and $\alpha_2 < 0$ imply that the roots of $\Delta\Pi(\alpha)$ do not occur over the range $\frac{t}{4bk(A-t)} < \alpha \leq \frac{t}{2bk(A-t)}$. Thus, the sign of $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha}$ is constant over this range. To see when $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} > 0$, note that:

$$\frac{\partial\Delta\Pi\left(\frac{t}{4bk(A-t)}\right)}{\partial\alpha} = \frac{k(A-t)^2\overbrace{\left(2A^2-4At+t^2\right)}^{\text{(+) if } t<\frac{2}{2+\sqrt{2}}A}}{2(2A-t)^2}$$

Clearly, the sign of $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha}$ over the mid range of productivity is dependent on the size of trade costs relative to the market, and is positive if $t < \frac{2}{2+\sqrt{2}}A$. Hence, for $t < \frac{2}{2+\sqrt{2}}A$, $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} > 0$ for $\alpha \in \left(\frac{t}{4bk(A-t)}, \frac{t}{2bk(A-t)}\right)$. Else, if $t > \frac{2}{2+\sqrt{2}}A$, $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} < 0$ for $\alpha \in \left(\frac{t}{4bk(A-t)}, \frac{t}{2bk(A-t)}\right)$.

Finally, for $\alpha \in \left(\frac{t}{2bk(A-t)}, \infty\right)$, I can write:

$$\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} = \frac{k\,(2A-t)\left(4-8b^2\alpha^2k^2\right)}{2(4\alpha kb+2)^2(2\alpha kb+2)^2}$$

The positive solution to $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} = 0$ is $\frac{\sqrt{2}}{2bk}$. It is also clear that for $\alpha < \frac{\sqrt{2}}{2bk}$, $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} > 0$, and for $\alpha > \frac{\sqrt{2}}{2bk}$, $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} < 0$. Again, this root is only relevant when $\frac{t}{2bk(A-t)} < \frac{\sqrt{2}}{2bk}$. This occurs if $t < \frac{2}{2+\sqrt{2}}A$.

Overall, I have shown that if $t < \frac{2}{2+\sqrt{2}}A$, $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} > 0$ for $\alpha \in \left(0, \frac{\sqrt{2}}{2bk}\right)$, and $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} < 0$ for $\alpha \in \left(\frac{\sqrt{2}}{2bk}, \infty\right)$. In contrast, if $t > \frac{2}{2+\sqrt{2}}A$, $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} > 0$ for $\alpha \in \left(0, \frac{\sqrt{2}}{4bk}\right)$, and $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} < 0$ for $\alpha \in \left(\frac{\sqrt{2}}{4bk}, \infty\right)$

To finish proving the properties in Lemma 5, I must show that $\frac{1}{2}\pi^B(\alpha) < \pi^N(\alpha) < \pi^B(\alpha)$ for $\alpha > 0$. The condition $\pi^N(\alpha) < \pi^B(\alpha)$ is trivial through lower costs. The condition $\frac{1}{2}\pi^B(\alpha) < \pi^N(\alpha)$ is true if $\Delta\Pi(\alpha) < \pi^N(\alpha)$. To see this, expanding $\Delta\Pi(\alpha)$, I get $\pi^B(\alpha) - \pi^N(\alpha) < \pi^N(\alpha)$, which simplifies to $\frac{1}{2}\pi^B(\alpha) < \pi^N(\alpha)$. Hence, I will now show that $\Delta\Pi(\alpha) < \pi^N(\alpha)$. To do

this, I will first show that $\Delta\Pi(0) = \pi^N(0)$, and then $\frac{\partial \Delta\Pi(\alpha)}{\partial \alpha} < \frac{\partial \pi^N(\alpha)}{\partial \alpha}$ for all finite α. Showing $\Delta\Pi(0) = \pi^N(0)$ is straightforward, given the multiplicative nature of α. For $\alpha \leq \frac{t}{4bk(A-t)}$, I can write $\frac{\partial \Delta\Pi(\alpha)}{\partial \alpha} - \pi^N(\alpha)$ as:

$$\frac{\partial \Delta\Pi(\alpha)}{\partial \alpha} - \pi^N(\alpha) = -\frac{2A^2\alpha^2k^2b}{(4\alpha kb+1)(2\alpha kb+1)} < 0$$

For $\frac{t}{4bk(A-t)} < \alpha \leq \frac{t}{2bk(A-t)}$, I can write $\frac{\partial \Delta\Pi(\alpha)}{\partial \alpha} - \frac{\partial \pi^N(\alpha)}{\partial \alpha}$ as:

$$\frac{\partial \Delta\Pi(\alpha)}{\partial \alpha} - \pi^N(\alpha) = -\frac{kt(2A-t)}{(4\alpha kb+2)^2} < 0$$

Finally, for $\frac{t}{2bk(A-t)} < \alpha$, I can write $\frac{\partial \Delta\Pi(\alpha)}{\partial \alpha} - \frac{\partial \pi^N(\alpha)}{\partial \alpha}$ as:

$$\frac{\partial \Delta\Pi(\alpha)}{\partial \alpha} - \pi^N(\alpha) = -\frac{4bk^2\alpha(2A-t)^2(3b\alpha k+2)}{(4\alpha kb+2)^2(2\alpha kb+2)^2} < 0$$

Proof of Lemma 6

Clearly, $\Delta\Pi^*(0) = -\delta$. In addition, it is straightforward to show that $\lim_{\alpha\to\infty} \Delta\Pi^*(\alpha) = \frac{(2A-t)t}{4b} - \delta$. To derive the slope properties of $\Delta\Pi^*(\alpha)$, note that for $\alpha \leq \frac{t}{2bk(A-t)}$, $\frac{A^2\alpha k}{2(2b\alpha k+1)}$ is equal to the no acquisition profit function in the closed economy, $\pi^N(\alpha)$. Thus, for $\alpha \leq \frac{t}{2bk(A-t)}$, $\frac{\partial \Delta\Pi^*(\alpha)}{\partial \alpha} > 0$. For $\alpha > \frac{t}{2bk(A-t)}$, with some work, one can show that the only positive root of $\Delta\Pi^*(\alpha)$ is $\widehat{\alpha} = \frac{\sqrt{2}(2A+t+\sqrt{2}t)}{2bk(2A-2t-\sqrt{2}t)}$. To start, differentiate $\Delta\Pi^*(\alpha)$ with respect to α, set equal to zero, and solve for α:

$$\widehat{\alpha}_1 = \frac{\overbrace{\left(\left(2\sqrt{2}-2\right)A+t\right)}^{+}}{2bk\underbrace{\left(2A-2t-\sqrt{2}t\right)}_{+ \text{ if } t<\frac{2}{2+\sqrt{2}}A}} \quad \widehat{\alpha}_2 = -\frac{\overbrace{\left(2A-\left(\sqrt{2}-1\right)t\right)}^{+ \text{ if } t<A}}{2bk\underbrace{\left(2(A-t)+\sqrt{2}t\right)}_{+}} < 0$$

Clearly, $\widehat{\alpha}_2$ is irrelevant. Regarding $\widehat{\alpha}_1$, it is only relevant if $t < \frac{2}{2+\sqrt{2}}A$. To see this, note that $\widehat{\alpha}_1 > \frac{t}{2bk(A-t)}$ if the following holds:

$$\widehat{\alpha}_1 - \frac{t}{2bk(A-t)} = \frac{(\sqrt{2}-1)(2A-t)A}{2bk(A-t)\left(\left(2-\sqrt{2}\right)A-t\right)} > 0$$

This clearly holds if $t < \left(2-\sqrt{2}\right)A$. Note that $t < \left(2-\sqrt{2}\right)A$ can be written as $t < \left(2-\sqrt{2}\right)\frac{2+\sqrt{2}}{2+\sqrt{2}}A = \frac{4-2+2\sqrt{2}-2\sqrt{2}}{2+\sqrt{2}}A = \frac{2}{2+\sqrt{2}}A$.

Now, all that is left is identifying the sign of $\frac{\partial \Delta\Pi^*(\alpha)}{\partial \alpha}$ on either side of $\widehat{\alpha}$. At $\alpha = \frac{t}{2bk(A-t)}$, $\frac{\partial \Delta\Pi^*(\alpha)}{\partial \alpha} = \frac{k(A-t)^2}{2} > 0$. The second derivative at $\widehat{\alpha}_1$ gives us guidance to the shape at the peak.

Precisely:

$$\frac{\partial^2 \Delta\Pi^*(\widehat{\alpha}_1)}{\partial \alpha^2} = -\frac{\overbrace{(17\sqrt{2}-24)}^{+} bk^2(2A-2t-\sqrt{2}t)^4}{4(2A-t)A}$$

Clearly, $\frac{\partial^2 \Delta\Pi^*(\widehat{\alpha}_1)}{\partial \alpha^2} < 0$ only if $t < \frac{2}{2+\sqrt{2}}A$. Thus, if $t < \frac{2}{2+\sqrt{2}}A$, $\widehat{\alpha}_1$ is a maximum, and $\frac{\partial \Delta\Pi^*(\alpha)}{\partial \alpha} < 0$ for $\alpha > \widehat{\alpha}_1$. As $t \to \frac{2}{2+\sqrt{2}}A$, $\widehat{\alpha}_1 \to \infty$, where $\frac{\partial^2 \Delta\Pi^*(\widehat{\alpha}_1)}{\partial \alpha^2} \to 0$.

To show that $\frac{1}{2}\pi^{B^*}(\alpha) \leq \pi^N(\alpha) < \pi^{B^*}(\alpha)$, note that for low productivity $\left(\alpha \leq \frac{t}{2bk(A-t)}\right)$, $\frac{1}{2}\pi^{B^*}(\alpha) = \pi^N(\alpha)$. For $\alpha > \frac{t}{2bk(A-t)}$, firms that do nothing can now export and $\pi^N(\alpha)$ is larger relative to $\pi^{B^*}(\alpha)$. Thus, $\frac{1}{2}\pi^{B^*}(\alpha) \leq \pi^N(\alpha)$ for all α.

Proof of Lemma 8

To show that $\alpha_S < \underline{\alpha}_B^* < \overline{\alpha}_B^*$, the analysis is similar. Equilibrium conditions dictate that:

$$\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B^*) - \delta = \pi^N(\alpha_S)$$

Subtracting $\frac{1}{2}\pi^{B^*}(\underline{\alpha}_B^*)$, we get:

$$\frac{1}{2}\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B^*) - \delta = \pi^N(\alpha_S) - \frac{1}{2}\pi^{B^*}(\underline{\alpha}_B^*)$$

Using the result in Lemma 7 that $\frac{1}{2}\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B^*) \leq 0$, the LHS of the above condition must be negative. Hence, the RHS, $\pi^N(\alpha_S) - \frac{1}{2}\pi^{B^*}(\underline{\alpha}_B^*)$ must also be negative. This can only be the case if $\underline{\alpha}_B^* > \alpha_S$. Finally, $\overline{\alpha}_B^* > \underline{\alpha}_B^*$ follows from the shape of $\Delta\Pi^*(\alpha)$ and the definitions of $\underline{\alpha}_B^*$ and $\overline{\alpha}_B^*$. Thus, $\alpha_S < \underline{\alpha}_B^* < \overline{\alpha}_B^*$ when both $\underline{\alpha}_B^*$ and $\overline{\alpha}_B^*$ are defined, and using the indifference conditions in (30) and (36), we have the result in Lemma 8. When $\overline{\alpha}_B^*$ is not defined, the results in Lemma 8 follow given that $\Delta\Pi^*(\alpha) < R_a$ for $\alpha < \underline{\alpha}_B^*$, and $\Delta\Pi^*(\alpha) > R_a$ for $\alpha > \underline{\alpha}_B^*$.

Proof of Lemma 9

To start, note that from Lemma 6, if $\frac{(2A-t)t}{4b} > \delta$, $\lim_{\alpha\to\infty} \Delta\Pi^*(\alpha) > 0$. Since by Lemma 5 $\Delta\Pi(0) = \lim_{\alpha\to\infty} \Delta\Pi(\alpha) = 0$, and by Lemma 6 $\Delta\Pi^*(0) = -\delta$, this guarantees that $\Delta\Pi^*(\alpha)$ and $\Delta\Pi(\alpha)$ cross at least once over the support of α. To prove that $\Delta\Pi^*(\alpha)$ and $\Delta\Pi(\alpha)$ cross only once at a unique value of α, I must show that $\frac{\Delta\Pi^*(\alpha,A)}{\partial\alpha} > \frac{\Delta\Pi(\alpha,A)}{\partial\alpha}$. To start, note that this condition simplifies to

$$\frac{\partial \pi^{B^*}(\alpha)}{\partial \alpha} > \frac{\partial \pi^B(\alpha)}{\partial \alpha}$$

Note that $\frac{\partial \pi^{B^*}(\alpha)}{\partial \alpha}$ is written as:

$$\frac{\partial \pi^{B^*}(\alpha)}{\partial \alpha} = \frac{A^2 k}{(2b\alpha k+1)^2}$$

and $\frac{\partial \pi^B(\alpha)}{\partial \alpha}$ is written as:

$$\frac{\partial \pi^B(\alpha)}{\partial \alpha} = \left\{ \begin{array}{ll} \frac{A^2 k}{(4b\alpha k+1)^2} & \alpha \leq \frac{t}{4bk(A-t)} \\ \\ \frac{A^2 k}{(2b\alpha k+1)^2} - \frac{tk(4A-t)}{4(2b\alpha k+1)^2} & \alpha > \frac{t}{4bk(A-t)} \end{array} \right\}$$

Clearly, $\frac{\partial \pi^{B^*}(\alpha)}{\partial \alpha} > \frac{\partial \pi^B(\alpha)}{\partial \alpha}$ for all α. Hence, when $\frac{(2A-t)t}{4b} > \delta$, $B^* \succ B$ if $\alpha > \alpha_{BB^*}$. If $\frac{(2A-t)t}{4b} < \delta$, $B \succ B^*$ for all α.

C Simulation Details

Parameters for Simulation of the model

Parameter	Value	Notes
θ	100	Maximum marginal utility of the differentiated good
η	1	Utility parameter
γ	0.1	Utility parameter
k	1	Initial capital endowment per firm
F_E	1	Fixed cost of entry
$G(\alpha)$	$1 - (\frac{1}{\alpha})^h$	Pareto productivity distribution, $\alpha \in (1, \infty)$
h	1.3	Pareto dispersion parameter
δ	varies	Fixed cost of foreign investment
t	varies	Per-unit cost of exports

The simulation itself uses Broyden's method for solving the system of nonlinear equations. The difficulty in solving the model is that to use a standard Newton-Raphson method, one needs to solve for the explicit form of the Jacobian of the system of equations. Broyden's method approximates this Jacobian after each iteration.

As there is an analytical solution for the case of free trade, I start with this case and incrementally adjust t and δ away from free trade, using the results from the previous case as the initial conditions of the simulation, and in particular, the initial guess of the Jacobian. The code of the simulation used to construct the figures in the paper is available upon request.

D Existence and Uniqueness of A

In both the closed and open economy, A is pinned down by the free entry condition. Generally, the free entry condition can be written as:

$$\int_{\alpha\in\Theta_S} R_a dG(\alpha)+\int_{\alpha\in\Theta_N} \pi^N(\alpha)dG(\alpha)+\int_{\alpha\in\Theta_B} (\pi^B(\alpha)-R_a)dG(\alpha)+\int_{\alpha\in\Theta_{B^*}} (\pi^{B^*}(\alpha)-R_a-\delta)dG(\alpha) = F_E$$

where the corresponding acquisition market clearing condition is written as:

$$\int_{\alpha\in\Theta_S} dG(\alpha) = \int_{\alpha\in\Theta_B} dG(\alpha) + \int_{\alpha\in\Theta_{B^*}} dG(\alpha)$$

Substituting the acquisition market clearing condition simplifies the free entry condition as follows:

$$\int_{\alpha\in\Theta_N} \pi^N(\alpha)dG(\alpha) + \int_{\alpha\in\Theta_B} \pi^B(\alpha)dG(\alpha) + \int_{\alpha\in\Theta_{B^*}} (\pi^{B^*}(\alpha) - \delta)dG(\alpha) = F_E$$

The measures of active firms, Θ_N, Θ_B, and Θ_{B*}, will be characterized shortly. Differentiating yields:

$$\int_{\alpha\in\Theta_N} \frac{\partial\pi^N(\alpha)}{\partial A}dG(\alpha) + \int_{\alpha\in\Theta_B} \frac{\pi^B(\alpha)}{\partial A}dG(\alpha) + \int_{\alpha\in\Theta_{B^*}} \frac{\partial\pi^{B^*}(\alpha)}{\partial A}dG(\alpha) + Indirect = F_E$$

To prove uniqueness, I must first show that the indirect effects (through changes to Θ_N Θ_B and Θ_{B^*}) are all zero. Once this result is established, all that is left are the direct effects, which are all positive. Precisely, $\frac{\partial\pi^N(\alpha)}{\partial A} > 0$, $\frac{\partial\pi^B(\alpha)}{\partial A} > 0$, and $\frac{\partial\pi^{B^*}(\alpha)}{\partial A} > 0$. Since expected profits are zero when $A = 0$, and given a smooth distribution of productivity, the intermediate value theorem guarantees a unique value of A provided that the fixed cost of entry is not too high.

There are seven cases of acquisition activity over which I must show that the indirect effects are zero. Using Lemmas 7 and 8, these seven cases, are listed the following table (with the productivity sorting listed below each).

Case	Θ_N	Θ_B	Θ_{B*}	Marking Clearing
1 S, N, B, N	$(\alpha_S, \underline{\alpha}_B) \cup (\overline{\alpha}_B, \infty)$	$(\underline{\alpha}_B, \overline{\alpha}_B)$	$\emptyset$	$G(\alpha_s) = G(\overline{\alpha}_B) - G(\underline{\alpha}_B)$
2a S, N, B^*, N	$(\alpha_S, \underline{\alpha}_{B^*}) \cup (\overline{\alpha}_{B^*}, \infty)$	$\emptyset$	$(\underline{\alpha}_{B^*}, \overline{\alpha}_{B^*})$	$G(\alpha_s) = G(\overline{\alpha}^*_B) - G(\underline{\alpha}^*_B)$
2b S, N, B^*	$(\alpha_S, \underline{\alpha}_B)$	$\emptyset$	$(\underline{\alpha}_{B^*}, \infty)$	$G(\alpha_s) = 1 - G(\underline{\alpha}^*_B)$
3a S, N, B, B^*, N	$(\alpha_S, \underline{\alpha}_B) \cup (\overline{\alpha}_{B^*}, \infty)$	$(\underline{\alpha}_B, \alpha_{BB^*})$	$(\alpha_{BB^*}, \overline{\alpha}_{B^*})$	$G(\alpha_s) = G(\overline{\alpha}^*_B) - G(\underline{\alpha}_B)$
3b S, N, B, B^*	$(\alpha_S, \underline{\alpha}_B)$	$(\underline{\alpha}_B, \alpha_{BB^*})$	(α_{BB^*}, ∞)	$G(\alpha_s) = 1 - G(\underline{\alpha}_B)$
4a S, N, B, N, B^*, N	$(\alpha_S, \underline{\alpha}_B) \cup (\overline{\alpha}_B, \underline{\alpha}_{B^*}) \cup (\overline{\alpha}_{B^*}, \infty)$	$(\underline{\alpha}_B, \overline{\alpha}_B)$	$(\underline{\alpha}_{B^*}, \overline{\alpha}_{B^*})$	$G(\alpha_s) = G(\overline{\alpha}_B) - G(\underline{\alpha}_B) + G(\overline{\alpha}^*_B) - G(\underline{\alpha}^*_B)$
4b S, N, B, N, B^*	$(\alpha_S, \underline{\alpha}_B) \cup (\overline{\alpha}_B, \underline{\alpha}_{B^*})$	$(\underline{\alpha}_B, \overline{\alpha}_B)$	$(\underline{\alpha}_{B^*}, \infty)$	$G(\alpha_s) = G(\overline{\alpha}_B) - G(\underline{\alpha}_B) + 1 - G(\underline{\alpha}^*_B)$

Case 1 is when only domestic acquisitions occur (this is also a proof which is relevant for the closed economy). Cases 2a and 2b are when only foreign acquisitions occur, where the former consists of two foreign productivity cutoffs (low trade costs) and the latter consists of one productivity cutoff

(high trade costs). Cases 3a and 3b consist of the case when both domestic and foreign acquisitions occur and there is one continuous region of acquisition activity. Cases 4a and 4b consist of when both domestic and foreign acquisitions occur, but where there is a region of no acquisitions that occurs in-between.

The strategy will be to derive the direct and indirect effects via leibniz rule, and then show that by imposing the derivative of the corresponding acquisition market clearing condition, and then the relevant equilibrium conditions, the indirect effects are equal to zero.

Case 1:

$$\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B}^{\overline{\alpha}_B} \frac{\pi^B(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B}^{\infty} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha)$$
$$-g(\alpha_S)\frac{\partial \alpha_S}{\partial A}\pi^N(\alpha_S) - g(\underline{\alpha}_B)\frac{\partial \underline{\alpha}_B}{\partial A}(\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B)) + g(\overline{\alpha}_B)\frac{\partial \overline{\alpha}_B}{\partial A}(\pi^B(\overline{\alpha}_B) - \pi^N(\overline{\alpha}_B))$$

Substituting for $g(\alpha_S)\frac{\partial \alpha_S}{\partial A}$ using the derivative of the acquisition market clearing condition, and simplifying, yields:

$$\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B}^{\overline{\alpha}_B} \frac{\pi^B(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B}^{\infty} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha)$$
$$-g(\underline{\alpha}_B)\frac{\partial \underline{\alpha}_B}{\partial A}(\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) - \pi^N(\alpha_S)) + g(\overline{\alpha}_B)\frac{\partial \overline{\alpha}_B}{\partial A}(\pi^B(\overline{\alpha}_B) - \pi^N(\overline{\alpha}_B) - \pi^N(\alpha_S))$$

Via the equilibrium conditions in (29), (31) and (32), it must be the case that $\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) - \pi^N(\alpha_S) = 0$ and $\pi^B(\overline{\alpha}_B) - \pi^N(\overline{\alpha}_B) - \pi^N(\alpha_S) = 0$. Hence, all that remains are the direct effects, and thus, $\frac{\partial LHS}{\partial A} > 0$.

Case 2a:

$$\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B^*} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B^*}^{\overline{\alpha}_B^*} \frac{\pi^{B^*}(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B^*}^{\infty} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha)$$
$$-g(\alpha_S)\frac{\partial \alpha_S}{\partial A}\pi^N(\alpha_S) - g(\underline{\alpha}_B^*)\frac{\partial \underline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B) - \delta) + g(\overline{\alpha}_B^*)\frac{\partial \overline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\overline{\alpha}_B^*) - \pi^N(\overline{\alpha}_B^*) - \delta)$$

Substituting for $g(\alpha_S)\frac{\partial \alpha_S}{\partial A}$ using the derivative of the acquisition market clearing condition, and simplifying, yields:

$$\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B^*} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B^*}^{\overline{\alpha}_B^*} \frac{\pi^{B^*}(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B^*}^{\infty} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha)$$
$$-g(\underline{\alpha}_B^*)\frac{\partial \underline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B^*) - \delta - \pi^N(\alpha_S)) + g(\overline{\alpha}_B^*)\frac{\partial \overline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\overline{\alpha}_B^*) - \pi^N(\overline{\alpha}_B^*) - \delta - \pi^N(\alpha_S))$$

Via the equilibrium conditions in (29), (34) and (35), it must be the case that $\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B^*) - \delta - \pi^N(\alpha_S) = 0$ and $\pi^{B^*}(\overline{\alpha}_B^*) - \pi^N(\overline{\alpha}_B^*) - \delta - \pi^N(\alpha_S) = 0$. Hence, all that remains are the direct effects, and thus, $\frac{\partial LHS}{\partial A} > 0$.

Case 2b: The proof is similar to 2a, except the upper cutoff does not exist for foreign acquisitions.

$$\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B^*} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B^*}^{\infty} \frac{\pi^{B^*}(\alpha)}{\partial A} dG(\alpha)$$
$$-g(\alpha_S)\frac{\partial \alpha_S}{\partial A}\pi^N(\alpha_S) - g(\underline{\alpha}_B^*)\frac{\partial \underline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B) - \delta)$$

Substituting for $g(\alpha_S)\frac{\partial \alpha_S}{\partial A}$ using the derivative of the acquisition market clearing condition, and simplifying, yields:

$$\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B^*}^{\infty} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha)$$
$$-g(\underline{\alpha}_B^*)\frac{\partial \underline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B^*) - \delta - \pi^N(\alpha_S))$$

Via the equilibrium conditions in (29) and (34), it must be the case that $\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B^*) - \delta - \pi^N(\alpha_S) = 0$. Hence, all that remains are the direct effects, and thus, $\frac{\partial LHS}{\partial A} > 0$.

Case 3a:

$$\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B}^{\alpha_{BB^*}} \frac{\pi^B(\alpha)}{\partial A} dG(\alpha) + \int_{\alpha_{BB^*}}^{\overline{\alpha}_B^*} \frac{\pi^{B^*}(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B^*}^{\infty} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha)$$
$$-g(\alpha_S)\frac{\partial \alpha_S}{\partial A}\pi^N(\alpha_S) - g(\underline{\alpha}_B)\frac{\partial \underline{\alpha}_B}{\partial A}(\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B))$$
$$-g(\alpha_{BB^*})\frac{\partial \alpha_{BB^*}}{\partial A}(\pi^{B^*}(\alpha_{BB^*}) - \pi^B(\alpha_{BB^*}) - \delta) + g(\overline{\alpha}_B^*)\frac{\partial \overline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\overline{\alpha}_B^*) - \pi^N(\overline{\alpha}_B^*) - \delta)$$

Note that that $\pi^{B^*}(\alpha_{BB^*}) - \pi^B(\alpha_{BB^*}) - \delta = 0$ by the definition of α_{BB^*} in Lemma 9. Further, substituting for $g(\alpha_S)\frac{\partial \alpha_S}{\partial A}$ using the derivative of the acquisition market clearing condition, and simplifying, yields:

$$\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B}^{\alpha_{BB^*}} \frac{\pi^B(\alpha)}{\partial A} dG(\alpha) + \int_{\alpha_{BB^*}}^{\overline{\alpha}_B^*} \frac{\pi^{B^*}(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B^*}^{\infty} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha)$$
$$-g(\underline{\alpha}_B)\frac{\partial \underline{\alpha}_B}{\partial A}(\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) - \pi^N(\alpha_S))$$
$$+g(\overline{\alpha}_B^*)\frac{\partial \overline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\overline{\alpha}_B^*) - \pi^N(\overline{\alpha}_B^*) - \delta - \pi^N(\alpha_S))$$

Via the equilibrium conditions in (29), (31) and (35), it must be the case that $\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) - \pi^N(\alpha_S) = 0$ and $\pi^{B^*}(\overline{\alpha}_B^*) - \pi^N(\overline{\alpha}_B^*) - \delta - \pi^N(\alpha_S) = 0$. Hence, all that remains are the direct effects, and thus, $\frac{\partial LHS}{\partial A} > 0$.

Case 3b: The derivation is similar to 3a, except the upper cutoff does not exist for foreign acqui-

sitions.

$$\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B}^{\alpha_{BB^*}} \frac{\pi^B(\alpha)}{\partial A} dG(\alpha) + \int_{\alpha_{BB^*}}^{\infty} \frac{\pi^{B^*}(\alpha)}{\partial A} dG(\alpha)$$
$$-g(\alpha_S)\frac{\partial \alpha_S}{\partial A}\pi^N(\alpha_S) - g(\underline{\alpha}_B)\frac{\partial \underline{\alpha}_B}{\partial A}(\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B)) - g(\alpha_{BB^*})\frac{\partial \alpha_{BB^*}}{\partial A}(\pi^{B^*}(\alpha_{BB^*}) - \pi^B(\alpha_{BB^*}) - \delta)$$

Again, $\pi^{B^*}(\alpha_{BB^*}) - \pi^B(\alpha_{BB^*}) - \delta = 0$ by the definition of α_{BB^*} in Lemma 9. Hence, substituting for $g(\alpha_S)\frac{\partial \alpha_S}{\partial A}$ using the derivative of the acquisition market clearing condition, and simplifying, yields:

$$\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B}^{\alpha_{BB^*}} \frac{\pi^B(\alpha)}{\partial A} dG(\alpha) + \int_{\alpha_{BB^*}}^{\infty} \frac{\pi^{B^*}(\alpha)}{\partial A} dG(\alpha)$$
$$-g(\underline{\alpha}_B)\frac{\partial \underline{\alpha}_B}{\partial A}(\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) - \pi^N(\alpha_S))$$

Via the equilibrium conditions in (29) and (31), it must be the case that $\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) - \pi^N(\alpha_S) = 0$. Hence, all that remains are the direct effects, and thus, $\frac{\partial LHS}{\partial A} > 0$.

Case 4a:

$$\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B}^{\overline{\alpha}_B} \frac{\pi^B(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B}^{\underline{\alpha}_B^*} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha)$$
$$+ \int_{\underline{\alpha}_B^*}^{\overline{\alpha}_B^*} \frac{\pi^{B^*}(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B^*}^{\infty} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha)$$
$$-g(\alpha_S)\frac{\partial \alpha_S}{\partial A}\pi^N(\alpha_S) - g(\underline{\alpha}_B)\frac{\partial \underline{\alpha}_B}{\partial A}(\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B)) + g(\overline{\alpha}_B)\frac{\partial \overline{\alpha}_B}{\partial A}(\pi^B(\overline{\alpha}_B) - \pi^N(\overline{\alpha}_B))$$
$$-g(\underline{\alpha}_B^*)\frac{\partial \underline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B) - \delta) + g(\overline{\alpha}_B^*)\frac{\partial \overline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\overline{\alpha}_B^*) - \pi^N(\overline{\alpha}_B^*) - \delta)$$

Substituting for $g(\alpha_S)\frac{\partial \alpha_S}{\partial A}$ using the derivative of the acquisition market clearing condition, and simplifying, yields:

$$\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B}^{\overline{\alpha}_B} \frac{\pi^B(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B}^{\underline{\alpha}_B^*} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha)$$
$$+ \int_{\underline{\alpha}_B^*}^{\overline{\alpha}_B^*} \frac{\pi^{B^*}(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B^*}^{\infty} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha)$$
$$-g(\underline{\alpha}_B)\frac{\partial \underline{\alpha}_B}{\partial A}(\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) - \pi^N(\alpha_S)) + g(\overline{\alpha}_B)\frac{\partial \overline{\alpha}_B}{\partial A}(\pi^B(\overline{\alpha}_B) - \pi^N(\overline{\alpha}_B) - \pi^N(\alpha_S))$$
$$-g(\underline{\alpha}_B^*)\frac{\partial \underline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B) - \delta - \pi^N(\alpha_S)) + g(\overline{\alpha}_B^*)\frac{\partial \overline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\overline{\alpha}_B^*) - \pi^N(\overline{\alpha}_B^*) - \delta - \pi^N(\alpha_S))$$

Via the equilibrium conditions in (29), (31), (32), (34) and (35), it must be the case that $\pi^B(\underline{\alpha}_B) -$

$\pi^N(\underline{\alpha}_B) - \pi^N(\alpha_S) = 0$, $\pi^B(\overline{\alpha}_B) - \pi^N(\overline{\alpha}_B) - \pi^N(\alpha_S) = 0$, $\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B^*) - \delta - \pi^N(\alpha_S) = 0$ and $\pi^{B^*}(\overline{\alpha}_B^*) - \pi^N(\overline{\alpha}_B^*) - \delta - \pi^N(\alpha_S) = 0$. Hence, all that remains are the direct effects, and thus, $\frac{\partial LHS}{\partial A} > 0$.

Case 4b: The derivation is similar to 4a, except the upper cutoff does not exist for foreign acquisitions.

$$\begin{aligned}\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B}^{\overline{\alpha}_B} \frac{\pi^B(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B}^{\underline{\alpha}_B^*} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B^*}^{\infty} \frac{\pi^{B^*}(\alpha)}{\partial A} dG(\alpha) \\ -g(\alpha_S)\frac{\partial \alpha_S}{\partial A}\pi^N(\alpha_S) - g(\underline{\alpha}_B)\frac{\partial \underline{\alpha}_B}{\partial A}(\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B)) \\ +g(\overline{\alpha}_B)\frac{\partial \overline{\alpha}_B}{\partial A}(\pi^B(\overline{\alpha}_B) - \pi^N(\overline{\alpha}_B)) - g(\underline{\alpha}_B^*)\frac{\partial \underline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B) - \delta)\end{aligned}$$

Substituting for $g(\alpha_S)\frac{\partial \alpha_S}{\partial A}$ using the derivative of the acquisition market clearing condition, and simplifying, yields:

$$\begin{aligned}\frac{\partial LHS}{\partial A} = \int_{\alpha_S}^{\underline{\alpha}_B} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B}^{\overline{\alpha}_B} \frac{\pi^B(\alpha)}{\partial A} dG(\alpha) + \int_{\overline{\alpha}_B}^{\underline{\alpha}_B^*} \frac{\partial \pi^N(\alpha)}{\partial A} dG(\alpha) + \int_{\underline{\alpha}_B^*}^{\infty} \frac{\pi^{B^*}(\alpha)}{\partial A} dG(\alpha) \\ -g(\underline{\alpha}_B)\frac{\partial \underline{\alpha}_B}{\partial A}(\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) - \pi^N(\alpha_S)) + g(\overline{\alpha}_B)\frac{\partial \overline{\alpha}_B}{\partial A}(\pi^B(\overline{\alpha}_B) - \pi^N(\overline{\alpha}_B) - \pi^N(\alpha_S)) \\ -g(\underline{\alpha}_B^*)\frac{\partial \underline{\alpha}_B^*}{\partial A}(\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B) - \delta - \pi^N(\alpha_S))\end{aligned}$$

Via the equilibrium conditions in (29), (31), (32), (34) and (35), it must be the case that $\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) - \pi^N(\alpha_S) = 0$, $\pi^B(\overline{\alpha}_B) - \pi^N(\overline{\alpha}_B) - \pi^N(\alpha_S) = 0$ and $\pi^{B^*}(\underline{\alpha}_B^*) - \pi^N(\underline{\alpha}_B^*) - \delta - \pi^N(\alpha_S) = 0$. Hence, all that remains are the direct effects, and thus, $\frac{\partial LHS}{\partial A} > 0$.