

# Lecture 4 - Economics 113

## Agenda

1. Administrative notes
2. Probability (lots of it)

# Office Hours

- ▶ Stella: Tuesday 2:00-4:00pm - 403F, E2
- ▶ Ren: Wednesday 9:30-11:30am - 403G E2
- ▶ Ambrish: Thursday 1:00- 3:00pm - 403F, E2
- ▶ MSI
  - Monday 2:00 - 3:10PM ARCenter 203
  - Wednesday 11:00 - 12:10PM ARCenter 203
  - Thursday 2:00 - 3:15PM Oakes Learning Center

# Probability

## Definitions

- ▶ What is probability?
  - ▶ A numerical measure of how likely an event is to occur.
- ▶ What are the key properties of probability?
- ▶ Probabilities range from 0 to 1.
- ▶ Important types of probability:
  - ▶ Probability of Compound Events
  - ▶ Conditional Probability

# Probability

## Probability of compound events

- ▶ Notation
- ▶  $Pr(A \text{ or } B) = Pr(A \cup B)$ 
  - ▶ Probability that either A, B, or both A and B occur.
  - ▶ “Union” of A and B
- ▶  $Pr(A \text{ and } B) = Pr(A \cap B)$ 
  - ▶ Probability that both A and B occur.
  - ▶ “Intersection” of A and B
- ▶ Example: Draw one card from a deck of cards
  - ▶ Event A: Drawing a face card
  - ▶ Event B: Drawing a spade
- ▶  $Pr(A \cup B)$

$$Pr(A \cup B) = (\text{Draw either a face card or spade or both})$$

- ▶  $Pr(A \cap B)$

$$Pr(A \cap B) = Pr(\text{Draw either a jack, queen, or king of spades})$$

# Probability

## Calculation of the union

- ▶ General formulation

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

- ▶ What if events are mutually exclusive?

$$Pr(A \cap B) = 0$$

- ▶ Thus

$$Pr(A \cup B) = Pr(A) + Pr(B)$$

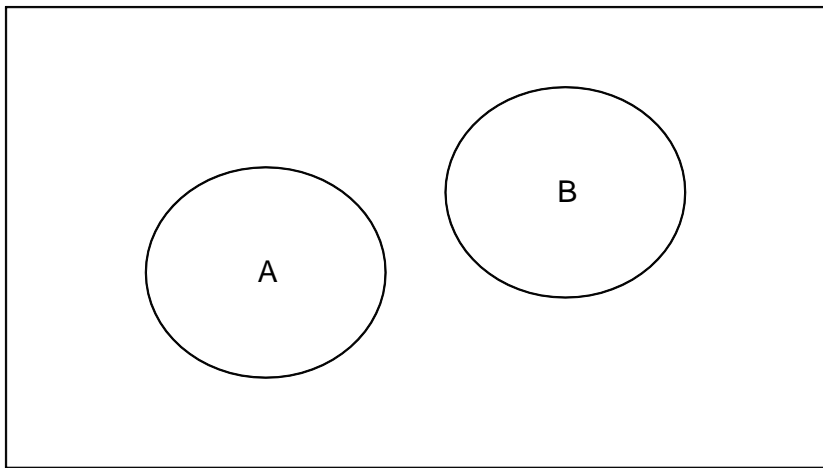
- ▶ Example: Event A is the getting 2 or less. Event B is getting 4 or more.
- ▶ What is  $Pr(A \cup B)$  ?

$$\begin{aligned} Pr(A \cup B) &= Pr(D \leq 2) + Pr(D \geq 4) \\ &= \frac{2}{6} + \frac{3}{6} \end{aligned}$$

# Probability

## Union of Events

- ▶ Union of Mutually Exclusive Events

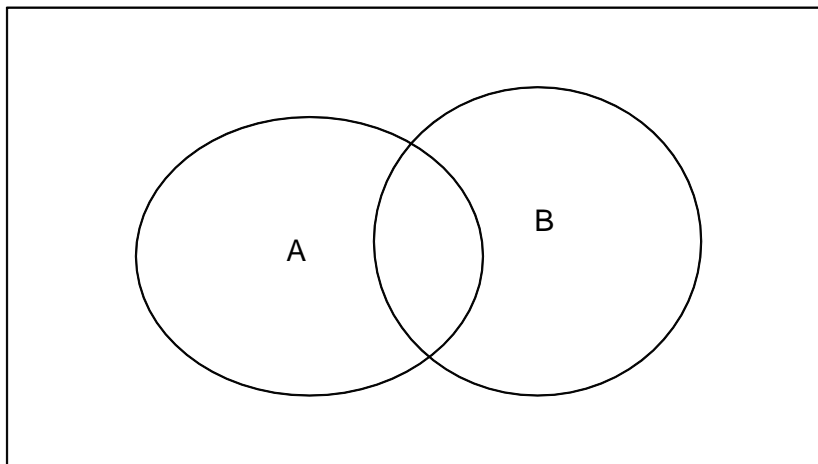


- ▶ Events A and B do not intersect.

# Probability

## Union of Events

- ▶ Events which are not mutually exclusive.

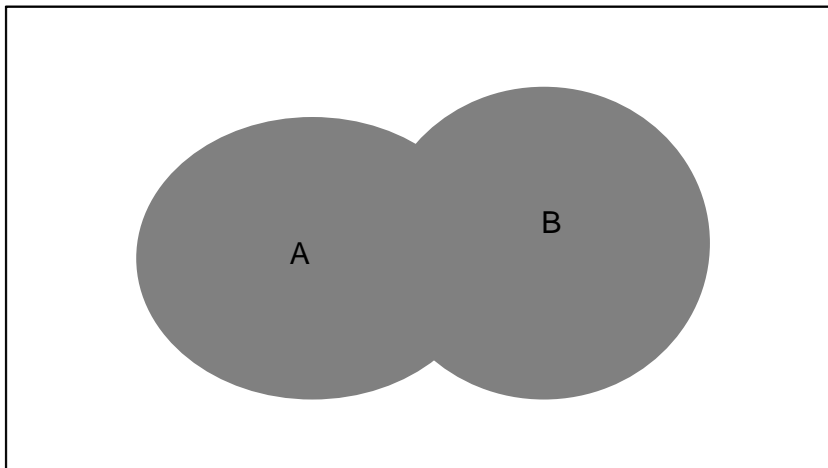


- ▶ Recall  $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$
- ▶ Why do we subtract  $Pr(A \cap B)$ ?

# Probability

## Union of Events

- ▶ Events which are not mutually exclusive.

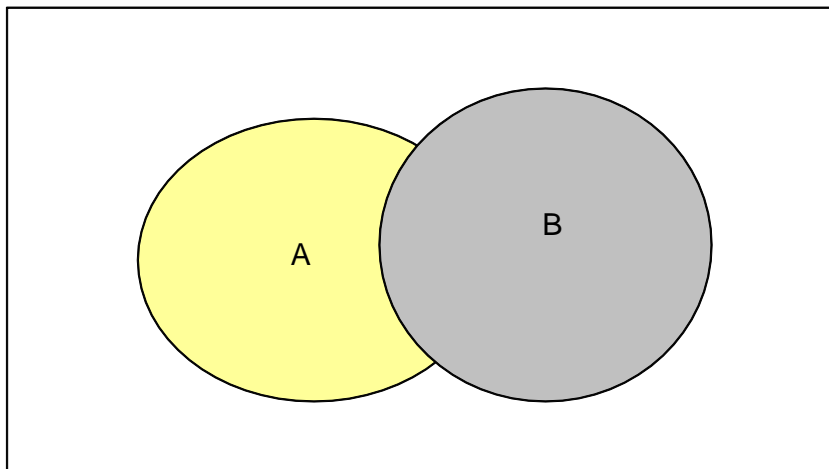


- ▶ Recall  $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$
- ▶ Why do we subtract  $Pr(A \cap B)$ ?

# Probability

## Union of Events

- ▶ Events which are not mutually exclusive.



- ▶ Recall  $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$
- ▶ Why do we subtract  $Pr(A \cap B)$ ?

# Probability

## Union

- ▶ Calculation of the union
- ▶ Event A is getting an even number; Event B is getting 4 or more
- ▶ What is  $\Pr(A)$ ?
  - ▶  $\Pr(A) = 3/6$
- ▶ What is  $\Pr(B)$ ?
  - ▶  $\Pr(B) = 3/6$
- ▶ What is  $\Pr(A \cap B)$ ?
  - ▶  $\Pr(A \cap B) = 2/6$
- ▶ Put it all together

$$\begin{aligned}\Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B) \\ &= 3/6 + 3/6 - 2/6\end{aligned}$$

# Probability

## Complements

- ▶ What is the complement of event A?

$$Pr(A) + Pr(A^c) = 1$$

- ▶ Event A: Roll a die and get 2 or lower:

$$A = \{1, 2\}$$

$$A^c = \{3, 4, 5, 6\}$$

- ▶ Calculate  $Pr(A^c)$ :

$$Pr(A^c) = 1 - Pr(A)$$

$$= 1 - 2/6$$

$$= 4/6$$

# Probability

## Conditional Probability

- ▶ Probability of event A occurring *given that event B has occurred*

$$Pr(A|B)$$

- ▶ Given B, you can shrink the sample space
- ▶ The equation

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

- ▶ Example: Flip a fair coin 3 times
  - ▶ Event A: Getting exactly two heads in three flips
  - ▶ Event B: Getting heads on the first flip

# Probability

## Conditional Probability

- ▶ Diagrams help!!!

HHH	HHT	HTH	THH
HTT	THT	TTH	TTT

- ▶ Diagram with conditioning

HHH	<b>HHT</b>	<b>HTH</b>	
HTT			

- ▶ What are  $Pr(A \cap B)$  and  $Pr(B)$ ?

$$Pr(A \cap B) = 2/8$$

$$Pr(B) = 4/8$$

- ▶ Put it together

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{\frac{2}{8}}{\frac{4}{8}} = 1/2$$

# Probability

## Statistical Independence

- ▶ **Definition:** Events A and B are *statistically independent* if the occurrence of one event does not effect the probability of the other event.
- ▶ Example: The experiment is you draw one card from a deck.
  - ▶ Event A is drawing a face card from the deck in the first draw
  - ▶ Event B is drawing a spade from a deck of cards.
- ▶ **Multiplicative rule for independent events:** If event A and event B are statistically independent then:

$$Pr(A \text{ and } B) = Pr(A \cap B) = Pr(A)Pr(B)$$

- ▶ How does this simplify the calculation of  $Pr(A \cup B)$ ?
- ▶ How does this simplify the calculation of  $Pr(A|B)$ ?

# Probability

## Statistical Independence

- ▶ Example: Flipping a rigged coin twice.
  - ▶ Event A is a head on the first flip.
  - ▶ Event B is a tail on the second flip.
  - ▶  $Pr(H) = 0.6$
- ▶ What is  $Pr(A \cap B)$ ?

$$Pr(A \cap B) = Pr(A)P(B) = (.6) \cdot (.4) = 0.24$$

- ▶ Example: What is the probability that I get four heads in four flips?
  - ▶ Event A is a head on the first flip, Event B is a head on the second flip, Event C is a head on the third flip, Event D is a head on the fourth flip
- ▶ What is the probability of getting 4 heads?

$$\begin{aligned}Pr(A \cap B \cap C \cap D) &= Pr(A)P(B)P(C)P(D) \\ &= (.6) \cdot (.6) \cdot (.6) \cdot (.6) = (.6)^4 \\ &= 0.1296\end{aligned}$$