Introduction to Econometrics

ID_____

Exam 3 – 65 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. No cell phones. Do not use your own scratch paper.

You must show your work to receive full credit

I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.

Signature_____

Problem 1 (40 Points)

We wish to predict wage outcomes using the following regression:

 $wage = \beta_0 + \beta_1 highschool + \beta_2 college + u$

Here, *wage* is the monthly wage in dollars, *highschool* is a dummy variable identifying respondents with between 12 and 15 years of education, and *college* is a dummy variable identifying respondents with 16 or more years of education.

| Source | SS. | df | MS | | Number of obs | = | 935 |
|--------------------------------|-------------------------------|----------------------------------|----------------------------------|--------------------------------------|---|----------------------|-------------------------------------|
| Model Residual | 12925398.1 139790770 | 2 6462 932 1499 | 2699.03 | | F(2, 932) Prob > F R-squared | = = = | 43.09 0.0000 0.0846 0.0827 |
| Total | 152716168 | 934 1635 | 507.675 | | Root MSE | = | 387.29 |
| wage | Coef. | Std. Err. | t | P> t | [95% Conf. | Int | terval] |
| highschool college _cons | 134.305 369.1184 774.25 | 44.20878 48.07994 41.28478 | XXXXXXXX XXXXXXXX XXXXXXXX | xxxxxxxxx xxxxxxxxx xxxxxxxxxx | <pre>xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx</pre> | XXXX XXXX XXXX | XXXXXXX XXXXXXX XXXXXXX |

a.) Please construct a 95% confidence interval for the coefficient on *college*, and interpret this confidence interval. Show your work! (10 Points)

For the next few regressions, we adjust the wage regression by taking natural logs, log(*wage*), and estimating:

$$\log(wage) = \beta_0 + \beta_1 college + \beta_2(age - 35) + \beta_3 college \cdot (age - 35) + u$$

where *age* is the respondent's age in years, and below, age35 = age-35. The results are the following:

| Source | SS | df | | MS | | Number of obs | = | 935 |
|---------------|------------|-------|------|---------|-----------|---|-----|---------|
| | + | | | | | F(3, 931) | = | 36.38 |
| Model | 17.3830024 | 3 | 5.79 | 9433414 | | Prob > F | = | 0.0000 |
| Residual | 148.273292 | 931 | .159 | 9262397 | | R-squared | = | 0.1049 |
| | + | | | | | Adj R-squared | = | 0.1020 |
| Total | 165.656294 | 934 | .177 | 7362199 | | Root MSE | = | .39908 |
| | | | | | | | | |
| lwage | Coef. | Std. | Err. | t | P> t | [95% Conf. | In | terval] |
| | + | | | | | | | |
| college | .3107355 | .0350 | 0239 | XXXXXX | XXXXXXXXX | ****** | ХХХ | XXXXXXX |
| age35 | .0140162 | .0047 | 7714 | XXXXXX | XXXXXXXXX | XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX | XXX | XXXXXXX |
| college age35 | .0325312 | .0100 | 0746 | XXXXXX | XXXXXXXXX | ****** | ХХХ | XXXXXXX |
| _ cons | 6.739551 | .0178 | 3392 | XXXXXX | XXXXXXXXX | ****** | XXX | XXXXXXX |
| | | | | | | | | |

b.) Usually as one gets older, higher wages are earned. Does this relationship depend on whether you've earned a college education? Form a hypothesis to test this statement, and then test this hypothesis at the 99% level. Briefly interpret your result, and show your work!! (10 points)

c.) Please interpret the coefficient on *college* precisely. (10 points)

d.) You're unhappy with the regression in 'b', since "college is all that matters". So you instead regress:

$$log(wage) = \beta_0 + \beta_1 college + u$$

where *urban* is a dummy variable identifying respondents from metropolitan areas.

| Source | SS | df | MS | | Number of obs | = | 935 |
|----------|------------|-----------|---------|--------------|---------------|------|--------|
| + | | | | | F(1, 933) | = | 70.42 |
| Model | 11.6258689 | 1 11. | 6258689 | | Prob > F | = | 0.0000 |
| Residual | 154.030425 | 933.1 | 6509156 | | R-squared | = | 0.0702 |
| + | | | | | Adj R-squared | = | 0.0692 |
| Total | 165.656294 | 934 .17 | 7362199 | | Root MSE | = | .40631 |
| | | | | | | | |
| | | | | | | | |
| lwage | Coef. | Std. Err. | t | P> t | [95% Conf. | Int | erval] |
| + | | | | | | | |
| college | .2529157 | .0301388 | XXXXXX | XXXXXXXXXXXX | ****** | XXXX | XXXXXX |
| _cons | 6.712191 | .0154906 | XXXXXX | XXXXXXXXXXXX | ****** | XXXX | XXXXXX |
| | | | | | | | |

Which regression is preferred, the regression in '1b' or '1d'? Please test this hypothesis at the 95% level, stating your null and alternative hypotheses. Briefly interpret your result, and show your work!! (10 Points)

Problem 2 (25 Points)

a.) For this problem, we wish to associate college education with location choice:

$$urban = \beta_0 + \beta_1 college + \beta_2 age + \beta_3 age^2 + u$$

Here, *urban* is a dummy variable identifying respondents that live in metropolitan areas, *college* is a dummy variable identifying 16 years of schooling or more, and *age* is the age of the respondent.

| Source | | SS | df | | MS | | Number of ob | s = | 935 |
|----------|----------------|------------|------|------|---------|-----------|--------------|------|--------------|
| | ·+-· | | | | | | F(3, 931 |) = | 1.80 |
| Model | | 1.09532894 | 3 | .36 | 5109648 | | Prob > F | = | 0.1447 |
| Residual | | 188.363495 | 931 | .2 | 0232384 | | R-squared | = | 0.0058 |
| | ·+ | | | | | | Adj R-square | d = | 0.0026 |
| Total | | 189.458824 | 934 | .20 | 2846706 | | Root MSE | = | .4498 |
| urban | | Coef. | Std. | Err. | t | P> t | [95% Conf | . Ir | nterval] |
| college | · - | .0601481 | .033 | 5656 | XXXXXX | XXXXXXXXX | | XXXX | xxxxxxxx |
| age | Ì | .1503527 | .119 | 5888 | XXXXXX | XXXXXXXXX | ***** | XXXX | xxxxxxx |
| age2 | Ì | 0022769 | .001 | 1796 | XXXXXX | XXXXXXXXX | ***** | XXXX | xxxxxxx |
| cons | | -1.758366 | 1.97 | 5546 | XXXXXX | XXXXXXXXX | ****** | XXXX | XXXXXXXX |
| | · | | | | | | | | |

Please interpret the coefficient on *college*. At the 95% confidence level, please test whether it is greater than zero using a one-sided test, and briefly interpret your result. Show your work!! **(10 Points)**

b.) There is a traditional way to impose the hypothesis in '2a' (which you should use), but there is also another way given the regression specification and the way the question has been asked. What is it? (5 Points)

c.) Please <u>derive</u> and <u>solve</u> for the age at which the likelihood of living in a city is maximized. How do we know that the solution is a maximum rather than a minimum? Show your work!! (10 Points)

| | Normal Dis | Normal Distribution | | | | | | | |
|--------------------|----------------------|---------------------|---------------|--------|--|--|--|--|--|
| | from _oo f | o 7 | | | | | | | |
| 8 | | 02 | | | | | | | |
| Z 0.00 0.01 | 0.02 0.03 0.04 | 0.05 0.06 | 0.07 0.08 | 0.09 | | | | | |
| 0.0 0.5000 0.504 | 0 0.5080 0.5120 0.51 | 60 0.5199 0.5239 | 0.5279 0.5319 | 0.5359 | | | | | |
| 0.1 0.5398 0.543 | 8 0.5478 0.5517 0.55 | 57 0.5596 0.5636 | 0.5675 0.5714 | 0.5753 | | | | | |
| 0.2 0.5793 0.583 | 2 0.5871 0.5910 0.59 | 48 0.5987 0.6026 | 0.6064 0.6103 | 0.6141 | | | | | |
| 0.3 0.6179 0.621 | 7 0.6255 0.6293 0.63 | 31 0.6368 0.6406 | 0.6443 0.6480 | 0.6517 | | | | | |
| 0.4 0.6554 0.659 | 1 0.6628 0.6664 0.67 | 00 0.6736 0.6772 | 0.6808 0.6844 | 0.6879 | | | | | |
| 0.5 0.6915 0.695 | 0 0.6985 0.7019 0.70 | 54 0.7088 0.7123 | 0.7157 0.7190 | 0.7224 | | | | | |
| 0.6 0.7257 0.729 | 1 0.7324 0.7357 0.73 | 89 0.7422 0.7454 | 0.7486 0.7517 | 0.7549 | | | | | |
| 0.7 0.7580 0.761 | 1 0.7642 0.7673 0.77 | 04 0.7734 0.7764 | 0.7794 0.7823 | 0.7852 | | | | | |
| 0.8 0.7881 0.791 | 0 0.7939 0.7967 0.79 | 95 0.8023 0.8051 | 0.8078 0.8106 | 0.8133 | | | | | |
| 0.9 0.8159 0.818 | 6 0.8212 0.8238 0.82 | 64 0.8289 0.8315 | 0.8340 0.8365 | 0.8389 | | | | | |
| 1.0 0.8413 0.843 | 8 0.8461 0.8485 0.85 | 08 0.8531 0.8554 | 0.8577 0.8599 | 0.8621 | | | | | |
| 1.1 0.8643 0.866 | 5 0.8686 0.8708 0.87 | 29 0.8749 0.8770 | 0.8790 0.8810 | 0.8830 | | | | | |
| 1.2 0.8849 0.886 | 9 0.8888 0.8907 0.89 | 25 0.8944 0.8962 | 0.8980 0.8997 | 0.9015 | | | | | |
| 1.3 0.9032 0.904 | 9 0.9066 0.9082 0.90 | 99 0.9115 0.9131 | 0.9147 0.9162 | 0.9177 | | | | | |
| 1.4 0.9192 0.920 | 7 0.9222 0.9236 0.92 | 51 0.9265 0.9279 | 0.9292 0.9306 | 0.9319 | | | | | |
| 1.5 0.9332 0.934 | 5 0.9357 0.9370 0.93 | 82 0.9394 0.9406 | 0.9418 0.9429 | 0.9441 | | | | | |
| 1.6 0.9452 0.946 | 3 0.9474 0.9484 0.94 | 95 0.9505 0.9515 | 0.9525 0.9535 | 0.9545 | | | | | |
| 1.7 0.9554 0.956 | 4 0.9573 0.9582 0.95 | 91 0.9599 0.9608 | 0.9616 0.9625 | 0.9633 | | | | | |
| 1.8 0.9641 0.964 | 9 0.9656 0.9664 0.96 | 71 0.9678 0.9686 | 0.9693 0.9699 | 0.9706 | | | | | |
| 1.9 0.9713 0.971 | 9 0.9726 0.9732 0.97 | 38 0.9744 0.9750 | 0.9756 0.9761 | 0.9767 | | | | | |
| 2.0 0.9772 0.977 | 8 0.9783 0.9788 0.97 | 93 0.9798 0.9803 | 0.9808 0.9812 | 0.9817 | | | | | |
| 2.1 0.9821 0.982 | 6 0.9830 0.9834 0.98 | 38 0.9842 0.9846 | 0.9850 0.9854 | 0.9857 | | | | | |
| 2.2 0.9861 0.986 | 4 0.9868 0.9871 0.98 | 75 0.9878 0.9881 | 0.9884 0.9887 | 0.9890 | | | | | |
| 2.3 0.9893 0.989 | 6 0.9898 0.9901 0.99 | 04 0.9906 0.9909 | 0.9911 0.9913 | 0.9916 | | | | | |
| 2.4 0.9918 0.992 | 0 0.9922 0.9925 0.99 | 27 0.9929 0.9931 | 0.9932 0.9934 | 0.9936 | | | | | |
| 2.5 0.9938 0.994 | 0 0.9941 0.9943 0.99 | 45 0.9946 0.9948 | 0.9949 0.9951 | 0.9952 | | | | | |
| 2.6 0.9953 0.995 | 5 0.9956 0.9957 0.99 | 59 0.9960 0.9961 | 0.9962 0.9963 | 0.9964 | | | | | |
| 2.7 0.9965 0.996 | 6 0.9967 0.9968 0.99 | 69 0.9970 0.9971 | 0.9972 0.9973 | 0.9974 | | | | | |
| 2.8 0.9974 0.997 | 5 0.9976 0.9977 0.99 | 77 0.9978 0.9979 | 0.9979 0.9980 | 0.9981 | | | | | |
| 2.9 0.9981 0.998 | 2 0.9982 0.9983 0.99 | 84 0.9984 0.9985 | 0.9985 0.9986 | 0.9986 | | | | | |
| 3.0 0.9987 0.998 | 7 0.9987 0.9988 0.99 | 88 0.9989 0.9989 | 0.9989 0.9990 | 0.9990 | | | | | |

TABLE G.3b 5% Critical Values of the F Distribution

r*

| Numerator Degrees of Freedom | | | | | | | | | | | |
|------------------------------|---|------|------|------|-------|------|------|------|------|------|------|
| | | 1 | .2 | 3 | 4 | 5 | 6 | 7 | 8 | . 9. | |
| | ~ | 3.84 | 3.00 | 2.60 | .2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 |

Example: The 5% critical value for numerator df = 4 and large denominator $df(\infty)$ is 2.37. *Source:* This table was generated using the Stata[®] function invFtail.