Lecture 7 - Economics 113

Professor Spearot

- Agenda
  1. Changing Units of Measurement
  2. Nonlinear effects
  3. OLS and unbiased estimates
  4. Variance of the OLS estimates

OLS

Changing Units of Measurement

- Data Scaling
  - Predictions in different units
  - Different interpretations

- Example:

$$wage = \beta_0 + \beta_1 educ + \beta_2 tenure + u$$

  - *educ* is in years
  - *tenure* is years on the job
  - *wage* is in dollars

- Estimates:

$$\widehat{wage} = \widehat{\beta}_0 + \widehat{\beta}_1 educ + \widehat{\beta}_2 tenure$$

- Again, the u vanishes since $E[u|educ, tenure] = 0$.

OLS

Changing Units of Measurement

- Wage in cents rather than dollars?

$$\implies \widehat{wage}_{dollars} = \tfrac{1}{100}\widehat{wage}_{cents}$$

- Original Equation:

$$\widehat{wage}_{dollars} = \widehat{\beta}_0 + \widehat{\beta}_1 educ + \widehat{\beta}_2 tenure$$

- Substitute:

$$\begin{aligned} \frac{1}{100}\widehat{wage}_{cents} &= \widehat{\beta}_0 + \widehat{\beta}_1 educ + \widehat{\beta}_2 tenure \\ \widehat{wage}_{cents} &= 100\widehat{\beta}_0 + 100\widehat{\beta}_1 educ + 100\widehat{\beta}_2 tenure \end{aligned}$$

- What if we want to measure tenure in months?

$$\implies tenure_{years} = \tfrac{1}{12} tenure_{months}$$

- Substitute

$$\begin{aligned} \widehat{wage} &= \widehat{\beta}_0 + \widehat{\beta}_1 educ + \widehat{\beta}_2 \left(\frac{1}{12} tenure_{months}\right) \\ \widehat{wage} &= \widehat{\beta}_0 + \widehat{\beta}_1 educ + \left(\frac{1}{12}\widehat{\beta}_2\right) tenure_{months} \end{aligned}$$

OLS

Handling non-linearity

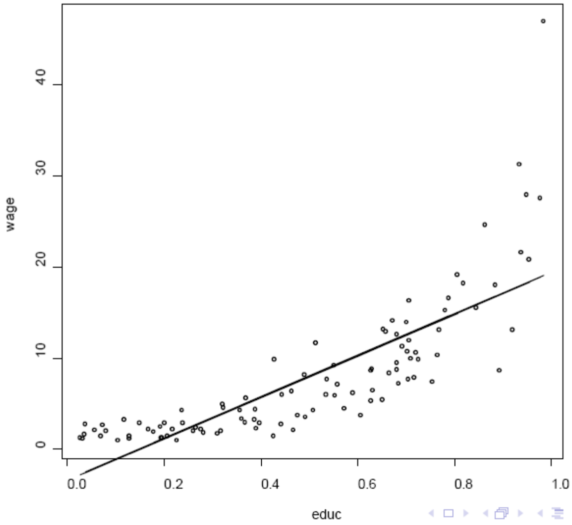
- Not everything linear in real life.
- Relationship between education and wage is linear?

$\Longrightarrow$ No.
$\Longrightarrow$ Which has the higher benefit?
1. 3 more years after 6th grade
2. 3 more years after undergrad?

- Common ways to easily handle non-linearity
    1. Take logs of the dependent variable
    2. Take logs of the independent variable
    3. Take logs of both

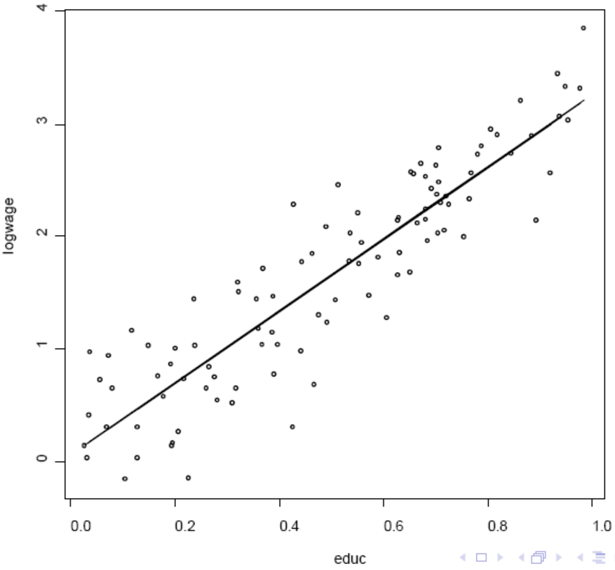
OLS

Wage in Levels



OLS

Wage in logs



OLS

Handling non-linearity

- If data are in levels:

$$\widehat{wage} = \widehat{\beta}_0 + \widehat{\beta}_1 educ$$

- How do we interpret $\widehat{\beta}_1$?
- Totally differentiate.

$$\partial \widehat{wage} = \widehat{\beta}_1 \partial educ$$

- Simplify

$$\frac{\partial \widehat{wage}}{\partial educ} = \widehat{\beta}_1$$

- Interpret $\widehat{\beta}_1$

$$\widehat{wage} = 15,432 + 1,324 educ$$

- For each additional year of education, you earn $1,324 more.

OLS

Handling non-linearity

- If *wage* is in logs

$$\log(\widehat{wage}) = \widehat{\beta}_0 + \widehat{\beta}_1 educ + \widehat{\beta}_2 tenure$$

- How do we interpret $\widehat{\beta}_1$?
- Totally differentiate.

$$\frac{\partial \widehat{wage}}{\widehat{wage}} = \widehat{\beta}_1 \partial educ$$

- Simplify

$$\underbrace{\frac{\partial \widehat{wage}}{\widehat{wage}} * 100}_{\text{\% change}} = \left(\widehat{\beta}_1 * 100\right) \underbrace{\partial educ}_{\text{unit change}}$$

- Interpret $\widehat{\beta}_1$ in the following results

$$\log(\widehat{wage}) = 9.64 + 0.08 educ$$

- A one-year increase in education yields an 8% increase in wage

OLS

Handling non-linearity

- If *wage* and *educ* in logs

$$\log(\widehat{wage}) = \widehat{\beta}_0 + \widehat{\beta}_1 \log(educ) + \widehat{\beta}_2 tenure$$

- How do we interpret $\widehat{\beta}_1$?
- Totally differentiate.

$$\frac{\partial \widehat{wage}}{\widehat{wage}} = \widehat{\beta}_1 \frac{\partial educ}{educ}$$

- Simplify

$$\underbrace{\frac{\partial \widehat{wage}}{\widehat{wage}} * 100}_{\text{\% change}} = \widehat{\beta}_1 \underbrace{\frac{\partial educ}{educ} * 100}_{\text{\% change}}$$

- Interpret $\widehat{\beta}_1$ in the following results

$$\log(\widehat{wage}) = 9.64 + 0.5 \log(educ)$$

- A 1% increase in education yields an 0.5% increase in wage

Simple Regression Model

Biased or unbiased

- When is $\widehat{\beta}_1$ a good estimate, where "good" is defined as unbiased?
- By unbiased, $E\left[\widehat{\beta}_1|x\right] = \beta_1$
  - $\widehat{\beta}_1's$ are centered around β_1
- $\widehat{\beta}_1$ Unbiased if the following assumptions hold!
  1. Linear in parameters: $y_i = \beta_0 + \beta_1 x_i$
  2. Random sample of size n. $\{(x_1, y_1), (x_2, y_2), (x_3, y_3) \ldots (x_n, y_n)\}$
  3. Zero conditional mean: $E(u|x) = 0$
  4. $\sigma_x^2 > 0$.

Simple Regression Model

Biased or unbiased

- Simple example
- Suppose that the population is characterized by:

$$y = 3 - 2x_1 + u$$

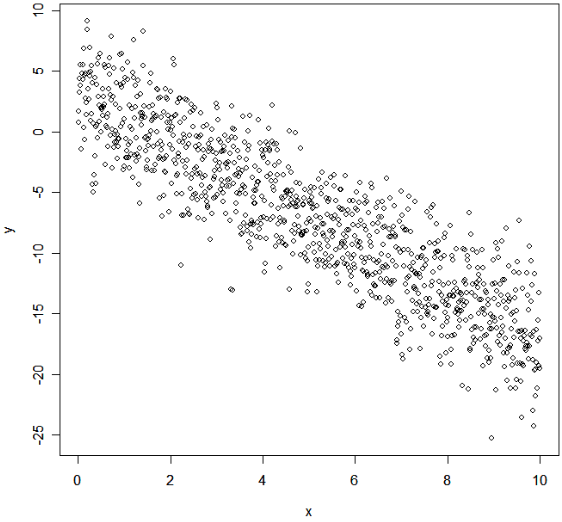
  - $\beta_0 = 3$
  - $\beta_1 = -2$
  - u distributed normal, mean 0 and sd 3
  - $x's$ are between 0.01 and 10, spaced evenly
  - 1000 people
- Estimate using:

$$y = \beta_0 + \beta_1 x_1 + u$$

- Plot y on x

Simple Regression Model

Biased or unbiased



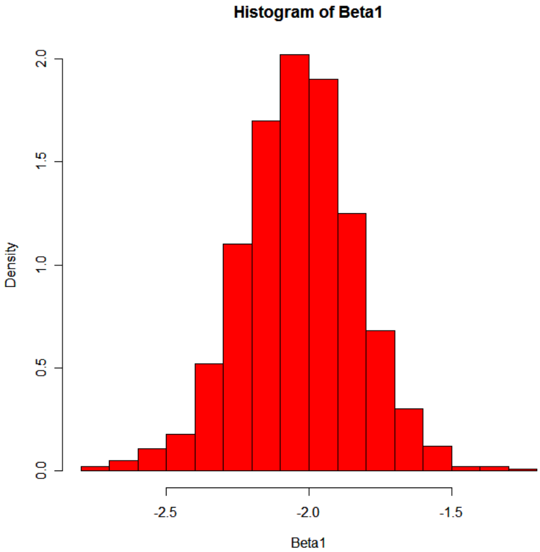
Simple Regression Model

Biased or unbiased

- Suppose that we sample 30 people from the population, and estimate β_1 via OLS
- First sample: $\widehat{\beta}_1 = -1.951$
- Second sample: $\widehat{\beta}_1 = -1.890$
- Third sample: $\widehat{\beta}_1 = -1.559$
- They're all wrong. Is this a problem?
- Keep sampling!!
- Sample 1000 times
- Plot a histogram of the estimates of $\widehat{\beta}_1$
- How does the distribution of estimates compare to -2?

Simple Regression Model

Biased or unbiased



OLS - Variance

Basics

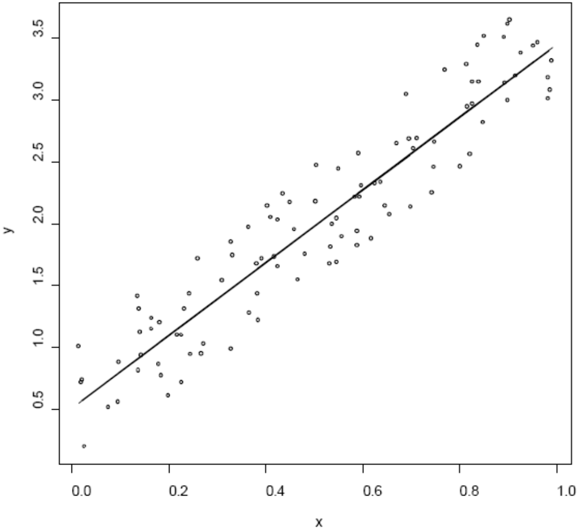
- If assumptions 1-4 hold, $\widehat{\beta}_1$ is centered around β_1.
  - *Central tendency says nothing about dispersion.*
- We are also interested in estimating $Var(\widehat{\beta}_1)$
  - Is the estimate of $\widehat{\beta}_1$precise/reliable?
- Assumption 5 - Homoskedastic Errors:

$$Var\left[u|x\right] = \sigma^2$$

  - Variance of errors is common across x.
  - Assumptions 1-5 are called the "Gauss-Markov Assumptions"
- If $Var\left[u|x\right] \neq Var\left[u\right]$, errors are heteroskedastic.

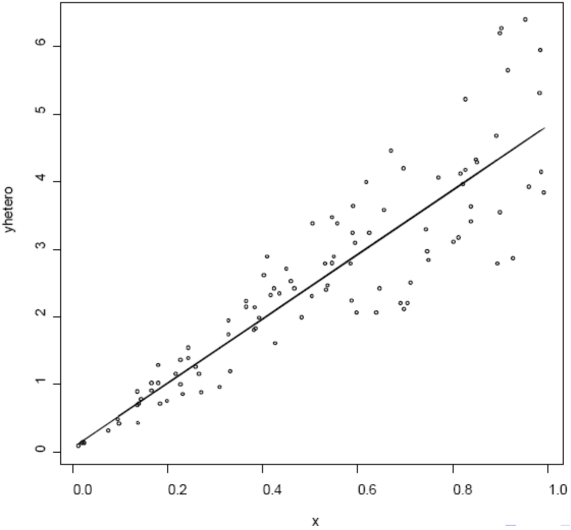
OLS - Variance

Homoskedastic Errors



OLS - Variance

Heteroskedastic Errors



OLS - Variance

Estimate Variance

- Variance of the slope parameter:

$$var\left(\widehat{\beta}_1\right) = \frac{\sigma^2}{\sum_{i=1}^{n}\left(x_i - \widehat{\mu}_x\right)^2}$$

- What do I need for these variance estimates?
    - An estimate of σ^2 :

$$\widehat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n}\widehat{u}_i^2$$

- Why $n-2$?
- $\widehat{\sigma}^2$ requires estimating $\widehat{\beta}_0$ and $\widehat{\beta}_1$.

OLS

Handling non-linearity

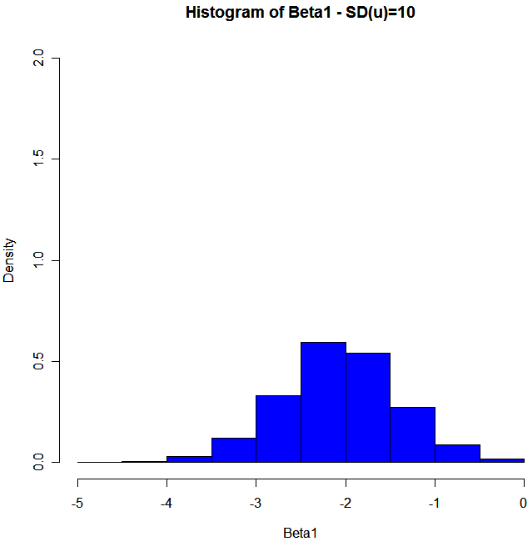
- *Standard error* of $\widehat{\beta}_1$:

$$se\left(\widehat{\beta}_1\right) = \sqrt{\frac{\widehat{\sigma}^2}{\sum_{i=1}^{n}\left(x_i - \widehat{\mu}_x\right)^2}}$$

- Dispersion of $\widehat{\beta}_1$ around β_1, same scale as β_1
- How does $\widehat{\sigma}$ effect the precision of our estimates? Why?
- Higher $\widehat{\sigma}$ yields more higher standard errors (lower precision).
- With higher $\widehat{\sigma}$, there is more noise, and thus it is harder to get a precise estimate of $\widehat{\beta}_1$
- Using the original example, what if u distributed normal, mean 0 and sd **10**

OLS - Variance

Estimate Variance



OLS - Variance

Estimate Variance