Lecture 17 - Economics 113

Professor Spearot

- Agenda
  1. F-Tests - General Regression Significance
  2. Quadratics
  3. Interactions
- Homework 1 due next friday. - LONG!!!
- Midterm 4 is next Friday, in class.

Multiple restrictions

The F-Test

- Compare a base model with a restricted model
- Use the F-Statistic

$$F_{stat} = \frac{\text{Average loss in explanatory power due to restrictions}}{\text{Average unexplained variation in base model}}$$

- If F is high $\Rightarrow$ we lose a ton by our restrictions.
- Under the null, F_{stat} is distributed according to an "F-distribution"
  - Even if restrictions are "good", large F_{stat} is randomly possible, but unlikely
- Compare $Fstat$ to the "F Distribution"
  1. Choose a significance level (say 5%)
  2. Using the 5% F-table, find the critical value, F_{crit}.
  3. If $F > F_{crit}$, reject the null (reject the restricted model in favor of the unrestricted model)

The F-Test

General Regression Significance

- Unrestricted model

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 exper + \beta_3 tenure + u$$

- Test $H_0 : \beta_1 = 0, \beta_2 = 0, \beta_3 = 0$
- Restricted model

$$\log(wage) = \beta_0 + u$$

- Create a new dependent variable
- Run regressions, conduct F Test.

Regressions

Quadratic terms

- The basic model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + u$$

- How should we interpret β_1 , β_2?
  - Together
- Take the derivative w.r.t x_1.

$$\frac{\partial y}{\partial x_1} = \beta_1 + 2\beta_2 x_1$$

- If $\beta_2 \neq 0$, then we can solve for the maximum or minimum

$$\begin{aligned} \beta_1 + 2\beta_2 x_1^* &= 0 \\ x_1^* &= -\frac{\beta_1}{2\beta_2} \end{aligned}$$

- What determines whether it is a maximum or minimum?

Example

Wages and Age

- Regression Equation

  $$\log(wage) = \beta_0 + \beta_{age} age + \beta_{age^2} age^2 + \beta_{educ} educ + \beta_{exper} exper + u$$

- Conditional on education and experience, how does age affect wages?
- Marginal effect of age on education

$$\frac{\partial \log(wage)}{\partial age} = \beta_{age} + 2\beta_{age^2} age$$

- Estimate!

Regressions

Quadratic terms

- Determining wages
- Base model

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 exper + \beta_3 tenure + u$$

- Squared model

$$\begin{aligned} \log(wage) &= \beta_0 + \beta_1 educ + \beta_2 exper + \beta_3 tenure \\ &\quad + \beta_4 exper^2 + \beta_5 tenure^2 \end{aligned}$$

- How do we interpret? Should we keep in the squared terms?
- $H_0: \ \beta_4 = 0, \, \beta_5 = 0$

Regressions

Interaction terms

- The basic model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_2 + u$$

- How do we interpret β_1 , β_2?
- Take the derivative w.r.t x_1.

$$\frac{\partial y}{\partial x_1} = \beta_1 + \beta_2 x_2$$

- How do we interpret $\frac{\partial y}{\partial x_1}$?

  $\Rightarrow$ If $\beta_2 > 0$, then higher x_2 increases $\frac{\partial y}{\partial x_1}$

  $\Rightarrow$ If $\beta_2 < 0$, then higher x_2 decreases $\frac{\partial y}{\partial x_1}$

Regressions

Interaction terms

- Interact *age* with *educ*

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 educ * age + \beta_3 age + \beta_4 exper + u$$

- How do we interpret these results?
- Do they make sense?
- Should we keep in the interaction term?