Lecture 11 - Economics 113

Professor Spearot

- Agenda
  1. Sampling Distribution
  2. T-statistic
  3. T-Tests - One-sided and Two-sided

Multivariate Regression

Gauss-Markov Theorem

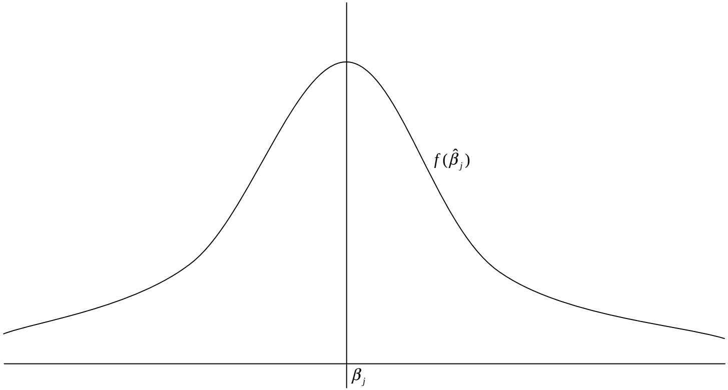
- If the following assumptions hold:
  1. Linear in parameters β_0, β_1,...,β_k
  2. Random Sampling
  3. Zero conditional mean
  4. No perfect collinearity
  5. Homoskedasticity
- OLS is the Best Linear Unbiased Estimator (OLS is BLUE)
- Assumptions 1-5 are referred to as the "Gauss-Markov" assumptions

Multivariate Regression

Inference!!!

- A sixth assumption: $u \tilde{} Normal(0, \sigma^2)$
- With this assumption:

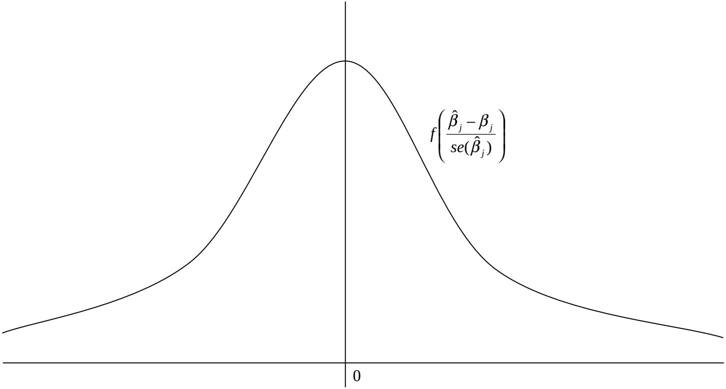
$$\widehat{\beta}_j \tilde{} Normal(\beta_j, Var(\widehat{\beta}_j))$$



Multivariate Regression

Normalization

- If the $\widehat{\beta}_j$'s are normal, what can we do with them?
- Normalize them! $\frac{\widehat{\beta}_j - \beta_j}{sd(\widehat{\beta}_j)} \sim Normal(0,1)$
- Distribution of $\frac{\widehat{\beta}_j - \beta_j}{se(\widehat{\beta}_j)}$ is standard normal distribution if N is large.



Multivariate Regression

The T-Distribution

- More generally:

$$\frac{\widehat{\beta}_j - \beta_j}{se(\widehat{\beta}_j)} \sim t_{n-k-1}$$

- t_{n-k-1} represents the "t-distribution" with $n-k-1$ degrees of freedom.
  - k slope parameters, 1 intercept term, n observations
- $se(\widehat{\beta}_j)$ is the standard error of $\widehat{\beta}_j$

$$se(\widehat{\beta}_j) = \sqrt{Var(\widehat{\beta}_j)}$$

- t_{n-k-1} looks similar to a standard normal distribution.
- This is the 'correct' distribution of $\widehat{\beta}_j' s$, centered around the population value β_j

The t-statistic

The breakdown

$$t(\widehat{\beta}_j | \beta_j) = \frac{\widehat{\beta}_j - \beta_j}{se(\widehat{\beta}_j)}$$

- $\widehat{\beta}_j$ and $se(\widehat{\beta}_j)$ are estimates
- β_j is the population parameter

  $\Longrightarrow$ We can't measure it, so we **form a hypothesis** about it

- If $H_0 : \beta_j = 0$

$$t(\widehat{\beta}_j | \beta_j = 0) = \frac{\widehat{\beta}_j}{se(\widehat{\beta}_j)}$$

- If $H_0 : \beta_j = \beta_H$

$$t(\widehat{\beta}_j | \beta_j = \beta_H) = \frac{\widehat{\beta}_j - \beta_H}{se(\widehat{\beta}_j)}$$

The t-statistic

The breakdown

- The farther $\widehat{\beta}_j$ is from the hypothesized value, the more likely the hypothesis is incorrect.
- $H_0 : \beta_j = 0$
- If H_0 is true, $\widehat{\beta}_j$ should be close to 0.

$$\Rightarrow \text{ small } \left| \frac{\widehat{\beta}_j}{se(\widehat{\beta}_j)} \right|$$

- if H_0 is false, $\widehat{\beta}_j$ should be far away from 0

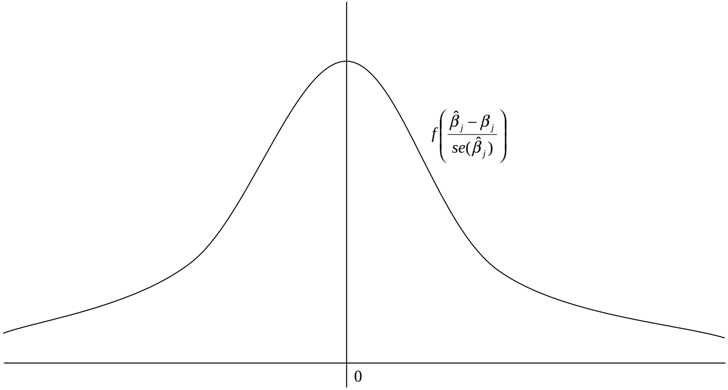
$$\Rightarrow \text{ large } \left| \frac{\widehat{\beta}_j}{se(\widehat{\beta}_j)} \right|$$

- What defines large and small?
- T-distribution defines large and small.

The t-statistic

The t-distribution

- Normalize the sampling distribution around β_j
- Distribution of $\frac{\widehat{\beta}_j - \beta_j}{se(\widehat{\beta}_j)}$ is standard normal distribution if N is large.



Hypothesis Testing

Basics

- "Null Hypothesis": The hypothesis that we are testing.
- Two outcomes:
  - Reject the null in favor of an alternative.
  - Fail to reject the null.
- For the moment, we are testing the following:

$$H_0 : \beta_j = 0$$

- Why will we never accept this hypothesis as the truth?

The t-statistic

An example - one sided test

- Experience and Wages

$$\log(wage) = \beta_0 + \beta_1 Educ + \beta_2 Exper + \beta_3 Tenure + u$$

- We estimate:

$$\begin{aligned}\widehat{\log(wage)} &= \underset{(0.104)}{0.284} + \underset{(0.007)}{0.092}\, Educ + \underset{(0.0017)}{0.0041}\, Exper + \underset{(0.003)}{0.022}\, Tenure \\ obs &= 526,\ R^2 = 0.316\end{aligned}$$

- $\underset{\text{(standard error)}}{estimate}$
- Suppose $H_0 : \beta_2 = 0$.
- T-statistic for this hypothesis?
- Use the formula

$$t(\widehat{\beta}_2 | \beta_2 = 0) = \frac{\widehat{\beta}_2 - 0}{se(\widehat{\beta}_2)} = \frac{0.0041}{0.0017} = 2.41$$

The t-statistic

An example - one sided test

- We need to compare our null hypothesis to some alternative, H_A.
- $H_A : \beta_2 > 0$
- When should we reject H_0 in favor of H_A?

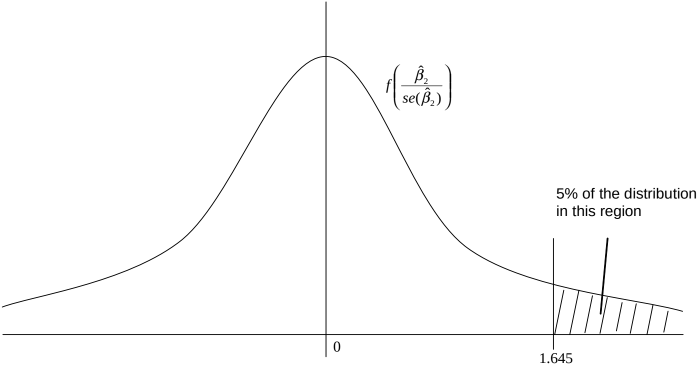
  $\Rightarrow$ If $\widehat{\beta}_2$ is sufficiently greater than zero.
- What determines "sufficiently greater"

  $\Rightarrow$ The t-distribution
- However, we could be wrong, since we do not measure β_2
- 95% "Confidence level" is a common desired level of accuracy.
- This means that we falsely reject the null no more than 5% of the time.

The t-statistic

One sided test - graphically

- The critical region



- Very unlikely (0.05 probability) to draw a random $\widehat{\beta}_2$ and end up in this region!!
- Thus, for $H_A : \beta_2 > 0$ we reject the null if we're in this region.

The t-statistic

An example - one sided test

- $H_A : \beta_2 > 0$ - 95% confidence
- Three steps
  1. Calculate $n - k - 1$
  2. If $n - k - 1$ large enough, find the appropriate critical value, t_{crit}, using the standard normal distribution.
  3. If $\frac{\widehat{\beta}_2 - 0}{se(\widehat{\beta}_2)} > t_{crit}$, reject H_0 in favor of H_A.

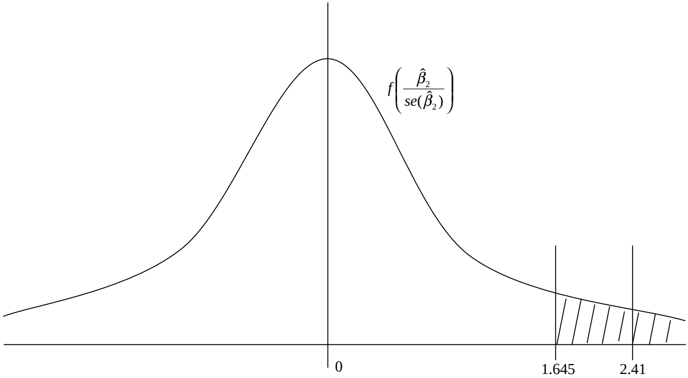
     $\Rightarrow$ Thus, if $\widehat{\beta}_2$ is sufficiently *greater* than zero, it is very unlikely that $\beta_2 = 0$.

     $\Rightarrow$ It would require a fluke for $\widehat{\beta}_2$ to be significantly greater than its true value.
- With enough observations, at 95% confidence, $t_{crit} = 1.645$
- Since $t(\widehat{\beta}_2 | \beta_2 = 0) = 2.41 > 1.645$, **reject** $\beta_2 = 0$ in favor of $\beta_2 > 0$.
- Experience has a **positive and statistically significant** effect on wages!

The t-statistic

One sided test - graphically

- We are in the rejection region.



- So we reject the null.
- We may falsely reject H_0, but on average, it will be less than 5% of the time.

The t-statistic

An example - two-sided tests

- Two-sided tests require a slightly different approach.
- Now, the alternative is $H_A : \beta_2 \neq 0$
- This is often used in economics.
- Three steps
  1. Calculate $n - k - 1$
  2. If $n - k - 1$ large enough, find the two-sided critical value, t_{crit}, using the standard normal distribution.
  3. If $\left| t(\widehat{\beta}_2 | \beta_2 = 0) \right| > t_{crit}$, reject H_0 in favor of H_A.

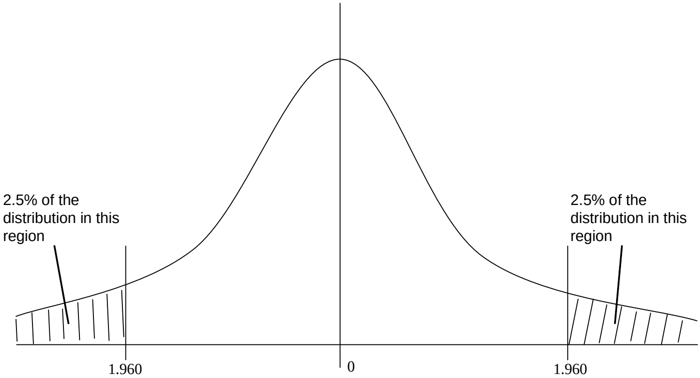
$\Rightarrow$ Thus, if $\widehat{\beta}_2$ is sufficiently far away from zero, it is very unlikely that $\beta_2 = 0$.

$\Rightarrow$ $\widehat{\beta}_2$ can now either be positive or negative.

The t-statistic

two sided test - graphically

- At 95% confidence, $t_{crit} = 1.960$



- Reject the null if we are in one of the shaded regions.
- Together, they equal 5% of the distribution.

The t-statistic

An example - two-sided test

- Test scores and school size. We estimate:

$$\widehat{math10} = \underset{(6.113)}{2.274} + \underset{(0.00010)}{0.00046}\, totcomp + \underset{(0.040)}{0.048}\, staff - \underset{(0.00022)}{0.0002}\, enroll$$

$$obs = 408,\ R^2 = 0.0541$$

- Suppose $H_0 : \beta_{enroll} = 0$.
- Let $H_A : \beta_{enroll} \neq 0$.
- T-statistic?
- $$t(\widehat{\beta}_{enroll} | \beta_{enroll} = 0) = \frac{\widehat{\beta}_{enroll} - 0}{se(\widehat{\beta}_{enroll})} = \frac{-0.0002}{0.00022} = -0.91$$
- $t_{crit} = 1.960$.
- Since $|-0.91| < 1.960$, we **cannot reject the null** that enrollment has no effect.

The t-statistic

two sided test - graphically

- $\widehat{\beta}_{enroll}$ is not far enough away from zero on either side to conclude that enrollment has an effect.