

Homework 2 - Answer Key

1a.

$$\Pr(S_n) = .8$$

Given that each roll is independent, we can multiply the probability of getting a strike by itself three times.

$$\Pr(S_1 \cap S_2 \cap S_3) = .8 * .8 * .8 = .512$$

1b.

Not getting the Turkey is the complement of getting the Turkey.

In math terms we can say, $\Pr(\text{Turkey}) + \Pr(\text{No Turkey}) = 1$.

$$\Pr(\text{No Turkey}) = 1 - .512 = .488$$

2a.

$S = \{\text{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT}\}$

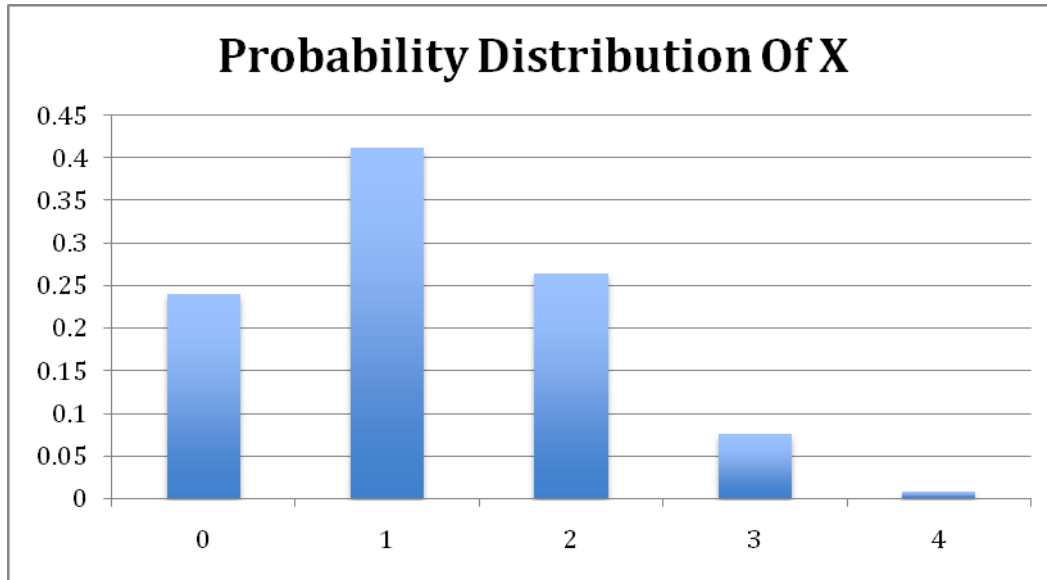
$$\Pr(X=4) = .3 * .3 * .3 * .3 = .0081$$

$$\Pr(X=3) = (4)(.3 * .3 * .3 * .7) = .0756$$

$$\Pr(X=2) = (6)(.3 * .3 * .7 * .7) = .2646$$

$$\Pr(X=1) = (4)(.3 * .7 * .7 * .7) = .4116$$

$$\Pr(X=0) = (.7 * .7 * .7 * .7) = .2401$$



2b.

$$E[X] = \sum X_i \cdot \Pr(X_i) = 4 \cdot 0.0081 + 3 \cdot 0.0756 + 2 \cdot 0.2646 + 1 \cdot 0.4116 + 0 \cdot 0.2401 = 1.2$$

3.

$$\Pr(A|B) = \Pr(A \cap B) / \Pr(B)$$

Define event A to be drawing from the all-red-card bowl.

Define Event B to be drawing a red card.

$\Pr(A \cap B) = 4/8$ because there are 4 instances of a red card coinciding with the all-red-card bowl (The starred entries in the diagram below).

| All-Red-Card Bowl | Other Bowl |
|-------------------|------------|
| Red* | Red |
| Red* | Black |
| Red* | Black |
| Red* | Black |

$\Pr(B) = 5/8$ because there are 5 red cards out of 8 total.

$$\Rightarrow \Pr(A|B) = (4/8) / (5/8) = 4/5 = .8$$

4a.

The probability that a wave is 6 ft tall is equal to 0. With all continuous variables, the probability of getting any single value is 0.

4b.

First we calculate the z-scores.

$$z(6)=(6-6)/2=0$$

$$z(9)=(9-6)/2=1.5$$

$$\Pr[z(6)<Z<z(9)]=\Pr(0<Z<1.5)=\Pr(Z<1.5)-\Pr(Z<0)$$

Then we look up the values for $\Pr(Z<1.5)$ and $\Pr(Z<0)$ on our tables and plug them in.

$$\Pr(0<Z<1.5) = .9332 - .5 = .4332$$

4c.

Calculate z-scores:

$$z(2)=(2-6)/2=-2$$

$$z(7)=(7-6)/2=.5$$

$$\Pr[z(2)<Z<z(7)]=\Pr(-2<Z<.5)=\Pr(Z<.5)-[1-\Pr(Z<2)]$$

Look up values and plug-in:

$$.6915 - .0227 = .6688$$

4d.

Define the first wave being between 4ft and 7ft as Event A.

Define the second wave being between 4ft and 9ft as Event B.

Find $\Pr(A)$:

$$z(4)=(4-6)/2=-1$$

$$z(7)=(7-6)/2=.5$$

$$\Pr[z(4)<Z<z(7)]=\Pr(-1<Z<.5)=\Pr(Z<.5)-[1-\Pr(Z<1)]$$

$$=.6915 - .1586 = .5329 = \Pr(A)$$

Find $\Pr(B)$:

$$z(4) = (4-6)/2 = -1$$

$$z(9) = (9-6)/2 = 1.5$$

$$\begin{aligned}\Pr[z(4) < Z < z(9)] &= \Pr(-1 < Z < 1.5) = \Pr(Z < 1.5) - [1 - \Pr(Z < 1)] \\ &= .9332 - .1587 = .7745\end{aligned}$$

Use formula for $\Pr(A \text{ or } B)$:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

Because the events are independent we can say:

$$\Pr(A \cap B) = \Pr(A) * \Pr(B)$$

Plugging values into $\Pr(A \text{ or } B)$:

$$\Pr(A \cup B) = .5329 + .7745 - (.5329 * .7745) = .8947$$