Lecture 3 - Economics 113

Agenda

1. Administrative notes
2. More Examples
3. Data Scaling
4. Basics of Probability
5. Quote of the Day:

Do not fear mistakes. There are none.

-Miles Davis

Office Hours

- Stella: Tuesday 2:00-4:00pm - 403F, E2
- Ren: Wednesday 9:30-11:30am - 403G E2
- Ambrish: Thursday 1:00- 3:00pm - 403F, E2

Computer Program R

- We will eventually use the statistics program "R" to work on computer problems
- By the end of next week, please download it to your computer from: www.r-project.org
- If you do not have a computer, it is available on the UCSC virtual lab

Simple Example

Mean

- Sample $X = \{2, 3, 4\}$
- Calculate the mean of X
- $\widehat{\mu}_x = \frac{1}{n} \sum_{i=1}^{n} x_i$
- $\widehat{\mu}_x = \frac{1}{3} (2 + 3 + 4) = 3$
- Calculate the variance of X
- $\widehat{\sigma}_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \widehat{\mu}_x)^2$
- $\widehat{\sigma}_x^2 = \frac{1}{2} \left((2-3)^2 + (3-3)^2 + (4-3)^2 \right) = \frac{1}{2} (1 + 0 + 1) = 1$

Scaling

Adding a constant

- What if we define a new variable, $Z = X + 3$
- Calculate the mean of $Z = \{5, 6, 7\}$
- What will happen to the mean, variance?
- $\widehat{\mu}_z = \frac{1}{3}(5 + 6 + 7) = 6$
- $\widehat{\sigma}_z^2 = \frac{1}{2}\left((5-6)^2 + (6-6)^2 + (7-6)^2\right) = \frac{1}{2}(1 + 0 + 1) = 1$
- Adding a constant **does** affect central tendency.
- Adding a constant **does not** affect dispersion.
- 1 $\underline{\mathbf{2} \quad \mathbf{3} \quad \mathbf{4}}$ 5 6 7 8 9
- 1 2 3 4 $\underline{\mathbf{5} \quad \mathbf{6} \quad \mathbf{7}}$ 8 9

Scaling

Multiplying by a constant

- What if we define a new variable, $J = 3X$
- Calculate the mean/variance of $J = \{6, 9, 12\}$
- What will happen to the mean, variance?
- $\widehat{\mu}_j = \frac{1}{3}(6 + 9 + 12) = 9$
- $\widehat{\sigma}_j^2 = \frac{1}{2}\left((9-6)^2 + (9-9)^2 + (12-9)^2\right) = \frac{1}{2}(9 + 0 + 9) = 9$
- Multiplying by a constant affects both central tendency and dispersion.
- 1 $\underline{\textbf{2} \quad \textbf{3} \quad \textbf{4}}$ 5 6 7 8 9 10 11 12
- 1 2 3 4 5 $\underline{\textbf{6} \quad 7 \quad 8 \quad \textbf{9} \quad 10 \quad 11 \quad \textbf{12}}$
- Generally, if $J = aX$, then $\widehat{\sigma}_j^2 = a^2\widehat{\sigma}_x^2$, $\widehat{\mu}_j = a\widehat{\mu}_x$

Covariance

Scaling

- Suppose $Z = aX$. What is $\widehat{\sigma}_{zy}$?
- Write $\widehat{\sigma}_{zy}$

$$\widehat{\sigma}_{zy} = \frac{1}{n-1} \sum_{i=1}^{n} (z_i - \widehat{\mu}_z) \left(y_i - \widehat{\mu}_y \right)$$

- Substitute for $\widehat{\mu}_z$ and z_i
- This gives:

$$\widehat{\sigma}_{zy} = \frac{1}{n-1} \sum_{i=1}^{n} (ax_i - a\widehat{\mu}_x) \left(y_i - \widehat{\mu}_y \right)$$

- Factor out a and simplify:

$$\widehat{\sigma}_{zy} = a \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \widehat{\mu}_x) \left(y_i - \widehat{\mu}_y \right) = a\widehat{\sigma}_{xy}$$

- Covariance is sensitive to scale!! Is this a problem?

Correlation

Scaling

- Is correlation sensitive to scale?
- Suppose $Z = aX$
- $\widehat{\rho}_{zy} = \frac{\widehat{\sigma}_{zy}}{\widehat{\sigma}_z \widehat{\sigma}_y} = \frac{a\widehat{\sigma}_{xy}}{a\widehat{\sigma}_x \widehat{\sigma}_y} = \frac{\widehat{\sigma}_{xy}}{\widehat{\sigma}_x \widehat{\sigma}_y} = \widehat{\rho}_{xy}$

Probability

Definitions

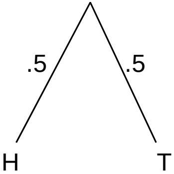
- Experiment: Any manipulation of the world or observation of the world.
  - Flipping a coin
  - Randomized Drug trials
- Sample Point: An outcome of the experiment
  - One sample point must occur
  - Heads or tails
  - Drug success or failure
- Sample points are mutually exclusive
- Sample space: Space of all possible outcomes

Probability

Basics

- How do we represent the sample space?
- Coin flip (fair coin:
  - $S = \{H, T\}$
- Venn diagram:

  - The entire box is the space of all possible outcomes
  - The space in "H" is the probability of H occurring.
  - The space in "F" is the probability of F occurring.
- Tree diagram:



Probability

Basics

- Event: A collection of sample points
  - An event either happens or does not.
  - One sample point is an event.
  - A group of sample points is an event.
- The *complement* of an event is a way to represent the event not occurring.
  - The complement of Event A is denoted A^C
  - Suppose that Event A is heads
    - A^C is tails.

Probability

Basics

- How do we represent the event space?
- New example: Flip a coin twice.
  - Sample space?
  - $S = \{HH, HT, TH, TT\}$
- Suppose that event A is getting heads on exactly one of the two flips.
  - Event space?
  - $A = \{HT, TH\}$
  - What is A^C?
  - $A^C = \{HH, TT\}$
- How do we represent this with Venn and Tree diagrams?

Probability

Basics

- Venn Diagram

HH	TH
HT	TT

- Tree Diagram