Lecture 12 - Economics 113

Professor Spearot

- Agenda
  1. T-Tests - One-sided and Two-sided
  2. Computer!!
- Exams can be picked up after class if you didn't pick them up on Monday.

Hypothesis Testing

Basics

- "Null Hypothesis": The hypothesis that we are testing.
- "Alternative Hypothesis": Hypothesis which we are testing the null against.
- Two outcomes:
  - Reject the null in favor of an alternative.
  - Fail to reject the null.
- We tend to reject the null when the estimate is "far away" from the null, toward the alternative hypothesis.
- "far away" is relative to the precision of our estimates.
- Given an estimate and standard error (precision), "far away" is determined by the t-distribution.

The t-statistic

An example - one sided test

- Experience and Wages

$$\log(wage) = \beta_0 + \beta_1 Educ + \beta_2 Exper + \beta_3 Tenure + u$$

- We estimate:

$$\begin{aligned} \widehat{\log(wage)} &= \underset{(0.104)}{0.284} + \underset{(0.007)}{0.092}\, Educ + \underset{(0.0017)}{0.0041}\, Exper + \underset{(0.003)}{0.022}\, Tenure \\ obs &= 526,\ R^2 = 0.316 \end{aligned}$$

- $\underset{\text{(standard error)}}{estimate}$
- $H_0 : \beta_2 = 0.$
- $H_A : \beta_2 > 0$
- Use a t-test to test the null against the alternative at 95% confidence

The t-statistic

An example - one sided test

- Three steps
  1. Calculate $n-k-1$
  2. If $n-k-1$ large enough, find the appropriate critical value, t_{crit}, using the standard normal distribution.
  3. If $\frac{\widehat{\beta}_2 - 0}{se(\widehat{\beta}_2)} > t_{crit}$, reject H_0 in favor of H_A.

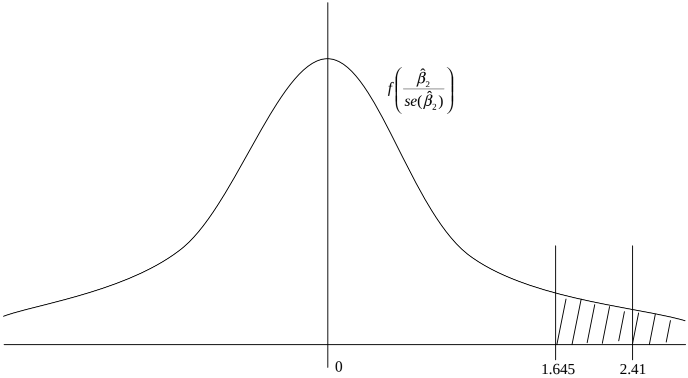
     $\Rightarrow$ Thus, if $\widehat{\beta}_2$ is sufficiently *greater* than zero, it is very unlikely that $\beta_2 = 0$.

     $\Rightarrow$ It would require a fluke for $\widehat{\beta}_2$ to be significantly greater than its true value.
- With enough observations, at 95% confidence, $t_{crit} = 1.645$
- Since $t(\widehat{\beta}_2 | \beta_2 = 0) = 2.41 > 1.645$, **reject** $\beta_2 = 0$ in favor of $\beta_2 > 0$.
- Experience has a **positive and statistically significant** effect on wages!

The t-statistic

One sided test - graphically

- We are in the rejection region.



- So we reject the null.
- We may falsely reject H_0, but on average, it will be less than 5% of the time.

The t-statistic

An example - two-sided tests

- Two-sided tests require a slightly different approach.
- Now, the alternative is $H_A : \beta_2 \neq 0$
- This is often used in economics.
- Three steps
  1. Calculate $n - k - 1$
  2. If $n - k - 1$ large enough, find the two-sided critical value, t_{crit}, using the standard normal distribution.
  3. If $\left| t(\widehat{\beta}_2 | \beta_2 = 0) \right| > t_{crit}$, reject H_0 in favor of H_A.

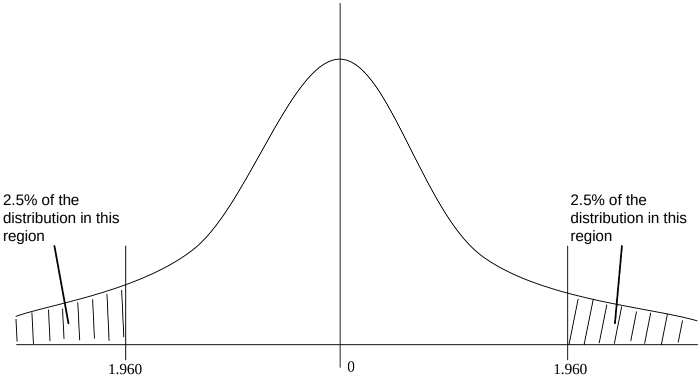
$\Rightarrow$ Thus, if $\widehat{\beta}_2$ is sufficiently far away from zero, it is very unlikely that $\beta_2 = 0$.

$\Rightarrow$ $\widehat{\beta}_2$ can now either be positive or negative.

The t-statistic

two sided test - graphically

- At 95% confidence, $t_{crit} = 1.960$



- Reject the null if we are in one of the shaded regions.
- Together, they equal 5% of the distribution.

The t-statistic

An example - two-sided test

- Test scores and school size. We estimate:

$$\widehat{math10} = \underset{(6.113)}{2.274} + \underset{(0.00010)}{0.00046}\, totcomp + \underset{(0.040)}{0.048}\, staff - \underset{(0.00022)}{0.0002}\, enroll$$
$$obs = 408,\ R^2 = 0.0541$$

- Suppose $H_0 : \beta_{enroll} = 0$.
- Let $H_A : \beta_{enroll} \neq 0$.
- T-statistic:

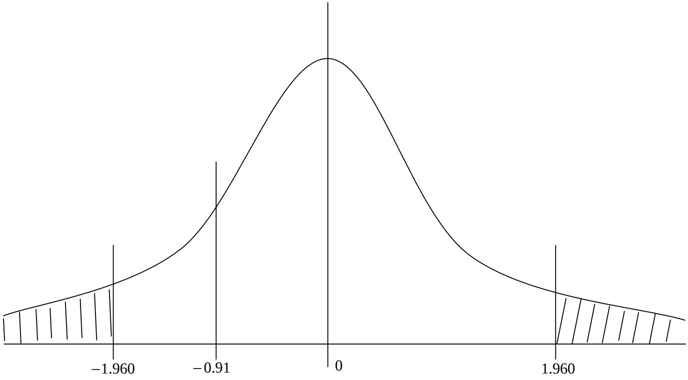
$$t(\widehat{\beta}_{enroll} | \beta_{enroll} = 0) = \frac{\widehat{\beta}_{enroll} - 0}{se(\widehat{\beta}_{enroll})} = \frac{-0.0002}{0.00022} = -0.91$$

- $t_{crit} = 1.960$.
- Since $|-0.91| < 1.960$, we **cannot reject the null** that enrollment has no effect.

The t-statistic

two sided test - graphically

- $\widehat{\beta}_{enroll}$ is not far enough away from zero on either side to conclude that enrollment has an effect on math scores.



The t-test

Non-zero hypothesis

- Crime and School size

$$\log(Crime) = \beta_0 + \beta_1 \log(Enroll) + u$$

- We estimate:

$$\begin{aligned}\widehat{\log(Crime)} &= \underset{(0.104)}{-6.63} + \underset{(0.11)}{1.27} \log(Enroll) \\ obs &= 197,\ R^2 = 0.585\end{aligned}$$

- Suppose $H_0 : \beta_1 = 1$
- How do we interpret this?
- 1% increase in enrollment causes a 1% increase in crime.
- T-stat:

$$t(\widehat{\beta}_2 | \beta_2 = 1) = \frac{\widehat{\beta}_2 - 1}{se(\widehat{\beta}_2)} = \frac{1.27 - 1}{0.11} = 2.45$$

The t-test

An example - two-sided tests

- $H_A : \beta_1 \neq 1$
- Three steps
  1. Calculate $n - k - 1 = 197 - 1 - 1 = 195$
  2. At the 5% level, t_{crit} is roughly 1.96
  3. $|2.45| > 1.96 \Rightarrow$ reject H_0 in favor of H_A.
- A 1% increase in enrollment yields a greater than 1% increase in the crime rate.