$\qquad$ ANSWER KEY $\qquad$ ID

## Final - 100 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but no graphing calculators. No cell phones. Do not use your own scratch paper.

## You must show your work to receive full credit

I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.

## Signature

## Problem 1 (50 Points)

We wish to predict real wage outcomes using the following regression:

$$
\log \left(r w_{i t}\right)=\beta_{0}+\beta_{1} \text { college }_{i}+\beta_{2} \text { female }_{i}+\beta_{3} \text { black }_{i}+\alpha_{t}+u_{i t}
$$

Here, $r w_{i t}$ is the real wage for respondent $i$ interviewed in year $t$, college $e_{i}$ takes on a value of 1 if respondent $i$ is a college graduate ( 0 otherwise), female $_{i}$ takes a value of 1 if respondent $i$ is female ( 0 otherwise), and black ${ }_{i}$ takes on a value of 1 if respondent $i$ is black ( 0 otherwise). The term $\alpha_{t}$ represents year fixed effects, which are suppressed in the following results:

| Source \| | SS | df | MS | $\begin{aligned} & \text { Number of obs }=598475 \\ & F(9,598465)=9167.91 \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| Model \| | 25658.4537 | 9 | 2850.9393 | Prob > F | 0.0000 |
| Residual \| | 186104.335 | 65 | . 310969454 | R-squared | 0.1212 |
|  |  |  |  | Adj R-squa | 0.1212 |
| Total \| | 211762.789 | 74 | . 353837908 | Root MSE | . 55765 |
| ln_rw \| | Coef. | Std. Err. |  | [95\% Conf. Interval] |  |
| college \| | . 3899047 | . 0018 | 27 xxxxx |  | xxxxxx |
| female \| | -. 2447752 | . 001 | 44 xxxxx | xxxxxxxxxxxxxxxx | xxxxxx |
| black \| | -. 1123664 | . 0024 | 96 xxxxx |  | xxxxxxx |
| cons I | 2.781718 | . 0020 | 19 xxxxxx | XXXXXXXXXXXXXXXX | $\mathbf{x x x x x x x}$ |

a.) Please interpret precisely the coefficient on college. (10 Points)

First, exponentiate the effect and subtract 1 .
$\exp (0.389)-1=0.476 \quad+4$
Within years, having a college degree increases wages by $47.6 \%$ relative to those without a college degree

```
    +2
                +2
                +2
```

b.) We wish to test whether there are any interactions between female and black and college using the following specification:

$$
\log \left(r w_{i t}\right)=\beta_{0}+\beta_{1} \text { college }_{i}+\beta_{2} \text { female }_{i}+\beta_{3} \text { black }_{i}+\beta_{4} \text { college }_{i} \cdot \text { female }_{i}+\beta_{5} \text { college }_{i} \cdot \text { black }_{i}+\alpha_{t}+u_{i t}
$$

The results from running this regression (again suppressing year estimates) are below:

| Source \| | SS | df | MS | Number of obs | 598475 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | F( 11,598463) | 7504.87 |
| Model \| | 25670.1273 | 11233 | . 64794 | Prob > F | 0.0000 |
| Residual \| | 186092.661 | 63.31 | 950987 | R -squared | 0.1212 |
|  |  |  |  | Adj R-squared | 0.1212 |
| Total 1 | 211762.788 | 74 | 837908 | Root MSE | . 55763 |
| ln_rw \| | Coef. | Std. Err. | t | [95\% Conf. | erval] |
| college \| | . 3968689 | . 0026744 | xxxxx | xxxxxxxxxxxxxxx | xxxxxxx |
| female \| | -. 2415216 | . 0015945 | xxxxx |  | xxxxxxx |
| college_female \| | -. 0182506 | . 0037573 | xxxxx | xxxxxxxxxxxxxxx | xxxxxxx |
| black \| | -. 1162495 | . 0026832 | x $\times$ x $\times$ x |  | xxxxxx |
| college_black \| | . 0293265 | . 0073784 | xxxxx | Exxxxxxxxxxxxxx | xxxxxx |
| - cons \| | 2.78049 | . 0020615 | xxxxx |  | xxxxxx |

Which regression is preferred, the regression in ' 1 'a' or the regression here in ' 1 b '? Please test this hypothesis at the $95 \%$ level, stating your null and alternative hypotheses. ( 10 points)

```
H0: \(B_{3}=0, B_{5}=0+1\)
HA: \(H_{0}\) not true +1
\(q=2 \quad+0.5\)
\(d f_{u r}=598463 \quad+0.5\)
\(S S R_{u r}=186092 \quad+0.5\)
\(S S R_{r}=186104.335+0.5\)
\(F_{\text {stat }}=((186104-186092) / 2) /(186092 / 598463)=19.30+3\)
\(F_{c r i t}=3 \quad+1\)
\(F_{\text {stat }}>F_{\text {crit }}=\gg\) Reject the null! +2
```

The interactions between female, black, and college are a jointly significant determinant of the real wage.
c.) Please write the Stata code required to generate college_female and college_black, and provide a different command than in ' 1 b ' to estimate the specification with year fixed effects. ( $\mathbf{1 0}$ points)
gen college_female $=$ college $*$ female $\quad+3$
gen college_black $=$ college $*$ black +3
xtreg ln_rw college female college_female black college_black, fe i(year)

$$
+4
$$

d.) Does the black-white wage gap depend on whether the respondent is college educated? Test this hypothesis at the $99 \%$ level, stating your null and alternative hypothesis. Show your work! ( $\mathbf{1 0}$ points)

```
H0: B B =0 +1
```

HA: $B_{5}!=0 \quad+1$
$T_{\text {stat }}=(.0293265 / .0073784)=3.97+3$
$T_{\text {crit }}=2.575 \quad+1$
$\left|T_{\text {stat }}\right|>\left|T_{\text {crit }}\right|=\gg$ reject the null!! +1

The black-white wage gap is significantly affected by a college education. +3
e.) What is the precise difference in predicted wages between a black college-educated male and a white female without a college degree? ( $\mathbf{1 0}$ points)

```
BM_C = 2.78049 + 0.3968689 - 0.1162495 + 0.0293265 +1
WF_NC=2.78049-0.2415216 +1
BM_C - WF_NC = 0.551 +2
    (Taking the difference properly is worth 4 total points. I don't care how one gets
it)
exp(0.551)-1=0.735 +3
```

A black, college educated male makes $73.5 \%$ more than white female without a college education.
$+3$

## Problem 2 (50 Points)

a.) We now use our wage panel dataset from 1980-1987 to examine the determinants of annual hours worked:

$$
\text { hours }_{i t}=\beta_{0}+\beta_{1} \text { educ }_{i}+\beta_{2} \text { manu }_{i t}+\beta_{3} \text { union }_{i t}+\alpha_{t}+u_{i t}
$$

Here, hours $_{i t}$ is annual hours worked for individual $i$ in year $t$, educ ${ }_{i}$ is the time-invariant education level of individual $i$, manu $_{i t}$ equals 1 if individual $i$ works in a manufacturing job in year $t$ ( 0 otherwise), and union ${ }_{i t}$ equals 1 if individual $i$ works in a union job in year $t(0$ otherwise). Note that manufacturing and union jobs are not mutually exclusive outcomes. Estimating this equation using Pooled OLS, we get the following.

| Source | SS | df | MS | Number of obs $=$ | 1200 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Model | 25891174.6 | 10 | 2589117.46 | Prob > F | 0.0000 |
| Residual | 422789266 | 1189 | 355583.908 | R -squared | 0.0577 |
| Total | 448680441 | 1199 | 374212.211 | Adj R-squared Root MSE | $0.0498$ |
| hours | Coef. | Std. | Err | [95\% Conf. In | terval] |
| educ | -23.3217 | 10.21 |  <br>  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX |  |  |
| union | -56.6942 | 42.35 |  |  |  |
| manuf | 60.6815 | 39.21 |  |  |  |
| year |  |  |  |  |  |
| 1981 | 162.335 | 68.86 |  |  |  |
| 1982 | 213.217 | 69.00 |  <br>  |  |  |
| 1983 | 291.039 | 68.90 |  |  |  |
| 1984 | 315.447 | 69.11 | XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX |  |  |
| 1985 | 358.572 | 68.91 |  |  |  |
| 1986 | 381.867 | 68.89 |  |  |  |
| 1987 | 454.073 | 69.04 | XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX |  |  |
| cons | 2230.84 | 130.5 | 198 xxxxx |  |  |

Please construct and interpret a $95 \%$ confidence interval for the constant in this regression. (10 Points)

$$
\begin{gathered}
2230.84-1.96 * 130.5198<B_{0}<2230.84+1.96 * 130.5198+2 \\
1975.021<B_{0}<2486.659
\end{gathered}
$$

With $95 \%$ confidence, a respondent with zero years of education that works in a non-union, non-manufacturing job,
worked between $\frac{1975.02 \text { and } 2486}{+2}$ hours in $\frac{1980}{+2}$.
b.) I claim that being in a union has a significant effect on annual hours worked. Using the results in ' 2 a ', what is the probability that I'm wrong? (10 Points)

$$
\begin{aligned}
t_{\text {stat }}= & -56.6942 / 42.35081=-1.34 \\
\text { Pvalue } & =\operatorname{Pr}\left(|T|>\left|t_{\text {stat }}\right|\right) \\
& =\operatorname{Pr}\left(T>\left|t_{\text {stat }}\right|\right)+\operatorname{Pr}\left(T<-\left|t_{\text {stat }}\right|\right) \\
& =2\left(1-\operatorname{Pr}\left(T<\left|t_{\text {stat }}\right|\right)\right) \\
& =2(1-\operatorname{Pr}(T<1.34))=2(1-0.9099)=\mathbf{0 . 1 8 0 2}
\end{aligned}+7
$$

c.) Hours worked cannot be negative, though pooled OLS may yield negative values for predictions. What are the two techniques we can use to remedy this issue? (5 Points)

Tobit and Poisson +2.5 each
d.) We now augment the regression equation in ' $2 a$ ' to include individual fixed effects, $\alpha_{i}$, but removing the time fixed effects.

$$
\text { hours }_{i t}=\beta_{0}+\beta_{2} \text { manu }_{i t}+\beta_{3} \text { union }_{i t}+\alpha_{i}+u_{i t}
$$

What happened to education, and why? (5 Points)

Education does not vary by time within the individual. So, it is absorbed in the fixed effect.
e.) After initializing the panel dimension of the dataset, we estimate the model from ' 2 d ':


Please interpret the coefficient on тanu, and test whether it is significantly different from zero at the $95 \%$ level. Show your work! (10 points)

Within individuals, being in a manufacturing job increases annual hours worked by 43.9 relative to nonmanufacturing jobs. +3
$H_{0}: B_{2}=0+1$
$H_{A}: B_{2}!=0+1$
$T_{\text {stat }}=(43.92626 / 46.45788)=0.945+2$
$T_{\text {crit }}=1.96+1$
$\left|T_{\text {stat }}\right|<\left|T_{\text {crit }}\right|=\gg$ fail to reject the null!! Within individuals, the effect of being in a manufacturing industry on hours worked is insignificant. +2
f.) Again assuming that the panel dataset is already initialized, please write out the code to estimate the following:

$$
\Delta \text { hours }_{i t}=\beta_{2} \Delta \text { manu }_{i t}+\beta_{3} \Delta \text { union }_{i t}+\Delta u_{i t}
$$

How does the interpretation for the coefficient on тапи change for this regression relative to 2 d ?

```
gen diff_hours \(=\) D.hours \(\quad+1\)
gen diff_manu \(=\) D.manu \(\quad+1\)
gen diff_union \(=\) D.union \(\quad+1\)
reg diff_hours diff_manu diff_union, noconstant
    \(+2+2\)
```

The interpretation is now "in the short run"
$+3$

|  |  | 0.01 | 0.02 | 0.03 | 0.04 |  |  | 0. 07 | 0.08 | 0. 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.5000 | 0.5040 | 0.5080 | 0.5 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 |  |
| 0 | 10.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 75 | 0.5714 |  |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 |  |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 |  |
| 0 | 0.6554 | 0.6591 | 0.6628 | 0 | 0.6700 | 0.6736 | 2 | 0.6808 | 4 |  |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | . 7054 | 8 | 3 | 0.7157 | 0.7190 |  |
| 0 | 0.7257 | 0.7291 | 0.7324 | 0 | 0.7389 | 0.7422 | 0.7454 | 486 | 0.7517 |  |
| 0 | 0.7580 | 0 | 0.7642 | 0 | 0.7704 | 0.7734 | 64 | 94 | 0.7823 |  |
| 0 | 0.7881 | 0 | 0 | 0 | 0.7995 | 0.8023 | 1 | 78 | 0.8106 |  |
| 0 | 0.8159 | 0.8186 | 0.82 | 0.8238 | 0.8264 | 0.8289 | 5 | 0.8340 | 0.8365 |  |
| 1 | 0.8413 | 0.8438 | 0.84 | 0.8 | 0.8508 | 0 | 4 | 0.8577 | 0.8599 |  |
|  | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0. | 0.8770 | 0.8790 | 0.8810 |  |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.890 | 0.8925 | 0.894 | 0.8962 | 0.8980 | 0.8997 |  |
| 1 | 0.9032 | 0.9049 | 0.906 | 0.9082 | 0.9099 | 0. | 0.9131 | 0.9147 | 0.9162 |  |
|  | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 |  |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 |  |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 |  |
| 1 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 |  |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 |  |
| 1.9 | 0.9713 | 0.9719 | 0.97 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 |  |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 |  |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 |  |
| 2.2 | \| 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.987 | 0.9881 | 0.9884 | 0.9887 |  |
| 2.3 | \| 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 |  |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 |  |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 |  |
| 2.6 | 10.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 |  |
| 2.7 | 10.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 |  |
| 2.8 | 10.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 |  |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 |  |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0 |  |

## TABLE G.3b

5\% Critical Values of the $F$ Distribution

| Nưnerator Degrees of Freedor |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9. | 10 |
| $\infty$ | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 |

Example: The $5 \%$ critical value for numerator $d f=4$ and large denominator $d f(\infty)$ is 2.37 .
Source: This table was generated using the Stata ${ }^{\mathscr{D}}$ function invFtail.

