Lecture 5 - Economics 113

Agenda

1. Administrative notes
2. Random Variables
3. Normal Distribution
4. Exam on Friday!!!

Office Hours

- Stella: Tuesday 2:00-4:00pm - 403F, E2
- Ren: Wednesday 9:30-11:30am - 403G E2
- Ambrish: Thursday 1:00- 3:00pm - 403F, E2
- MSI

  Monday 2:00 - 3:10PM ARCenter 203
  Wednesday 11:00 - 12:10PM ARCenter 203
  Thursday 2:00 - 3:15PM Oakes Learning Center

Random Variables

Discrete

- Definition: A discrete random variable is a random variable that can take on only a finite number of values.
- Examples?
  - # heads on a series of coin flips
  - # of foreclosures
  - # strikes per game
  - # waves per hour
  - # gaffes per second (the Biden/Palin variable)
- Probability distribution is a list of all possible pairs $[x, Pr(x)]$
  - x is a value of a random variable X
  - $Pr(x)$ is the probability that x occurs.
- Important property: $\sum_{x \in X} \text{Pr(x)=1}$

Random Variables

Discrete

- Example: Flip a fair coin 4 times.
- The random variable X is the number of heads.
- How many values can this take on?

$$(0, 1, 2, 3, 4)$$

- Sample Space

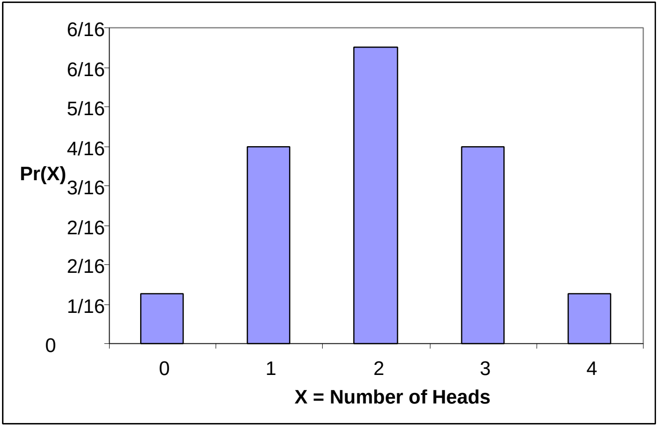
$$S = \{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THTH, THHT, TTHH, HTTT, THTT, TTHT, TTTH, TTTT\}$$

- $Pr(x = 0) = \frac{1}{16}$ or $[0, Pr(0)] = [0, \frac{1}{16}]$
- $Pr(x = 1) = \frac{4}{16}$ or $[1, Pr(1)] = [1, \frac{4}{16}]$
- $Pr(x = 2) = \frac{6}{16}$ or $[2, Pr(2)] = [2, \frac{6}{16}]$
- $Pr(x = 3) = \frac{4}{16}$ or $[3, Pr(3)] = [3, \frac{4}{16}]$
- $Pr(x = 4) = \frac{1}{16}$ or $[4, Pr(4)] = [4, \frac{1}{16}]$

Random Variables

Discrete

- Probability Distribution Function (PDF)



Random Variables

Discrete - Summary Measures

- Expected value: The mean of the probability distribution

$$\mu_x = E(X) = \sum_{x \in X} x \Pr(x)$$

- Previous example?

$$\mu_x = 0 * \frac{1}{16} + 1 * \frac{4}{16} + 2 * \frac{6}{16} + 3 * \frac{4}{16} + 4 * \frac{1}{16} = 2$$

- Variance: Weighted average of the squared deviations around the mean

$$\begin{aligned}
\sigma_x^2 &= E(X - \mu_x)^2 = \sum_{x \in X} (x - \mu_x)^2 \Pr(x) \\
\sigma_x^2 &= (0-2)^2 * \frac{1}{16} + (1-2)^2 * \frac{4}{16} + (2-2)^2 * \frac{6}{16} + (3-2)^2 * \frac{4}{16} \\
&\quad + (4-2)^2 * \frac{1}{16} \\
&= \frac{4}{16} + \frac{4}{16} + 0 + \frac{4}{16} + \frac{4}{16} = 1
\end{aligned}$$

Random Variables

Continuous

- Definition: A variable that can take on an infinite number of values.
- Examples?
  - Consumer expenditures
  - Durations
  - Shares
- Probability distribution:
  - We would like to list out the probability of each event. Two problems:
    1. Too many of them
    2. Chance of any given outcome happening is 0.
  - What is the chance that a person will run a mile in exactly 6:23.3454675678566858567896793345676654467?

Random Variables

Continuous

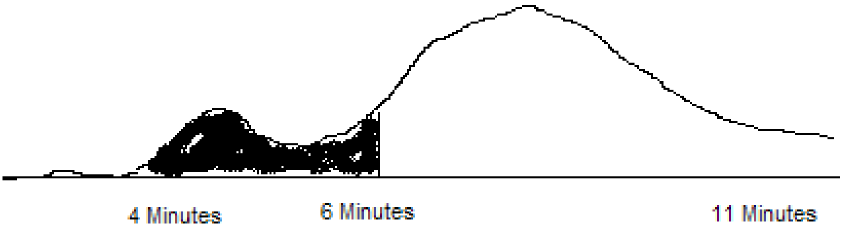
- Example: Distribution of one-mile times by college students

- What can we say about this distribution?

Random Variables

Continuous

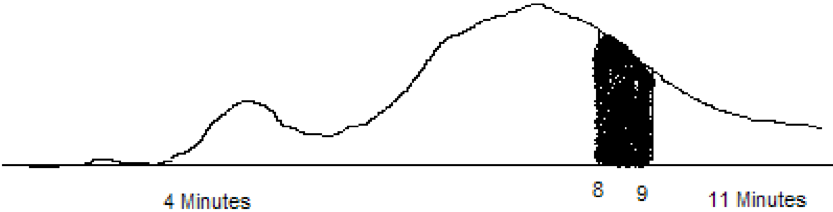
- What is the probability that a randomly selected person can run a mile in less than 6 minutes?
- Shade in the area below six minutes



Random Variables

Continuous

- What is the probability that a randomly selected person runs a mile in between 8 and 9 minutes?
- Shade in the area above 8 and below 9:



Random Variables

Normal Distribution

- The most important distribution in statistics is the normal distribution
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$
- $Mean = \mu$
- $Std.Deviation = \sigma$
- Comes up often in nature
- Symetric Bell Shape
- Infinite support (every real number is possible)
- Small and large numbers are possible, but extremely unlikely (important!!!!)

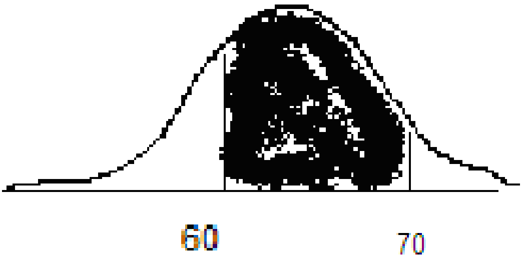
Random Variables

Normal Distribution

- Example: Assume heights of women are distributed normally.
- Mean = 63 and St. Dev = 3
- We seek Pr(60<Woman<70). What do we do?
- Integrate!

$$\Pr(60 \leq x \leq 70) = \int_{60}^{70} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-63}{3}\right)^2}$$

- On a graph:



Random Variables

Standard Normal Distribution

- Define a new random variable

$$z = \frac{(x - \mu)}{\sigma}$$

- Mean of z is zero, st. dev =1, Distribution of z is normal
- Compute z's for 70 and 60

$$\begin{aligned} z_{60} &= \frac{(60 - 63)}{3} = -1 \\ z_{70} &= \frac{(70 - 63)}{3} = 2.33 \end{aligned}$$

- Calculate probability using the Z distribution

$$\begin{aligned} \Pr(60 &\leq x \leq 70) = \Pr(z_{60} < Z < z_{70}) \\ &= \Pr(Z < z_{70}) - \Pr(Z < z_{60}) \\ &= \Pr(Z < 2.33) - \Pr(Z < -1) \\ &= \Pr(Z < 2.33) - (1 - \Pr(Z < 1)) \\ &= .9901 - .1587 = .8314 \end{aligned}$$