Economics 113 - Spring 2008
Introduction to Econometrics
Name John P. Answer Key

Professor Spearot

Midterm Exam (120 points)

The exam is closed book and closed notes. You may use your calculators but please show your work step by step. No graphing calculators or cell phones. **You must show your work to receive full credit**

Problem 1 (40 Points)
Professor Spearot is prone to typos. In any given lecture the probability of a typo is independent of other lectures. Precisely, Pr(Typo)=0.7. Let the random variable X be the number of typos over three classes.

a. Please solve for and diagram the probability distribution of X. (10 points)

T = Typo
N = No Typo

X		Pr(X)
0	N N N	$(.3)^3 \times 1 = 0.027$
1	N N T N T N T N N	$(.3)^2(.7) \times 3 = 0.189$
2	N T T T N T T T N	$(.7)^2(.3) \times 3 = 0.441$
3	T T T	$(.7)^3 \times 1 = 0.343$

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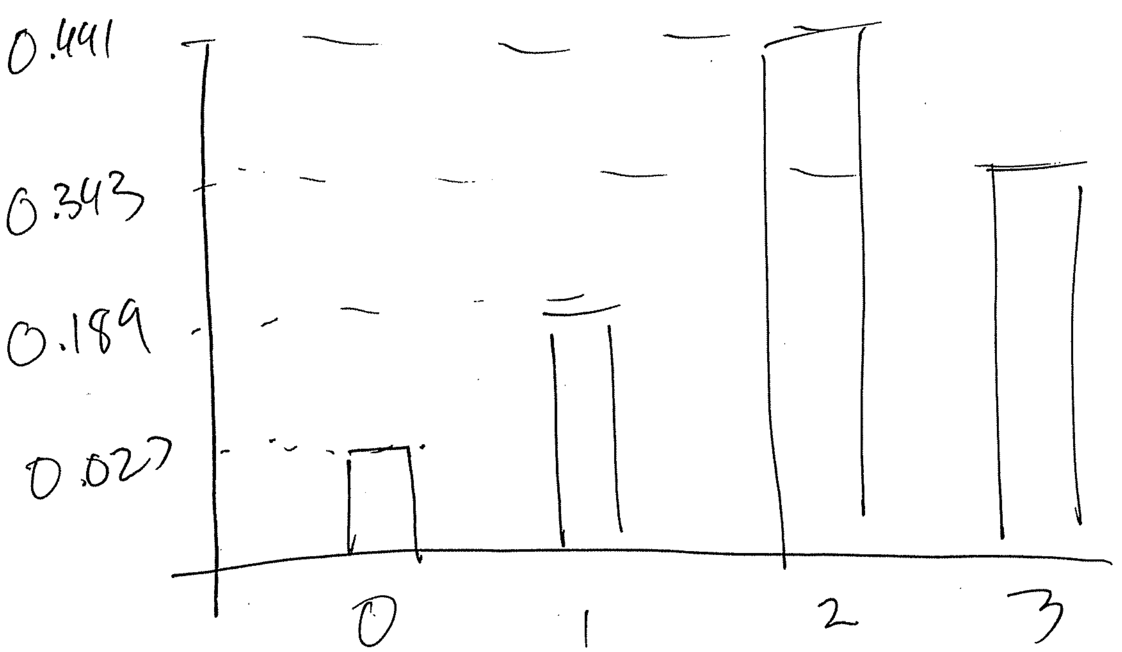


b. What is the expected value of X? (5 points)

$$E(X) = 0 \times 0.027 + 1 \times 0.0189 + 2 \times 0.441$$

$$+ 3 \times 0.343$$

$$\boxed{E(x) = 2.1}$$

(Harder question – diagrams help)

Momentum matters when teaching. For example, suppose that the probability of a typo in the first class is still 0.7. However, if Professor Spearot is typo-free during the first class, the probability of a typo in each remaining class is 0.3. If the first class contains a typo, then the probability of another in each remaining class is 0.8.

c. Given that Professor Spearot had no typos in his first lecture, what is the probability of having one typo over the next two lectures? (10 points)

~~N~~ N

N T

T N

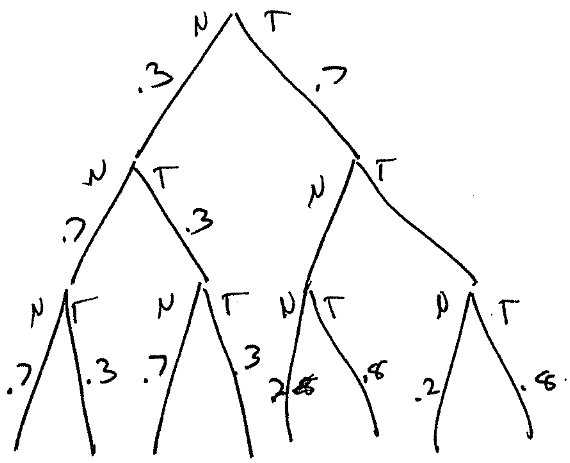
T T

5 points

$(.3)\cdot(.7)$
$(.7)\cdot(.3)$ $= .21 \times 2 = \boxed{0.42}$

5 points

d. Suppose that event A is the probability that Professor Spearot has a typo in his first OR third lectures. What is the probability of this event? (15 points)



$$Pr(A) = Pr(\text{First}) + P(\text{Third}) - Pr(\text{First} \mathbin{\&} \text{Third})$$

$$Pr(\text{First}) = 0.7$$

$$Pr(\text{Third}) = Pr(NNT \text{ or } NTT \text{ or } TNT \text{ or } TTT)$$

$$= Pr(NNT) + Pr(NTT) + Pr(TNT) + Pr(TTT)$$

$$= (.3)^2(.7) + (.3)^3 + (0.7)(.2)(.8) + .7(.8)^2$$

$$Pr(\text{Third}) = 0.65$$

$$P(\text{First} \mathbin{\&} \text{Third}) = Pr(TNT \text{ or } TTT)$$

$$= Pr(TNT) \text{ or } Pr\,TTT$$

$$= (0.7)(.2)(.8) + (.7)(.8)(.8)$$

$$= 0.56$$

$$Pr(A) = .7 + .65 - .56 \Rightarrow \boxed{Pr(A) = 0.79}$$

Problem 2 (30 Points)
The average number of beers consumed per week for UCSC graduate students is characterized by a normal distribution with mean 5.5 and standard deviation 2.

a. Is the normal distribution a good way to characterize this particular random variable? Explain BRIEFLY why or why not. (5 points)

No. You could drink negative beers

b. What is the probability that a randomly selected UCSC graduate student drinks between 4 and 7 beers per week? (10 points)

$$Z_7 = \frac{7-5.5}{2} = \frac{1.5}{2} = 0.75$$

$$Z_4 = \frac{4-5.5}{2} = -\frac{1.5}{2} = -0.75$$

$$Pr(Z_4 < Z < Z_7) = Pr(Z < Z_7) - Pr(Z < Z_4)$$

$$\cancel{= Pr(Z < \ldots) - Pr\ Z <}$$

$$= Pr(Z < 0.75) - Pr(Z < -0.75)$$

$$= 0.7734 - (1 - 0.7734)$$

$$\boxed{= 0.5468}$$

In contrast, the average number of beers consumed per week by Wisconsin graduate students is characterized by a normal distribution with mean 20 and standard deviation 5.

c. Suppose that a randomly selected Wisconsin graduate student drinks 10 beers per week. What is the probability that a randomly selected UCSC graduate student drinks the same number of beers in a given week? (5 points)

0. A continuous random variable.

d. Suppose that event A is when the randomly selected UCSC student drinks between 2 and 6 beers per week AND the randomly selected Wisconsin student drinks between 25 and 15 beers per week. Calculate the probability of event A. (10 points)

$$z_6 = \frac{6 - 5.5}{2} = 0.25 \qquad z_2 = \frac{(2 - 5.5)}{2} = -\frac{3.5}{2} = -1.75$$

$$Pr(z_2 < Z < z_6) = Pr(Z < 0.25) - (1 - Pr(Z < 1.75))$$

$$= 0.5987 - 1 + 0.9599$$

$$= 0.5586$$

$$z_{25} = \frac{25-20}{5} = 1 \qquad z_{15} = \frac{15-20}{5} = -1$$

$$Pr(z_{15} < Z < z_{25}) = Pr(Z < 1) - (1 - Pr(Z < 1))$$

$$= 2 Pr(Z < 1) - 1$$

$$\boxed{= 0.6826}$$

$$\boxed{Pr(A \cap B) = (0.6826) \cdot (0.5586) = 0.3813}$$

Problem 3 (50 Points)

TA Bob Baden was once a boxer. Some have even called him "the greatest of all time". Though he has taken an interest in econometrics this quarter, he is now contemplating a comeback.

In an effort to maximize his earnings, he wishes to first establish the relationship between "Earnings" and "Rounds" using the following equation:

$$Earnings = \beta_0 + \beta_1 Rounds + u$$

Over his *illustrious* career, he collected data on the Rounds boxed, "Rounds", and his TV receipts, "Earnings".

Rounds	Earnings (millions)
7	\$40
9	\$50
2	\$30

a. Compute the mean of *Rounds* and *Earnings* (5 points)

$$\tilde{\mu}_R = \frac{1}{3}(7+9+2) = \frac{18}{3} = 6$$

+2

$$\tilde{\mu}_E = \frac{1}{3}(40+50+30) = \frac{120}{3} = 40$$

+2

+1 for work

b. Compute the variance of *Rounds* and *Earnings.* (5 points)

$$\hat{\tilde{\sigma}}_R = \frac{1}{2}\left((7-6)^2 + (9-6)^2 + (2-6)^2\right)$$

$$= \frac{1}{2}(1+9+16) = \frac{26}{2} = 13$$

$$\hat{\sigma}^2_E = \frac{1}{2}\left((40-40)^2 + (50-40)^2 + (30-40)^2\right)$$

$$= \frac{1}{2}(100+100) = 100$$

c. Compute the covariance of *Rounds* and *Earnings*. (5 points)

$$\hat{\sigma}_{RE} = \frac{1}{2}\Big(\big(40-40\big)(2-6) + (50-40)(9-6) + (30-40)(2-6)\Big)$$

$$= \frac{1}{2}\big(10\cdot 3 + 10\cdot 4\big) = \frac{70}{2} = 35$$

d. Please compute $\hat{\beta}_0$ and $\hat{\beta}_1$ (5 points)

$$\hat{\beta}_1 = \frac{\hat{\sigma}_{RE}}{\hat{\sigma}_R^2} = \frac{35}{13} = 2.69$$

$$\hat{\beta}_0 = \hat{\mu}_E - 2.69\,\hat{\mu}_R$$

$$= 40 - (2.69)\cdot 6 = 23.86$$

e. If assumptions 1-4 hold, how much money does Bob make if he boxes a two round fight? (5 points)

$$\hat{E}(R=2) = 23.86 + 2.69\cdot 2$$

$$= 23.86 + 5.38$$

$$\boxed{= 29.24}$$

f. If assumptions 1-4 hold, how much more money does Bob make if he boxes two more rounds? (5 points)

$$2 \cdot 2.69 = \boxed{5.38}$$

$$\hat{E} = 20 - 2.69 \cdot R$$

g. Compute and interpret the R-Squared. (5 points)

R	E		$\hat{u}$
7	40	42.69	-2.69
9	50	48.07	1.93
2	30	29.24	0.76

$$SST = \sum_{i=1}^{3} (E_i - \mu_E)^2 = (n-1)\hat{\sigma}_E^2 = 2 \cdot 100 = 200$$

$$SSR = (-2.69)^2 + (1.93)^2 + (0.76)^2$$

$$= 11.538$$

$$R^2 = 1 - \frac{11.538}{200} = \boxed{0.942}$$

h. Suppose Bob forgot to include a measure of opponent quality, *quality*, in his analysis. Since Bob is "the greatest of all time", fighting a higher quality opponent tends to result in more rounds boxed. However, fighting a higher quality opponent tends to increase the TV payout to Bob. In what direction is the bias? Will this strengthen or weaken the result in d? (5 points)

Earn = Rounds + u

T

The bias is positive. This will weaken the result. The pro-earning benefit of a better opponent will be embodied in $\hat{\beta}_1$

i. Suppose that Bob decided to include *eyecolor* as an independent variable. It has no relationship with any variable mentioned thus far. Why should we be wary of the decision to include this variable? (5 points)

It will not produce any bias, but it will weaken the efficiency of the estimates. That is, variance of all estimates will be higher.

j. Suppose that Bob estimates $se(\hat{\beta}_1)$ so that he can test the hypothesis that β_1=0. However, after estimation, he decides that he wants to measure *Rounds* in minutes. There are 3 minutes in each round. What will happen to $\hat{\beta}_1$ and the t-statistic if *Rounds* is measured in minutes? (5 points)

Both $\hat{B}_1$ and $\widehat{se(B_1)}$ will be lower by $\frac{1}{3}$. Thus, the t-statistic will not change.

Extra Credit: Prove that $\sum_{i=1}^{n}(x_i x_i) - \sum_{i=1}^{n}(x_i \hat{\mu}_x) = \sum_{i=1}^{n}(x_i - \hat{\mu}_x)^2$. (10 points)

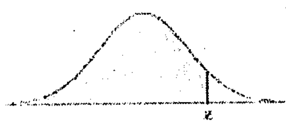
See Lecture

Helpful Formulas

$$\hat{\sigma}_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \hat{\mu}_x\right)^2 \qquad \hat{\sigma}_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \hat{\mu}_x\right)\left(y_i - \hat{\mu}_y\right) \qquad \hat{\rho}_{xy} = \frac{\hat{\sigma}_{xy}}{\hat{\sigma}_x \hat{\sigma}_y}$$

$$\hat{\beta}_0 = \hat{\mu}_y - \hat{\beta}_1 \hat{\mu}_x \qquad \hat{\beta}_1 = \frac{\sum_{i=1}^{n}\left(x_i - \hat{\mu}_x\right)\left(y_i - \hat{\mu}_y\right)}{\sum_{i=1}^{n}\left(x_i - \hat{\mu}_x\right)^2}$$

$$R^2 = 1 - \frac{SSR}{SST} \qquad SSR = \sum_{i=1}^{n}\left(\hat{u}_i\right)^2 \qquad SST = \sum_{i=1}^{n}\left(y_i - \hat{\mu}_y\right)^2$$



Normal Distribution from -oo to Z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990