Lecture 16 - Economics 113

Professor Spearot

- Agenda
  1. Basic F-tests
  2. General regression significance
  3. General Linear Restrictions
- Homework 5 due next friday. - LONG!!!
- Midterm 4 is next Friday, in class.

Multiple restrictions

The F-Test

- Baseball player salaries:

$$\begin{aligned}\widehat{\log(salary)} &= \underset{(0.29)}{11.19} + \underset{(0.0121)}{0.0689}\,years + \underset{(0.0026)}{0.0126}\,gamesyr \\ &\quad + \underset{(0.00110)}{0.00098}\,avg + \underset{(0.0161)}{0.0144}\,hrun + \underset{(0.0072)}{0.0108}\,rbi \\ obs &= 353,\ SSR = 183.186,\ R^2 = 0.6278\end{aligned}$$

- Call this model the "unrestricted model".
- How do we test $H_0 : \widehat{\beta}_{avg} = \widehat{\beta}_{hrun} = \widehat{\beta}_{rbi} = 0$?
- What is the alternative hypothesis?

$$\Rightarrow\ H_A :\ \text{Some combination is significant.}$$

- Restricted Baseball model

$$\begin{aligned}\widehat{\log(salary)} &= \underset{(0.11)}{11.22} + \underset{(0.0125)}{0.0689}\,years + \underset{(0.0013)}{0.0126}\,gamesyr \\ obs &= 353,\ SSR = 198.311,\ R^2 = 0.5971\end{aligned}$$

- Restricted model imposes the null hypothesis

Multiple restrictions

The F-Test

- We compare models by computing an F-statistic:

$$F = \frac{\frac{(SSR_R - SSR_{UR})}{q}}{\frac{SSR_{UR}}{n-k-1}}$$

  - SSR_R is the SSR for the restricted model.
  - SSR_{UR} is the SSR for the unrestricted model.
  - q is the number of restrictions
  - $n - k - 1$ is DOF for the unrestricted model

- Intuition

$$F = \frac{\text{Average loss in explanatory power under } H_0}{\text{Average unexplained variation under } H_A}$$

- If F is high $\Rightarrow$ we lose a ton by our restrictions.
- For our model:

$$F_{stat} = \frac{\frac{(SSR_R - SSR_{UR})}{q}}{\frac{SSR_{UR}}{n-k-1}} = \frac{\frac{(198.311 - 183.186)}{3}}{\frac{183.186}{348}} = 9.55$$

Multiple restrictions

The F-Test

- Under the null, F_{stat} is distributed according to an "F-distribution"
  - Even if restrictions are "good", large F_{stat} is randomly possible, but unlikely
- Compare *Fstat* to the "F Distribution"
  1. Choose a significance level (say 5%)
  2. Using the 5% F-table, find the critical value, F_{crit}.
  3. If $F > F_{crit}$, reject the null!!
- Looking at the table, $F_{crit} = 2.60$
- Reject the null! Some combination of those variables is significant.

The F-Test

General Restrictions

- Unrestricted model

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 \log(exper) + \beta_3 log(tenure) + u$$

- Test $H_0 : \beta_1 = 1, \beta_2 = 0.5$
- New restricted model

$$\log(wage) - \log(educ) - 0.5 \log(exper) = \beta_0 + \beta_3 log(tenure) + u$$

- Create a new dependent variable
- Run regressions, conduct F Test.