Economics 113 Professor Spearot
Introduction to Econometrics
Fall 2007 – Final Exam
Name ____________________

JEAN PAUL'S ANSWER KEY

Final Exam Version B – 80 Points

You must answer all the questions. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. Do not use your own scratch paper.

You must show your work to receive full credit

You have plenty of time to finish. Take your time and relax. And, have a safe and Happy Holiday!

1. You have four kids that that weigh 50, 60, 70, and 80 pounds. Their respective heights are 2, 3, 4, and 3 ft.

a. What is the covariance between height and weight? **(5 points)**

Grading

$$\mu_W = \frac{50+60+70+80}{4} = \frac{260}{4} = 65$$ +1

$$\mu_H = \frac{2+3+4+3}{4} = \frac{12}{4} = 3$$ +1

$$\sigma_{HW} = \frac{1}{(4-1)}\left((50-65)(2-3) + (60-65)(3-3) + (70-65)(4-3) + (80-65)(3-3)\right)$$ +1

$$= \frac{1}{3}(15+5) = \frac{20}{3}$$ +1

+1 for some work

b. Suppose I estimate $Height = \beta_0 + \beta_1 Weight + u$ using a different sample (not your answer from a). The sample covariance of *Height* and *Weight* is 10. The sample variance of *Weight* is 2. What is the estimate of β_1? **(5 points)**

$$\hat{\beta}_1 = \frac{\sigma_{hw}}{\sigma_w^2} = \frac{10}{2} = 5$$

+1 for work

+4 for answer

2. You wish to predict the effects of education, experience, and tenure on wage outcomes. Specifically, you estimate the following specification:

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 tenure + \beta_3 \text{exper} + u$$

The results from running this regression are below:

Source	SS	df	MS
Model	25.6953278	3	8.56510927
Residual	139.960966	931	.150334013
Total	165.656294	934	.177362199

Number of obs =	935
F(3, 931) =	56.97
Prob > F =	0.0000
R-squared =	0.1551
Adj R-squared =	0.1524
Root MSE =	.38773

lwage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
educ	.0748638	.0065124	xxxx	xxxxx	xxxxxxxx	xxxxxxxx
tenure	.0133748	.0025872	xxxx	xxxxx	xxxxxxxx	xxxxxxxx
exper	.0153285	.0033696	xxxx	xxxxx	xxxxxxxx	xxxxxxxx
_cons	5.496696	.1105282	xxxx	xxxxx	xxxxxxxx	xxxxxxxx

a. Do a two sided t-test at the 5% level to determine if experience **(exper)** is a statistically significant determinant of the log wage. Please state the null and alternative hypotheses, and interpret the result. **(5 points)**

$H_0: \beta_3 = 0$ +1

$H_A: \beta_3 \neq 0$ +1

$$t_{stat} = \frac{0.01532 - 0}{0.0033696} = 4.55$$ +1

$t_{crit} = 1.96$ +1

$|t_{stat}| > t_{crit} \Rightarrow$ Reject H_0 +1

Experience is significant in determining the wage.

b. Perhaps you've heard the phrase, "I was trained in the school of hard knocks…it's just as good as school". Write down the hypothesis that states that education (**educ**) and experience (**exper**) have equal effects on the log wage. Also provide a two-sided alternative. **(2 Points)**

$$H_0: \quad \beta_1 = \beta_3 \qquad \underline{+1}$$

$$H_A: \quad \beta_1 \neq \beta_3 \qquad \underline{+1}$$

If the state the hypothesis as $\theta = \beta_1 - \beta_3 = 0$ that $\theta \neq 0$ also fine

c. Please manipulate the regression equation in (a) so that your null hypothesis in (b) can be tested using a t-test. Show your work!! **(8 Points)**

$$\theta = \beta_1 - \beta_3 \Rightarrow \theta + \beta_3 = \beta_1 \qquad \underline{+2}$$

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 tenure + \beta_3 exper + u$$

$$= \beta_0 + (\theta + \beta_3) educ + \beta_2 tenure + \beta_3 exper + u$$

$\underline{+4}$

$$= \beta_0 + \theta educ + \beta_2 tenure + \beta_3 (educ + exper) + u$$

$\underline{+2}$ for some work

Suppose that I adjust the specification in (a) as follows,

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 tenure + \beta_3 \text{exper} + \beta_4 \text{exper}^2 + \beta_5 age + \beta_6 age^2 + u$$

where *age* is the age of the respondent (in years).

Source	SS	df	MS
Model	26.2843766	6	4.38072944
Residual	139.371918	928	.150185256
Total	165.656294	934	.177362199

Number of obs	=	935
F(6, 928)	=	29.17
Prob > F	=	0.0000
R-squared	=	0.1587
Adj R-squared	=	0.1532
Root MSE	=	.38754

lwage	Coef.	Std. Err.	t	P>|t|	[99% Conf. Interval]
educ	.0703793	.0069319	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx		
tenure	.0127232	.0026224	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx		
exper	.0230826	.013667	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx		
exper2	-.0005216	.0006	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx		
age	.0069779	.1034685	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx		
age2	.0000507	.0015552	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx		
_cons	5.26504	1.697521	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx		

d. Please construct a 99% confidence interval for the coefficient on tenure, β_2. **(5 points)**

$CI = \hat{\beta}_2 \pm se(\hat{\beta}_2) t_{cr.t}$ +1

$t_{cr.t} = 2.576$ +1

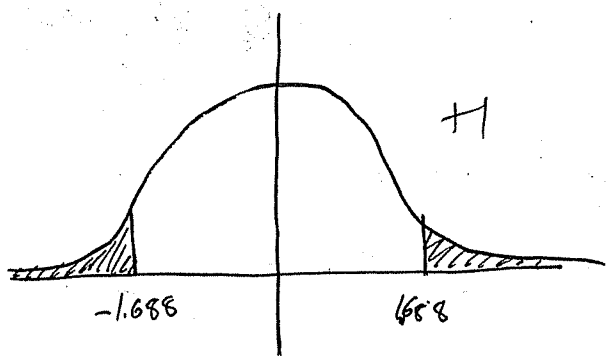
$CI = 0.0127 \pm 0.00262(2.576)$ +1

$\beta_2 \in [0.0060, 0.0194]$ +1

+1 for any work

e. What is the p-value for the linear experience term (**exper**)? Please draw the distribution under the null and compute the two-sided p-value (please give the range and an approximate value). **(5 points)**

$$t_{stat} = \frac{0.02308 - 0}{0.01367} = 1.688 \qquad +1$$



+1

$.05 \le p \le 0.10 \qquad +1$

if they have one, OK $\quad p \approx .09 \qquad +1$

+1 for reasonable work

f. Please test the hypothesis that adding *age*, *age*^{2}, and *exper*^{2} makes no difference in predicting the log wage. That is, please test whether these variables are jointly insignificant. Do this at the 95% level, stating the null and alternative hypotheses. **(10 points)**

$$F_{stat} = \frac{\frac{SSR_R - SSR_{UR}}{\#\ restrict}}{\frac{SSR_{UR}}{(n-k-1)_{UR}}} = \frac{\frac{139.96 - 139.37}{3}}{\frac{139.37}{928}}$$

$= 1.31$

+5 (+2 if incorrect)

$F_{crit} = 2.60 \quad +1$

$H_0: B_4 = 0,\ B_5 = 0,\ B_6 = 0$

$H_A: H_0$ not true

$F_{stat} < F_{crit} \quad +2$

Fail to reject H_0

+2

g. Professor Spearot will be 29 in January. He feels old, though is shamelessly hoping that his wage makes up for it. Assuming that he gains no additional experience and no additional tenure, what is the predicted effect on the wage in going from age 28 to age 29? Interpret briefly. **(5 points)**

Two possible answers

$$\frac{\partial \log(\text{wage})}{\partial \text{wage}} = \beta_5 + 2\beta_6 \text{ age}$$

$$= 0.00698 + 2 \cdot 0.0000507 \cdot 28$$

$$= 0.0098171$$

+3

Wage increases by 0,982% +2

$$F(\text{Age}) = \beta_5 \text{Age} + \beta_6 \text{Age}^2$$

$$= 0.00698 \text{Age} + (0.0000507) \cdot \text{Age}^2$$

$$F(29) - F(28)$$

$$= 0.00986$$

+3

0,986% +2

h. At what experience level is the wage maximized? Show your work! **(5 points)**

$$F(\text{exper}) = \beta_3 \text{exper} + \beta_4 \text{exper}^2$$ +1

$$\frac{\partial F}{\partial \text{exper}} = \beta_3 + 2\beta_4 \widehat{\text{exper}} = 0$$ +2

$$\widehat{\text{exper}} = -\frac{\beta_3}{2\beta_4}$$

$$= -\frac{0.0231}{2 \cdot (-0.000522)}$$

$$\boxed{\widehat{\text{exper}} = 22.12}$$ +2

3. Suppose that I run the following regression predicting the effects of candidate expenditures and other factors on election outcomes:

$$voteA = \beta_0 + \beta_1 \text{lexpendA} + \beta_2 \text{lexpendB} + \beta_3 prtystr + u$$

In the regression equation, lexpendA is the log expenditures of candidate A, lexpendB is the log expenditures of candidate B, and *prtystr* is the relative strength of party A. The results from running this regression are below:

```
. regress votea lexpenda lexpendb prtystra

      Source |       SS       df       MS              Number of obs =     173
-------------+------------------------------           F(  3,   169) =  215.23
       Model |  38405.1089     3  12801.703            Prob > F      =  0.0000
    Residual |  10052.1396   169  59.4801161           R-squared     =  0.7926
-------------+------------------------------           Adj R-squared =  0.7889
       Total |  48457.2486   172  281.728189           Root MSE      =  7.7123

------------------------------------------------------------------------------
       votea |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lexpenda |   6.083316     .38215    15.92   0.000     5.328914    6.837719
    lexpendb |  -6.615417   .3788203   -17.46   0.000    -7.363247   -5.867588
    prtystra |   .1519574   .0620181     2.45   0.015     .0295274    .2743873
       _cons |   45.07893   3.926305    11.48   0.000     37.32801    52.82985
------------------------------------------------------------------------------
```

a. Are the variables in the model (lexpendA, lexpendB, and *prtystr*) a significant determinant of the votes candidate A receives? Please test this hypothesis at the 95% level. Write the null hypothesis, the alternative, and briefly interpret the result. **(5 Points)**

$H_0: B_1 = 0, B_2 = 0, B_3 = 0$ +1

H_A: H_0 not true +1

$F_{stat} = 215.23$ +1

$F_{crit} = 2.60$ +1

$F_{stat} > F_{crit}$ +1

Reject the null, H_0 contains poor restrictions,

b. Suppose that Candidate A is Mike Gravel and Candidate B is Mitt Romney. Mitt spends much more than Mike. I want to predict the outcome of this hypothetical race, and produce confidence intervals for this prediction. Please write an equation for the prediction if lexpendA=1 (Mike), lexpendB=10 (Mitt), and prtystra=50 (citizens hate both parties equally!!). Derive a new estimating equation to generate the prediction and its standard error. Please also write the necessary commands to generate any new variables in STATA. **(10 Points)**

$$\theta = \beta_0 + \beta_1 \cdot 1 + \beta_2 \cdot 10 + \beta_3 \cdot 50 \quad +2$$

$$VoteA = \beta_0 + \beta_1 lexpendA + \beta_2 lexpendB + \beta_3 Prtystra$$

$$\beta_0 = \theta - \beta_1 \cdot 1 - \beta_2 \cdot 10 - \beta_3 \cdot 50 \quad +2$$

for work

$$VoteA = \theta - \beta_1 \cdot 1 - \beta_2 \cdot 10 - \beta_3 \cdot 50 + \beta_1 lexpendA + \beta_2 lexpendB + \beta_3 prtystra$$

$$+3$$

$$= \theta + \beta_1(lexpendA - 1) + \beta_2(lexpendB - 10) + \beta_3(prtystra - 50)$$

can use any name

generate lexpendA_1 = lexpendA - 1 +1

generate lexpendB_10 = lexpendB - 10 +1

generate prtysta_50 = prtysta - 50 +1

c. Suppose that as campaign expenditures rise, unobserved factors affecting voting outcomes tend to become more variable. What kind of problem is this? What should be done about it? **(5 Points)**

Heteroskedasticity +3

Use Robust standard errors +2

d. Suppose that candidate A spends more money because he/she has better ideas, and better ideas get you more votes. What assumption is violated in our current model? In what direction is β_1 biased? **(5 Points)**

$E(u|x) \neq 0$ +2

B_1 will be biased upward +3

Extra Credit:

Look back at the full model in problem #2. Age discrimination is perceived to be commonplace in American society. Age discrimination occurs if there exists an age above which wages go down, independent of other attributes (education, experience, tenure). Using the results in Problem #2, briefly discuss whether there is evidence of age discrimination. **(2 points)**

There is no positive age ~~[illegible]~~ ~~[illegible]~~ above which wage decreases.

No discrimination.

~~[illegible]~~

Could also solve for $\hat{Age}$. +2

Helpful formulas

$$\hat{\sigma}_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{\mu}_x)^2 \qquad \hat{\sigma}_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{\mu}_x)(y_i - \hat{\mu}_y) \qquad \hat{\rho}_{xy} = \frac{\hat{\sigma}_{xy}}{\hat{\sigma}_x \hat{\sigma}_y}$$

$$\hat{\beta}_0 = \hat{\mu}_y - \hat{\beta}_1 \hat{\mu}_x \qquad \hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \hat{\mu}_x)(y_i - \hat{\mu}_y)}{\sum_{i=1}^{n}(x_i - \hat{\mu}_x)^2}$$

$$R^2 = 1 - \frac{SSR}{SST} \qquad SSR = \sum_{i=1}^{n}(\hat{u}_i)^2 \qquad SST = \sum_{i=1}^{n}(y_i - \hat{\mu}_y)^2$$

$$\text{Adj}\, R^2 = 1 - \frac{\frac{SSR}{n-k-1}}{\frac{SST}{n-1}} \qquad \text{Fstat} = \frac{\frac{\mathrm{SSR}_{\mathrm{R}} - SSR_{UR}}{q}}{\frac{SSR_{UR}}{n-k-1}}$$