Economics 113

Professor Spearot

Winter 2009

Please report all regression output and commands.

Using Wage2.dta from the course website, please run the following regression.

$$log(wage) = \beta_0 + \beta_{Educ}Educ + \beta_{Exper}Exper + \beta_{IQ}IQ + \beta_{Order}BrthOrd + u \quad (1)$$

Here, $wage$ is the monthly wage, $Educ$ is years of education, $Exper$ is years of experience, IQ is IQ, and $BrthOrd$ is the order of birth of the respondent (within their family).

```
. regress lwage educ exper iq brthord

      Source |       SS       df       MS              Number of obs =     852
-------------+------------------------------           F(  4,   847) =   41.59
       Model |  24.1005736     4  6.02514339           Prob > F      =  0.0000
    Residual |  122.715501   847  .144882528           R-squared     =  0.1642
-------------+------------------------------           Adj R-squared =  0.1602
       Total |  146.816075   851  .172521827           Root MSE      =  .38063

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |   .0573578    .007679     7.47   0.000     .0422857    .0724299
       exper |   .0177944    .003369     5.28   0.000     .0111817    .0244071
          iq |   .0051764   .0010361     5.00   0.000     .0031427    .0072101
     brthord |  -.0164096    .008387    -1.96   0.051    -.0328714    .0000522
       _cons |   5.323603   .1343107    39.64   0.000     5.059982    5.587224
------------------------------------------------------------------------------
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a. What is the R^2 for this regression?

$R^2 = 0.1642$

b. Does IQ significantly affect wages? That is, can you conclude that β_{IQ} is significantly different from zero? Test this hypothesis at the 95% level.

Yes.

Null hypothesis H_0: $\beta_{IQ} = 0$

There are three ways to check if we can reject the null or not.

i). **Conduct a T-Test** To reject the null at 95% level from both side, we need

$$t - stat = |\frac{\beta_{IQ} - 0}{Std.\beta_{IQ}}| \geq 1.96 \quad (2)$$

1.96 is the critical value at 95% level (obtained from the standard normal distribution).

The regression output tells us that the t-statistic for β_{IQ} is 5.00, which is larger than 1.96. Therefore, the null $\beta_{IQ} = 0$ is rejected at 95% level in favor of the alternative.

ii).**Check the p-value**

The p-value tells us the probability that we are incorrect when we reject the null hypothesis. According to the regression output, the p-value is 0.000. That means, there is 0.000% probability that we reject the null hypothesis incorrectly. To reject the null at the 95% level means we will reject the null if the probability of rejecting it incorrectly is equal or less than 5%. Therefore, the current p-value, which is lower than 5% tells us that we can reject the null at 95% level.

iii). **Check the Confidence Interval**

The confidence interval gives us a range of values which we cannot reject as a null hypothesis. At the 95% level, the confidence interval for the null β_{IQ} satisfies:

$$|\frac{\widehat{\beta}_{IQ} - \beta_{IQ}}{Std.\beta_{IQ}}| \leq 1.96 \quad (3)$$

The regression output already gives us the Confidence Interval at 95% level, which is [0.0031427, 0.0072101]. The null $\beta_{IQ} = 0$ is not within this interval. Therefore, we reject the null at the 95% level.

c. Please construct a 99% confidence interval for β_{Educ}. Please interpret your results.

The t-distribution table gives that at 99% level, the t-statistics is 2.576. Thus the Confidence

Interval for the null β_{educ} satisfies:

$$|\frac{\widehat{\beta}_{Educ} - \beta_{educ}}{Std.\beta_{Educ}}| = |\frac{0.0573578 - \beta_{educ}}{.007679}| \leq 2.576, \tag{4}$$

Solving the above expression, we obtain the following confidence Interval for β_{educ} at 99% level: $0.03769956 < \beta_{educ} < 0.07701604$. This implies that with 99% confidence, each additional year of education increases your wage between 3.7 and 7.7%.

d. Suppose that I reject the hypothesis that $\beta_{Order} = 0$ in favor of a two-sided alternative. What does this mean? What is the probability that I'm wrong? Interpret the result. Should we include BrthOrd in the regression?

If we reject the hypothesis that $\beta_{Order} = 0$ in favor of a two-sided alternative, that means *Order* has a significant effect on wage, either positive or negative. The p-value is the probability of being wrong when rejecting the null hypothesis. In the regression output, p-value for $\beta_{BrthOrd}$ is 0.051. Thus, there is a low chance of concluding that birth order matters and being wrong, and we should keep it in the regression.

e. Suppose that I claim the effect of a one year increase in education is the same as a one year increase in experience. Derive an equation to test this hypothesis, and estimate the new equation using stata. Am I correct?

If experience and education have the same effect, it must be the case that

$$\theta \equiv \beta_{Educ} - \beta_{Exper} = 0$$

Solving for β_{Educ} and plugging into the regression equation, we get:

$$log(wage) = \beta_0 + \left(\theta + \beta_{Exper}\right) Educ + \beta_{Exper} Exper + \beta_{IQ} IQ + \beta_{Order} BrthOrd + u$$

simplifying:

$$log(wage) = \beta_0 + \theta Educ + \beta_{Exper} \left(Educ + Exper\right) + \beta_{IQ} IQ + \beta_{Order} BrthOrd + u$$

Running this regression, we get:

```
. gen educexper=educ+exper

. regress lwage educ educexper iq brthord

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   educexper |   .0177944    .003369     5.28   0.000     .0111817    .0244071
          iq |   .0051764   .0010361     5.00   0.000     .0031427    .0072101
     brthord |  -.0164096    .008387    -1.96   0.051    -.0328714    .0000522
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------------------------------------------------------------------------------
```

The new coefficient on θ represents the estimated difference between the effect of education and the effect of experience. Clearly, this is positive and highly significant, where the effect of an additional year of education is 3.9% greater than an additional year of experience.