

Name KEY

ID KEY

**Midterm 3 –90 Points**

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. No cell phones. Do not use your own scratch paper.

**You must show your work to receive full credit**

*I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.*

Signature KEY

Suppose that you wish to predict housing prices as a function of lot size and floor space via the following specification:

$$\log(\text{price}) = \beta_0 + \beta_1 \log(\text{lotsize}) + \beta_2 \log(\text{sqrft}) + u$$

Here, *price* is measured in dollars, *lotsize* is measured in square feet, and *sqrft* (floorspace) is also measured in square feet. The results from estimating this equation are below:

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	5.26768	0.60188	XXXXXXXXXXXXXXXXXXXX	XXXXXXXXXXXXXXXXXXXX
log(lotsize)	0.16846	0.03846	XXXXXXXXXXXXXXXXXXXX	XXXXXXXXXXXXXXXXXXXX
log(sqrft)	0.76237	0.08089	XXXXXXXXXXXXXXXXXXXX	XXXXXXXXXXXXXXXXXXXX

Multiple R-squared: 0.6353, Adjusted R-squared: 0.6267  
 F-statistic: 74.04 on 2 and 185 DF, SSR: 2.924

- a.) Using the 96% confidence level, test whether the coefficient on *log(lotsize)* is significantly different from zero. Please state your null and alternative hypotheses, and briefly interpret the result. (10 Points)

$$\begin{aligned}
 H_0: \beta_1 = 0 \\
 H_A: \beta_1 \neq 0
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} +2$$

$$t_{\text{stat}} = \frac{0.168 - 0}{0.0384} = 4.380$$

$$t_{\text{crit}} = 2.055 \quad (|t_{\text{stat}}| > t_{\text{crit}})$$

Reject the null!

*log(lotsize)* has a positive and statistically significant effect on the log wage.

b.) Please construct and interpret a 98% confidence interval for the coefficient on  $\log(\text{sqft})$ . Show your work!

(10 Points)

$$t_{\text{crit}} = 2.32 \quad 0.762 - 0.081 \cdot 2.32 < B_2 < 0.762 + 0.081 \cdot 2.32$$

$$\frac{\quad}{+2} \quad \quad \quad 0.574 < B_2 < 0.950$$

With 98% confidence, a 1% increase in  $\text{sqft}$  ~~was~~ yields <sup>+4</sup>  
 between a 0.574% and 0.95% increase in the price  
 of the home +9

c.) Suppose I claim that the effects of  $\log(\text{lotsize})$  and  $\log(\text{sqft})$  are identical. Please state a null and alternative hypothesis, and derive an equation that allows me to test the null against the alternative. Show your work!! (10 Points)

$$\theta = B_1 - B_2 \Rightarrow H_0: \theta = 0 \quad \left. \begin{array}{l} \\ H_A: \theta \neq 0 \end{array} \right\} + 4$$

$\Downarrow$

$$B_1 = \theta + B_2$$

$$\log(\text{price}) = B_0 + B_1 \log(\text{lotsize}) + B_2 \log(\text{sqft}) \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} + 6$$

$$= B_0 + (\theta + B_2) \log(\text{lotsize}) + B_2 \log(\text{sqft})$$

$$\Rightarrow \log(\text{price}) = B_0 + \theta \log(\text{lotsize}) + B_2 (\log(\text{lotsize}) + \log(\text{sqft})) + u$$

- d.) Suppose that we add the number of bedrooms, *bdrms*, and a variable, *colonial*, that takes on a value of 1 when the house is a colonial and 0 otherwise.

$$\log(\text{price}) = \beta_0 + \beta_1 \log(\text{lotsize}) + \beta_2 \log(\text{sqrft}) + \beta_3 \text{bdrms} + \beta_4 \text{colonial} + u$$

The results are below:

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	5.55817	0.65104	XXXXXXXXXXXXXXXXXXXXXXX	
log(lotsize)	0.16782	0.03818	XXXXXXXXXXXXXXXXXXXXXXX	
log(sqrft)	0.70719	0.09280	XXXXXXXXXXXXXXXXXXXXXXX	
bdrms	0.02683	0.02872	XXXXXXXXXXXXXXXXXXXXXXX	
colonial	0.05380	0.04477	XXXXXXXXXXXXXXXXXXXXXXX	

Multiple R-squared: 0.6491, Adjusted R-squared: 0.6322  
F-statistic: 38.38 on 4 and 183 DF, SSR=2.814

Suppose I claim that *bdrms* has a significant effect on  $\log(\text{price})$ . What is the probability that I'm wrong? Please state the null and alternative hypotheses, and show your work! (10 Points)

$H_0: \beta_3 = 0$   
 $H_A: \beta_3 \neq 0$

$t_{stat} = \frac{0.0268 - 0}{0.0287}$   
 $+2$

$P_{value} = 2(1 - P(T < |t_{stat}|))$   
 $= 2(1 - 0.824)$   
 $= \boxed{0.352}$

$+2$   
 $+6$

- e.) Is the model in 'd' preferred to the model in 'a'? If a hypothesis test is warranted, test this hypothesis at the 95% level, stating your null and alternative hypotheses. If not, provide other evidence for your answer. (10 Points)

Nested Model,  $q = 2$ ,  $F_{crit} = 3$

$H_0: \beta_3 = 0, \beta_4 = 0$   
 $H_A: H_0 \text{ not true}$

$F_{stat} = \frac{\frac{SSR_R - SSR_{UR}}{q}}{\frac{SSR_{UR}}{df_{UR}}} = \frac{\frac{2.924 - 2.814}{2}}{\frac{2.814}{183}} = \boxed{3.577}$

$F_{stat} > F_{crit} \Rightarrow \text{Reject } H_0!$   
 $+2$

f.) Dr. Spearot is in the market for a house, and would like one that is 1000 square feet in floorspace, 5000 square feet in lot size, 3 bedrooms, and a colonial (*colonial*=1). Please derive an equation that would allow me to estimate a predicted value for such a house with a standard error. Show your work!! (10 Points).

$$\Theta = \beta_0 + \beta_1 \log(5000) + \beta_2 \log(1000) + \beta_3 \cdot 3 + \beta_4 \cdot 1$$

$$\Rightarrow \beta_0 = \Theta - \beta_1 \log(5000) - \beta_2 \log(1000) - \beta_3 \cdot 3 - \beta_4 \cdot 1$$

$$\log(\text{price}) = \beta_0 + \beta_1 \log(\text{lot size}) + \beta_2 \log(\text{sqrft}) + \beta_3 \text{bedrms} + \beta_4 \text{colonial} + u$$

$$\log(\text{price}) = \Theta - \beta_1 \log(5000) - \beta_2 \log(1000) - \beta_3 \cdot 3 - \beta_4 \cdot 1 + \beta_1 \log(\text{lot size}) + \beta_2 \log(\text{sqrft}) + \beta_3 \text{bedrms} + \beta_4 \text{colonial} + u$$

$$\log(\text{price}) = \Theta + \beta_1 (\log(\text{lot size}) - \log(5000)) + \beta_2 (\log(\text{sqrft}) - \log(1000)) + \beta_3 (\text{bedrms} - 3) + \beta_4 (\text{colonial} - 1) + u$$

+6

g.) Do the variables in 'd' tell use anything about the log price? If a hypothesis test is warranted, test this hypothesis at the 95% level, stating your null and alternative hypotheses. If not, provide other evidence for your answer.

(10 Points)

~~XXXXXX~~  $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$  (+3)

$H_1: H_0 \text{ not true}$

$F_{stat} = 38.38$  (+3)       $F_{crit} = 2.37$  (+2)

$F_{stat} > F_{crit} \Rightarrow \text{Reject the null.}$  (+2)

h.) Suppose that we use a different specification, regressing the log housing price on the log of the assessed housing value, assess.

$$\log(\text{price}) = \beta_0 + \beta_1 \log(\text{assess}) + u$$

The results are below:

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.25409	0.76353	XXXXXXXXXXXXXXXXXXXX	XXXXXXXXXXXXXXXXXXXX
log(assess)	1.01341	0.06046	XXXXXXXXXXXXXXXXXXXX	XXXXXXXXXXXXXXXXXXXX

Multiple R-squared: 0.7656, Adjusted R-squared: 0.7629  
F-statistic: 280.9 on 1 and 186 DF, SSR: 1.88

Is the model in 'h' preferred to the model in 'd'? If a hypothesis test is warranted, test this hypothesis at the 95% level, stating your null and alternative hypotheses. If not, provide other evidence for your answer. (10 Points)

Non-nested  $\Rightarrow$  Adj.  $R^2$

Model h: 0.7629 +11

Model d: 0.6322 +9

$\Rightarrow$  Model h is preferred

+2

+3 if correct F-test applied incorrectly

- i.) Occupy Santa Cruz is angry about many things, but they are especially angry about the housing market. Whether they know it or not, they claim that housing assessments are unfair, in that the ratio  $\frac{\text{price}}{\text{assess}}$  depends on the value of *assess*. Using the results from 'g', please test this hypothesis as the 95% level, stating your null and alternative hypotheses, and briefly interpret the result. (10 Points)

$$H_0: \beta_1 = 1$$
$$H_1: \beta_1 \neq 1$$

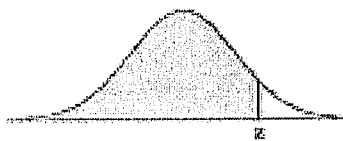
$$t_{\text{stat}} = \frac{1.0134 - 1}{0.06046} = -0.2216$$

$$t_{\text{crit}} = 1.96$$

$$|t_{\text{stat}}| < t_{\text{crit}}$$

Fail to reject the Null

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## Normal Distribution from $-\infty$ to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

TABLE G.3b

5% Critical Values of the  $F$  Distribution

		Numerator Degrees of Freedom									
		1	2	3	4	5	6	7	8	9	10
D e n o m i n a t o r  D e g r e e s  o f  F r e e d o m	10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
	11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
	12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
	13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
	14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
	15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
	16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
	17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
	18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
	19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
	20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
	21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
	22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
	23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
	24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
	25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
	26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
	27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
	28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	

Example: The 5% critical value for numerator  $df = 4$  and large denominator  $df (\infty)$  is 2.37.

Source: This table was generated using the Stata<sup>®</sup> function invFtail.