Economics 113 Professor Spearot
Introduction to Econometrics
Fall 2010 – Midterm 2
Name Answer Key ID

Midterm 2 – 70 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. Do not use your own scratch paper.

You must show your work to receive full credit

I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.

Signature

1. (45 Points) Suppose that you wish to predict wage outcomes via the following specification:

$$wage = \beta_0 + \beta_{drugs} drugs + u$$

wage is measured in dollars per month, and *drugs* (taking on values from 0 to 7) is measured in days of hard drug use per week.

a.) Suppose you estimate $\hat{\beta}_{drugs} = -100$ and $\hat{\sigma}_{drugs} = 5$. What is the covariance between wages and drug use? (5 Points)

$$\hat{\beta}_{drugs} = \frac{\hat{\sigma}_{wage,drugs}}{\hat{\sigma}^2_{drugs}} \quad +2$$

$$-100 = \frac{\hat{\sigma}_{wage,drugs}}{25} \Rightarrow \boxed{\hat{\sigma}_{wage\ drugs} = -2{,}500} \quad +2$$

+1 for squaring

b.) Supposing again that $\hat{\beta}_{drugs} = -100$, and further, that $\hat{\mu}_{drugs} = 1$ and $\hat{\mu}_{wage} = \$2{,}000$. What is the value of $\hat{\beta}_0$? (5 points)

$$\hat{\mu}_{wage} = \hat{\beta}_0 + \hat{\beta}_{drugs}\hat{\mu}_{drugs} \quad +2$$

$$\Rightarrow \hat{\beta}_0 = \hat{\mu}_{wage} - \hat{\beta}_{drugs}\hat{\mu}_{drugs}$$

$$= 2{,}000 - (-100)(1)$$

+1 setting the sign

$$\boxed{\hat{\beta}_0 = 2{,}100} \quad +2$$

c.) Please state the Gauss-Markov assumptions (5 Points)

+1 for each

1. Linear in parameters
2. Random Sample
3. $E(u|X)=0$ (zero conditional mean)
4. $\hat{\sigma}_x > 0$
5. Homoskedasticity

d.) "But Dr. Spearot, you should be taking logs of wages ~~and~~ drugs before running your regression. The coefficients from log-log regressions are much easier to interpret!!!!" What is fundamentally wrong with this statement? (5 Points)

+5

Cannot take logs of a variable that has zeros.

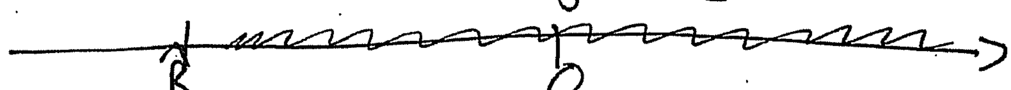
e.) In running the regression, I forgot to include *crime*, which is a variable measuring aggregate months spent in prison over the last 5 years. The variable *crime* is positively correlated with *drugs* and negatively correlated with *wage*. In what direction, if any, is the estimate $\hat{\beta}_{drugs}$ biased? Supposing that the original estimate of $\hat{\beta}_{drugs}$ is negative, what can be said about the sign of β_{drugs}? (5 Points)

$-$

$$wage = \beta_0 + \beta_1 \underbrace{drugs + u}_{+}$$

+3 downward bias

Cannot conclude anything about β_{drugs} +2



f.) Suppose that, on average, somebody with *drugs=0* has a positive predicted level of *crime*. Given the information in (e), in what direction, if any is $\hat{\beta}_0$ biased? Supposing that the original estimate of $\hat{\beta}_0$ is positive, what can be said about the sign of the true parameter β_0? (5 Points)

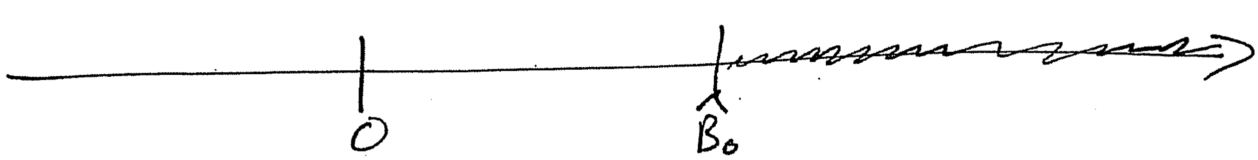
$-$

$$wage = \beta_0 + \underbrace{\beta_1 drugs + u}_{+}$$

+3

downward bias

We can conclude that β_0 is positive +2



g.) Suppose that the variance of the unobservable is large when *drugs* is low or high, but small when *drugs* in a mid-range. What kind of errors are these? (5 points)

Heteroskedastic +5

$Var(u|X) \neq Var(u) \leftarrow$ (not required)

h.) I report that the R^2 for the above regression is 0.65. What does this say about the model? What does this not say about the model? (5 points)

The model captures 65% of variation in wage. +2

Does not say anything about causality. +3

i.) Suppose that I double the size of the sample by replicating all observations once. Does anything related to the estimate $\hat{\beta}_{drugs}$ or its properties change? (5 points)

$$\text{Old Var} = \frac{\frac{1}{n-2}\sum_{i=1}^{n}\hat{u}_i^2}{\sum_{i=1}^{n}(X_i-\hat{\mu}_X)^2} \qquad \text{New Var} = \frac{\frac{2}{2n-2}\sum_{i=1}^{n}\hat{u}_i^2}{2\sum_{i=1}^{n}(X_i-\hat{\mu}_X)^2}$$

No work needed

New Var < Old Var if

$$\frac{\frac{2}{2n-2}\sum_{i=1}^{n}\hat{u}_i^2}{2\sum_{i=1}^{n}(x_i-\hat{\mu}_x)^2} < \frac{\frac{1}{n-2}\sum_{i=1}^{n}\hat{u}_i^2}{\sum_{i=1}^{n}(x_i-\hat{\mu}_x)^2}$$

$$\frac{\frac{2}{2n-2}}{2} < \frac{1}{n-2} \Rightarrow \frac{2}{2n-2} < \frac{2}{n-2} \checkmark$$

Variance of $\hat{\beta}_{drugs}$ goes down +5
Precision goes up.

2. (25 Points) Using a sample of workers in California, I wish to estimate

$$\log(Wealth) = \beta_0 + \beta_{educ} educ + \beta_{exper} exper + u$$

Educ and *Exper* are measured in years and *Wealth* in thousands of dollars.

a.) Suppose you estimate that $\hat{\beta}_{educ} = 0.2$. Please interpret this estimate. (5 Points)

Holding experience constant, a one year increase in educ yields a 20% increase in the wage.

+1 +4

b.) Suppose that we have two individuals with the same predicted values of *Wealth*. The first individual has 12 years of education and 10 years of experience. The second individual has 15 years of education and 7 years of experience. Again assuming that $\hat{\beta}_{educ} = 0.2$, what is the estimate of $\hat{\beta}_{exper}$? (5 Points)

$$(0.2)(12) + \hat{\beta}_{exper}(10) = (0.2)(15) + \hat{\beta}_{exper}(7) \quad +5$$

$$\hat{\beta}_{exper}(10 - 7) = 0.2(15 - 12)$$

$$\hat{\beta}_{exp.} = 0.2\left(\frac{15-12}{10-7}\right) = 0.2 \quad +2$$

c.) Suppose that the true parameter β_{educ} actually equals 0.15. Given that $\hat{\beta}_{educ} = 0.2$, is this sufficient to conclude that there is a bias in the estimate $\hat{\beta}_{educ}$? If so, what kind of bias is this? If not, why? (5 Points)

+2

No. It is natural to get estimates in a random sample that are different from the population. In fact, you will never get the exact population parameter

+3

d.) Suppose that instead of the above equation, I estimate the following:

$$Wealth = \beta_0 + \beta_{educ}\log(educ) + \beta_{exper}\log(exper) + u$$

Please derive and interpret the coefficient on *log(educ)* (10 Points)

$$\partial Wealth = \beta_{educ} \frac{\partial educ}{educ}$$

$$\partial Wealth = \frac{\beta_{educ}}{100} \cdot \left(\frac{\partial educ}{educ} \cdot 100 \right)$$

+5

A 1% change in educ yields a $\beta_{educ}/100$ change in Wealth

+5