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## Exam 3-65 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. No cell phones. Do not use your own scratch paper.

## You must show your work to receive full credit

I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.

Signature $\qquad$

## Problem 1 (40 Points)

We wish to predict wage outcomes using the following regression:

$$
\text { wage }=\beta_{0}+\beta_{1} \text { highschool }+\beta_{2} \text { college }+u
$$

Here, wage is the monthly wage in dollars, highschool is a dummy variable identifying respondents with between 12 and 15 years of education, and college is a dummy variable identifying respondents with 16 or more years of education.

a.) Please construct a $95 \%$ confidence interval for the coefficient on college, and interpret this confidence interval. Show your work! (10 Points)

For the next few regressions, we adjust the wage regression by taking natural $\log s, \log ($ wage $)$, and estimating:

$$
\log (\text { wage })=\beta_{0}+\beta_{1} \text { college }+\beta_{2}(\text { age }-35)+\beta_{3} \operatorname{college} e \cdot(\text { age }-35)+u
$$

where age is the respondent's age in years, and below, age $35=$ age- 35 . The results are the following:

b.) Usually as one gets older, higher wages are earned. Does this relationship depend on whether you've earned a college education? Form a hypothesis to test this statement, and then test this hypothesis at the $99 \%$ level. Briefly interpret your result, and show your work!! (10 points)
c.) Please interpret the coefficient on college precisely. (10 points)
d.) You're unhappy with the regression in 'b', since "college is all that matters". So you instead regress:

$$
\log (\text { wage })=\beta_{0}+\beta_{1} \text { college }+u
$$

where urban is a dummy variable identifying respondents from metropolitan areas.

| Source | SS | df MS |  |  |  | Number of obs $=935$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | F( 1, 933) |  | 70.42 |
| Model | 11.6258689 | 1 | 11. | 258689 |  | Prob > F |  | 0.0000 |
| Residual | 154.030425 | 933 |  | 509156 |  | R -squared |  | 0.0702 |
|  |  |  |  |  |  | Adj R-squared |  | 0.0692 |
| Total | 165.656294 | 934 | . 17 | 362199 |  | Root MSE | $=$ | . 40631 |
| lwage | Coef. | Std. | Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. | I | erval] |
| college | . 2529157 | . 0301 | 388 | XXXXX | XXXXXX | SXXXXXXXXXXXXX | X | XXXXXXX |
| _cons | 6.712191 | . 0154 | 906 | XXXXX | XXXXXX | XXXXXXXXXXXXXX | XX | XXXXXXX |

Which regression is preferred, the regression in ' 1 b ' or ' 1 d '? Please test this hypothesis at the $95 \%$ level, stating your null and alternative hypotheses. Briefly interpret your result, and show your work!! (10 Points)

## Problem 2 (25 Points)

a.) For this problem, we wish to associate college education with location choice:

$$
\text { urban }=\beta_{0}+\beta_{1} \text { college }+\beta_{2} \text { age }+\beta_{3} \text { age } e^{2}+u
$$

Here, urban is a dummy variable identifying respondents that live in metropolitan areas, college is a dummy variable identifying 16 years of schooling or more, and age is the age of the respondent.


Please interpret the coefficient on college. At the $95 \%$ confidence level, please test whether it is greater than zero using a one-sided test, and briefly interpret your result. Show your work!! (10 Points)
b.) There is a traditional way to impose the hypothesis in ' 2 a ' (which you should use), but there is also another way given the regression specification and the way the question has been asked. What is it? ( $\mathbf{5}$ Points)
c.) Please derive and solve for the age at which the likelihood of living in a city is maximized. How do we know that the solution is a maximum rather than a minimum? Show your work!! ( $\mathbf{1 0}$ Points)

|  |  | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.5 | 0.5040 | 0. |  |  |  |  | 0.5279 |  |  |
| 0. | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 |  |
| 0 | 0.579 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 |  |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 |  |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 |  |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0. | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 |  |
| 0.6 | 10.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 |  |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 |  |
| 0.8 | 10.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 |  |
| 0.9 | 10.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 |  |
| 0 | 10.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 |  |
| 1.1 | 0.8643 | 0.8665 | 0.868 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 |  |
| 1 | 0.8849 | 0.8869 | 0.888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 |  |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 |  |
| 1. | 0.9192 | 0.9207 | 0.922 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 |  |
| 1.5 | 0.9332 | 0.9345 | 0.935 | 0.9 | 0.9382 | 0. | 0.9406 | 0.9418 | 0.9429 |  |
| 1.6 | 0.945 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.950 | 0.9515 | 0.9525 | 0.9535 |  |
| 1.7 | 0.955 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 |  |
| 1.8 | 0.964 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.967 | 0.9686 | 0.9693 | 0.9699 |  |
| . 9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 9756 | 0.9761 |  |
| 2.0 | 0.97 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.979 | 0.9803 | 0.9808 | 0.9812 |  |
| 2.1 | 0.982 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 |  |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 |  |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 |  |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 |  |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 |  |
| 2.6 | 10.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 |  |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 |  |
| 2.8 | 10.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 |  |
| 2.9 | 10.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 |  |
| 3.0 | 0.9987 | 0.9987 | 0 | 0.9988 | 8 | 0.9989 | 0.9989 |  |  |  |

TABLE G.3b

## 5\% Critical Values of the $F$ Distribution

| Nünerator Degrees of Freedom: |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1. | 2 | 3 | 4. | 5 | 6 | 7 | 8 | 9. | 10 |
| $\infty$ | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 |

Example: The $5 \%$ critical value for numerator $d f=4$ and large denominator $d f(\infty)$ is 2.37 .
Source: This table was generated using the Stata ${ }^{\oplus}$ function invFtail.

