

Name ANSWER KEY

ID _____

Midterm 3 – 65 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. No cell phones. Do not use your own scratch paper.

You must show your work to receive full credit

I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.

Signature _____

Problem 1 (50 Points)

Suppose that you wish to predict wage outcomes via the following specification:

$$\log(\text{wage}) = \beta_0 + \beta_1 \text{educ} + \beta_2 \text{exper} + \beta_3 \text{tenure} + \beta_4 \text{Sibs} + \beta_5 \text{Brthord} + \beta_6 \text{feduc} + \beta_7 (\text{meduc} + \text{feduc}) + u$$

wage is measured in dollars per month, educ, exper, tenure, meduc (mother's education) and feduc (father's education) are measured in years. Sibs measures number of siblings, and Brthord measures the order in which the respondent was born (1=first, 2=second, etc). The results from estimating this equation are below:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	5.4707580	0.1413698	xxxxxxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxxxxxx
educ	0.0620606	0.0080466	xxxxxxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxxxxxx
exper	0.0185869	0.0040521	xxxxxxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxxxxxx
tenure	0.0096864	0.0030322	xxxxxxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxxxxxx
sibs	0.0017676	0.0080854	xxxxxxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxxxxxx
brthord	-0.0168955	0.0121860	xxxxxxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxxxxxx
feduc	-0.0006832	0.0105281	xxxxxxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxxxxxx
I(meduc + feduc)	0.0119228	0.0065391	xxxxxxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxxxxxx

Residual standard error: 0.3745 on 655 degrees of freedom.
 Multiple R-squared: 0.1835, Adjusted R-squared: 0.1748
 F-statistic: 21.03 on 7 and 655 DF, SSR = 91.84

a.) Please construct and interpret a 95% confidence interval for the coefficient on educ. (10 Points)

$$\hat{\beta}_{\text{educ}} - \text{se}(\hat{\beta}_{\text{educ}}) \cdot t_{\text{crit}} < \beta_{\text{educ}} < \hat{\beta}_{\text{educ}} + \text{se}(\hat{\beta}_{\text{educ}}) \cdot t_{\text{crit}} + 2$$

$$0.0621 - 0.00804 \cdot 1.96 < \beta_{\text{educ}} < 0.0621 + 0.00804 \cdot 1.96 + 2$$

$$0.0463 < \beta_{\text{educ}} < 0.0779 + 4$$

With 95% confidence, a 1 year increase in education +2 has between a 4.6% and 7.8% effect on wages.

b.) Using the 93% confidence level, test whether the coefficient on *tenure* is significantly different from zero. Please state your null and alternative hypotheses, and briefly interpret the result. (10 Points)

$$+1 \quad H_0: \beta_3 = 0 \quad t_{crit} = 1.81 \quad +2$$

$$+1 \quad H_A: \beta_3 \neq 0 \quad t_{stat} = \frac{0.00969}{0.00303} = 3.198 \quad +2$$

$|t_{stat}| > t_{crit}$ Reject H_0 in favor of H_A +2

+2 Tenure has a positive and statistically significant effect on the wage.

c.) Suppose that I claim that mother's education has a significant effect on wages. What is the probability that I'm wrong? Please state the null and alternative hypotheses, and show your work! (10 Points)

$$+2 \quad t_{stat} = \frac{0.0119 - 0}{0.0065} = 1.831 \quad H_0: \beta_7 = 0 \quad +2$$

$$H_A: \beta_7 \neq 0 \quad +2$$

$$P_{value} = Pr(|T| > t_{stat}) = 2 \cdot Pr(T > t_{stat}) = 2 \cdot (1 - Pr(T < t_{stat}))$$

$$= 2(1 - 0.9664) = 2 \cdot 0.0336$$

$$= \sqrt{0.0672} \quad +4$$

d.) Using the 99% confidence level, please test the hypothesis that the effect of *feduc* is significantly different than the effect of *meduc*. Please state your null and alternative hypotheses, and show your work! (10 Points)

$$+2 \quad H_0: \beta_6 = 0 \quad t_{stat} = \frac{-0.000683 - 0}{0.01053} = -0.0649 \quad +2$$

$$+2 \quad H_A: \beta_6 \neq 0 \quad t_{crit} = 2.575 \quad +2$$

$|t_{stat}| < t_{crit} \Rightarrow$ Fail to reject the null, +2

e.) The variables *Sibs* and *Brthord* take on only integer values, and thus taking derivatives is a bit coarse. So, you decide to leave them out and see what happens to other estimates. The results are below:

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	5.403359	0.129296	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx
educ	0.062781	0.007983	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx
exper	0.018796	0.004041	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx
tenure	0.009691	0.003032	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx
feduc	-0.001786	0.010475	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx
I(meduc + feduc)	0.013567	0.006404	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx

Residual standard error: 0.3745 on 657 degrees of freedom
 Multiple R-squared: 0.1806, Adjusted R-squared: 0.1744
 F-statistic: 28.96 on 5 and 657 DF, SSR = 92.17

Do the variables *Sibs* and *Brthord* have a joint effect on wages? Please state your null and alternative hypotheses, and test the null against the alternative at the 95% level. (10 Points)

+1 $H_0: \beta_4 = \beta_5 = 0$

+1 $H_A: H_0 \text{ not true}$

$SSR_{UR} = 91.84$

$SSR_R = 92.87$

$q = 2$

$df_{UR} = 655$

$$F_{stat} = \frac{\frac{SSR_R - SSR_{UR}}{q}}{\frac{SSR_{UR}}{df_{UR}}} = \frac{\frac{92.87 - 91.84}{2}}{\frac{91.84}{655}} = 1.176 \Big| +5$$

$F_{crit} = 3$

$F_{stat} \neq F_{crit}$

Fail to reject $H_0!$

+1

+2

Problem 2 (15 Points)

The *Residential Energy Consumption Survey* is put out by the US Department of Energy every few years. It surveys a representative sample of residents, their energy use, their appliance ownership, and their characteristics (income, family size, etc...). Using the 2005 version of the survey, and restricting the sample to California residents, I estimate the following equation:

$$\log(KWH) = \beta_0 + \beta_1 \log(Income) + \beta_2 Size + u$$

Here, *KWH* is kilowatt-hours of energy consumption, *Income* is household income, and *Size* is the number of people in the household. The results from estimating the above equation are as follows:

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	5.52078	0.38881	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx
log(Income)	0.26806	0.03710	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx
Size	0.09560	0.01719	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx

 Residual standard error: 0.5533 on 405 degrees of freedom
 Multiple R-squared: 0.1929, Adjusted R-squared: 0.1889
 F-statistic: 48.4 on 2 and 405 DF; SSR = 123.97

a.) Do the variables of the model explain $\log(KWH)$? Please state your null and alternative hypotheses, and test the null against the alternative at the 95% level (5 Points).

$$H_0: \beta_1 = \beta_2 = 0$$

$$H_a: H_0 \text{ not true}$$

$$+ \frac{1}{2} F_{stat} = 48.4$$

$$F_{stat} > F_{crit} \quad + \frac{3}{2}$$

$$+ \frac{1}{2} F_{crit} = 3$$

Reject H_0 in favor of H_a

The variables of the model are a significant determinant of the ~~the~~ $\log(KWH)$ $+ \frac{3}{2}$

b.) I wish to predict energy consumption for a family that earns \$100,000 a year and has 5 people in the household. Please derive a new equation that allows me to generate this prediction and a standard error for the prediction. Show your work!!! (10 Points)

$$\Theta = \beta_0 + \beta_1 \log(100,000) + \beta_2 \cdot 5 + \varepsilon$$

$$\Rightarrow \beta_0 = \Theta - \beta_1 \log(100,000) - \beta_2 \cdot 5 + \varepsilon$$

$$\log(\text{kWh}) = \beta_0 + \beta_1 \log(\text{Income}) + \beta_2 \text{Size} + \varepsilon$$

$$\log(\text{kWh}) = \Theta - \beta_1 \log(100,000) - \beta_2 \cdot 5 + \beta_1 \log(\text{Income}) + \beta_2 \text{Size} + \varepsilon$$

+6

$$\log(\text{kWh}) = \Theta + \beta_1 \left(\log\left(\frac{\text{Income}}{100,000}\right) \right) + \beta_2 (\text{SIZE} - 5) + \varepsilon$$

\uparrow
 $\log\left(\frac{\text{Income}}{100,000}\right) \text{ ok}$