

**Homework #2**  
**Economics 113**  
**Introduction to Econometrics**  
**Professor Spearot**  
**Due Friday, October 10th, 2008 – Beginning of class**

1. The Dude is an avid bowler. Since he is a master of relaxation, the probability of rolling a strike in any given frame is independent of all the others. Suppose that the probability of rolling a strike is 0.7 for each frame:

- a. A perfect game requires 12 strikes in a row. What is the probability that The Dude rolls 12 consecutive strikes?

*Given that each strike is independent, you simply multiply the  $Pr(\text{Strike})$  12 times. That is  $Pr(12 \text{ strikes}) = Pr(\text{Strike})^{12} = \underline{0.0138}$*

- b. What is the probability that The Dude fails to roll a strike in at least one of the twelve attempts?

*Failing to get 12 strikes is the complement of getting 12 strikes. So, the probability if not getting a strike in at least one frame is written as:*

*$Pr(\text{at least one frame without a strike}) = 1 - Pr(12 \text{ strikes}) = 1 - 0.0138 = \underline{0.9862}$*

2. Suppose that you join a game in which a coin is flipped three times. The probability of getting heads is 0.6. The random variable X is defined as the number of heads throughout the game.

- a. Please solve for and diagram the probability distribution of X.

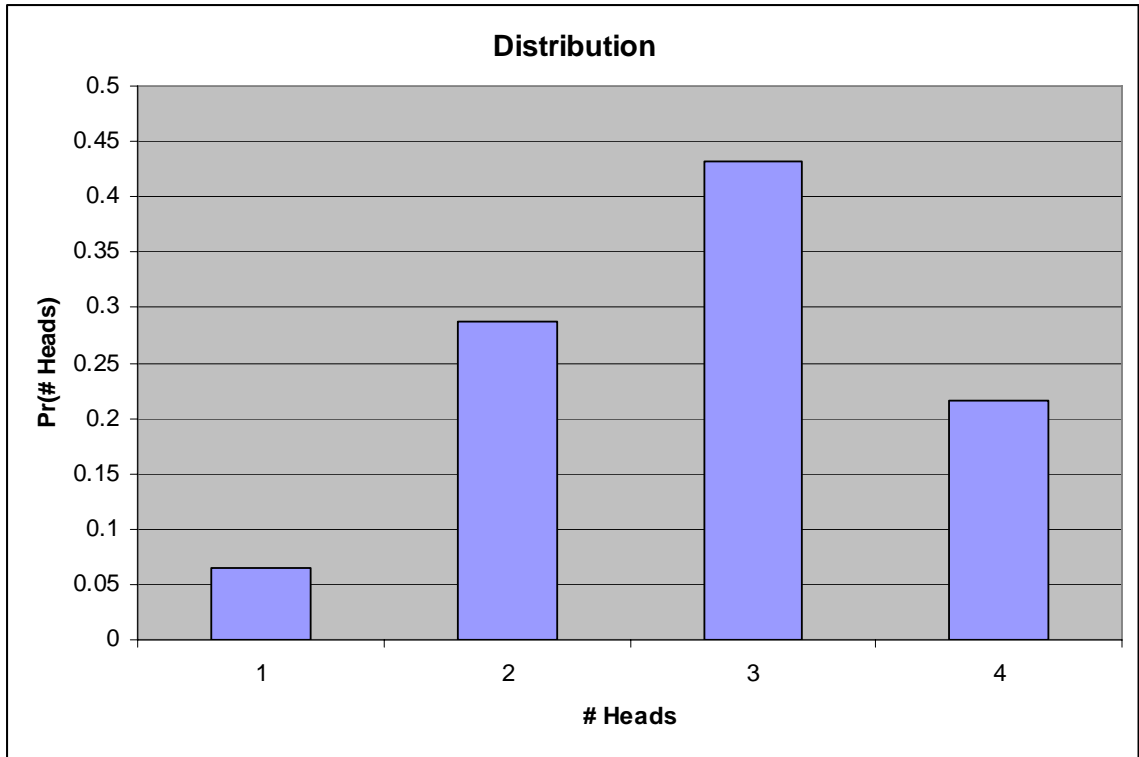
$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$$Pr(H=0) = 1 * (.4 * .4 * .4) = 0.064$$

$$Pr(H=1) = 3 * (.6 * .4 * .4) = 0.288$$

$$Pr(H=2) = 3 * (.6 * .6 * .4) = 0.432$$

$$Pr(H=3) = 1 * (.6 * .6 * .6) = 0.216$$



b. What is the expected value of X?

$$E(X) = 0 * 0.064 + 1 * 0.288 + 2 * 0.432 + 3 * 0.216 = \underline{1.8}$$

3. Suppose that there exists a 10-sided die with numbers 1-5 in Red and 6-10 in Blue. Calculate the probability of rolling an odd number given that the number you roll is blue.

$$S = \{1R, 2R, 3R, 4R, 5R, 6B, 7B, 8B, 9B, 10B\}$$

$$Pr(\text{Odd}|\text{Blue}) = Pr(\text{Odd} \& \text{Blue}) / Pr(\text{Blue}) = (2/10) / (5/10) = \underline{2/5}$$

4. Wave height is distributed normally with mean 5ft and standard deviation 2 ft.

a. What is the probability that a wave is 6ft tall?

*Zero. For a continuous random variable, the probability of being any specific value is zero.*

b. What is the probability that a wave is between 6 and 7 feet tall?

First, compute Z scores.

$$z(6) = (6-5)/2 = 0.5$$

$$z(7)=(7-5)/2=1$$

$$Pr(z(6)<Z<z(7))= Pr(Z<z(7))- Pr(Z<z(6))=0.8413-0.6915=\underline{0.1498}$$

- c. What is the probability that a wave is between 2 and 7 feet tall?

$$z(2)=(2-5)/2=-1.5$$

$$Pr(Z<z(2)) = Pr(Z<(-1.5)) = Pr(Z>(-1.5))= 1-Pr(Z<1.5)$$

$$Pr(z(6)<Z<z(7))= Pr(Z<z(7))- (1-Pr(Z<1.5))=0.8413-(1-0.9332)= \underline{0.7745}$$

- d. Suppose that two waves are coming, and independent from one another. What is the probability that wave one is between 3 and 7 feet tall OR wave two is between 4 and 6 feet tall?

*Call the first wave between 3 and 7 as event A*

*Call the second wave between 4 and 6 as event B*

$$z(4)=(-0.5)$$

$$z(6)=(0.5)$$

$$\begin{aligned} Pr(A)=Pr(z(4)<Z<z(6)) &= Pr(Z<z(6))- Pr(Z<z(4)) = Pr(Z<0.5)- Pr(Z<-0.5) \\ &= Pr(Z<0.5)- (1-Pr(Z<0.5)) \\ &= 2*Pr(Z<0.5)- 1 \\ &= 2*0.6915- 1 \\ &= 0.383 \end{aligned}$$

$$z(3)=(-1)$$

$$z(7)=(1)$$

$$\begin{aligned} Pr(B)=Pr(z(3)<Z<z(7)) &= Pr(Z<z(7))- Pr(Z<z(3)) = Pr(Z<1)- Pr(Z<-1) \\ &= Pr(Z<1)- (1-Pr(Z<1)) \\ &= 2*Pr(Z<1)- 1 \\ &= 2*0.8413- 1 \\ &= 0.6826 \end{aligned}$$

$$\begin{aligned} Pr(A \text{ OR } B) &= Pr(A)+Pr(B)-Pr(A\&B) \\ &= Pr(A)+Pr(B)-Pr(A)Pr(B) \\ &= 0.383+0.6826-0.383*0.6826 \\ &= \underline{0.8042} \end{aligned}$$